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FUNDAMENTALS OF ELECTRICAL DESIGN

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OF
ELECTRICAL DESIGN

BY

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PREFACE

This book, and the "design" course in which it may be used, will not teach anyone how to design electrical machines and apparatus. Undergraduates in electrical engineering must not expect to learn how to design, in college. Real knowledge of and skill at designing involve factors usually to be supplied only by actual experience in professional work done after graduation. This book is offered as a text in which are presented certain fundamental subjects which apply in two ways: they are basic to much electrical design as carried out professionally, and the mastering of these subjects may be considered a necessary part of the training of all electrical engineering students. Thus the earlier chapters will be found to deal with the electromagnetic system, fields and field mapping, attraction, and the thermal problems of heat storage, heat flow, heat radiation, heat dissipation, and temperature rise.

A casual inspection will show that the later chapters are devoted exclusively to the direct-current machine. There is no intention of ascribing to direct-current machinery any undue importance. But it happens that this type is the most familiar to the usual student; and furthermore, it offers an excellent array of problems in which the studies and analyses of the earlier chapters find a ready application. In large part, the knowledge of subject-matter and the training in method, as acquired by applying the fundamental theories and studies of the earlier chapters to the problems of the direct-current machine, will be found to be applicable to machines and apparatus of other types and kinds.

Several years ago, the author began to swing the content and treatment of his course in electrical machine design toward the scheme which is represented by the contents of this book. It was soon found that no textbook even approximated the point of view and treatment desired in the modified course. To meet this need, the author consequently developed first, a set of mimeographed notes and then a complete manuscript which, in its final form, was used a full year before publication.

The author wishes to express his appreciation for the kindly cooperation of the publishers in the preparation of the book; and for the interest of his teaching associates whose encouragement has meant much, especially at those times when the work on the manuscript has seemed more laborious than interesting. And very sincerely, the author recognizes his debt to his students; the inspiring contacts made with them in class and computing room furnishes a primary motivation not often recognized, and for which due credit should be given.

A. D. M.

The University of Michigan,
Ann Arbor, Michigan,
June, 1927.

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LIST OF SYMBOLS

(This list is not complete; some symbols which serve in only one part of the book, and for an immediate purpose, are omitted. When such a symbol is omitted here, it will be found that its meaning is fully given in the place where it appears.)

A = area in square inches
A = effective pole-face area or gap-section area
A = rate of heat production, units per second
A_1 = dissipating area, outer armature surface
A_2 = dissipating area, duct and end surfaces
A_3 = dissipating area, inside core surface
B = rate of heat dissipation, units per second per degree rise
B = flux density
B_g = flux density, in gap, averaged over a slot pitch
B_{ga} = flux density, in gap, averaged over pole arc
B_{gm} = flux density, in gap, maximum value of B_g
B_r = flux density, in tooth root
B_{rm} = flux density, maximum value of B_r
B_s = flux density, slot bottom, due to main gap field
B_s = flux density, across slot top, due to commutated slot current
C = heat storage capacity, units per degree rise
C = Carter's coefficient for air gaps
C = constant of integration
C = mean turn length of coil, inches
C_o = output coefficient for armatures
D = armature diameter, inches
D = bare wire diameter
D' = diameter of insulated wire
D_1 = diameter at tooth roots, armature
D_2 = diameter inside core, armature
d = slot depth; also tooth depth
E = volts or abvolts
E_g = volts generated or induced
e = volts or abvolts
e = natural base of logarithms
e = emissivity coefficient, heat radiation
F = force
f = space factor of coil
f = frequency, cycles per second
f_e = coefficient for eddy current loss, slot conductor loss
g = gap length, inches

g_a = gap length averaged over pole arc
g_c = constant gap length, located between tapered ends of main gap
g_t = gap length at pole tip
g_1 = least gap length chosen with which to produce gap and tooth saturation curve
g_2 = mean gap length chosen with which to produce gap and tooth saturation curve
g_3 = greatest gap length chosen with which to produce gap and tooth saturation curve
H = magnetic intensity
H = height of a coil, inches
I = current, amperes or abamperes
I_a = total armature current: (paths) $\times$ (current per path)
I_c = armature conductor current
I_{cs} = current in commutated slot
I_f = shunt field current
I_s = current in active slot
i = instantaneous current, amperes or abamperes
J = combined thermal conductivity of coils
k = tooth ratio, t/t_r
k_{cs} = I_{cs}/I_s
k_d = dissipation coefficient, watts per square inch per degrees Centigrade rise
k_f = thermal conductance
k_f = duct fringing coefficient
k_{fd} = see Art. 227
k_i = coefficient, yoke saturation effect: $M_{ig}k_i = M_e$
k_1 = ratio, tooth root to gap section area
k_2 = see Art. 239
L = length, inches
L = inductance, henries
L_a = core length of armature, including ducts
L_g = gross iron length of armature, or (L_a) minus (total of ducts)
L_i = total axial interpole length over two sides of a commutated turn
L_p = axial pole core length
L_y = yoke path length, for NI drop in yoke
l = number of layers in coil
M = strength of a magnetic pole
M = conductor section, circular mils
M = m.m.f. in ampere-turns
M_1 = circular mils section of larger of two adjacent wire sizes
M_2 = circular mils section of smaller of two adjacent wire sizes
M_e = excess NI of interpole winding, above amount needed to equalize M_r
M_g = gap NI drop
M_{gt} = gap and tooth NI drop, furnished by main field
M'_{gt} = part of M_{gt} furnished by shunt field winding
M_{ig} = interpole gap NI drop
M_m = main pole winding m.m.f. in ampere-turns

M_p = pole core NI drop
M_r = armature reaction m.m.f. at commutating zone
M_t = tooth NI drop
M_t = armature reaction m.m.f. at pole tip
M_y = yoke NI drop
m.m.f. = magnetomotive force, gilberts or ampere-turns
N = slot pitches per pole
N = number of turns in a coil
n = number of ducts
NI = abampere-turns
NI = ampere-turns
P = inside coil perimeter
P = pounds attraction
P = fraction of total heat emission
P = number of poles
P_p = pole pitch, inches
P' = outside coil perimeter
q = ampere-conductors per inch of periphery
R = radius, inches
R = reluctance, oersteds
R = resistance, ohms or abohms
R_a = armature resistance
S = surface area
S = number of slots
s = slot width
T = time, seconds
T = temperature, absolute (Kelvin)
T = thickness of field coil, inches
T = pole arc, inches
T_1 = thickness of inner section of two-wire coil
T_2 = thickness of outer section of two-wire coil
t = time, seconds
t = turns per layer
t = tooth width, top, inches
t_a = tooth width, average
t_r = tooth width, root
V = peripheral velocity, feet per minute
W = iron loss, watts per pound per cycle per second
W = unit tooth loss, per cycle per second
W_d = watts dissipated
W_i = interpole core width, at thickest part
W_l = watts loss
W_p = pole core width, peripherally
w = duct width
X = per cent of pole arc covered by constant gap
X = length of straight end extension of armature coil
Y = axial length of bent part of end connections, excluding curl
Z = total active armature conductors

GREEK LETTER SYMBOLS

Θ = angle of end connections
Θ = temperature or temperature rise, degrees Centigrade
Θ_H = highest temperature rise attained in heating-cooling cycle
Θ_L = lowest temperature rise attained in heating-cooling cycle
Θ_0 = ultimate temperature rise of heated body
Θ_p = preliminary estimate, armature temp. rise
θ_1 = rise of air, average, which passes armature surface
θ_2 = rise of air, average, which passes through ducts
θ_3 = rise of air, average, which passes inside core
Φ = flux
Φ = flux per pole
Φ_l = leakage flux

(Below are given the unit or basic values of commutation fluxes; a cap. Φ with the same subscript as one of these, denotes the *total* of that particular quantity.)

ϕ_s = slot flux (effective) per axial inch
ϕ_{s1} = flux over commutated slot, under interpole, per NI per axial inch
ϕ_{s2} = flux over commutated slot, not under interpole, per NI per axial inch
ϕ_a = armature reaction flux not under interpole
ϕ_{cr} = armature reaction flux of end-connections
ϕ_{cc} = commutated coil side flux at end-connections
ϕ_e = sum of above two fluxes
ϕ_r = resultant reversing interpole flux
ϕ_i = resultant total (nearly) interpole gap flux (at P–Q level)
ϕ_{il} = interpole leakage flux, basic value
ϕ_{ml} = main pole leakage flux, basic value
μ = permeability

FUNDAMENTALS OF ELECTRICAL DESIGN

CHAPTER I

DEVELOPMENT OF THE ABSOLUTE SYSTEM OF ELECTROMAGNETIC UNITS

By the time the student is ready for the usual course in electrical machine design, he has had a smattering of electricity and magnetism; but his work in this difficult subject has been of necessity so hasty that he can be forgiven for not having a good grasp on the many principles and relations involved. This book therefore begins with a very brief development of the electromagnetic units.[1] There are three systems of units available:

1. The **e. s. u.,** or **electrostatic unit system,** based on the definition of a **unit point charge.**

2. The **e. m. u.,** or **electromagnetic unit system,** also called the **absolute system,** based on the definition of a **unit point pole.**

3. The **practical system.**

Little engineering work is carried out in the electrostatic system. Most of it is done in terms of practical units, much of it in terms of absolute units, and quite a good deal in a mixture of the two. The practical system is based directly on the absolute system, and many of the absolute units are carried over bodily into the practical system.

1. Magnetism.—A magnet exhibits the quality of acting upon other magnets with a mechanical force. If it were not for electrical tests, the only direct way of finding if a piece of iron is magnetized is to see if it attracts or repels another piece of magnetic material.

[1] PENDER, HAROLD, "Electricity and Magnetism," McGraw-Hill Book Company, Inc.

With long, slim magnets most of the source of the force is found by experiment to reside in the ends of the magnet. Suppose a given magnet is used as a temporary standard. By measuring the forces with which it acts on other magnets, one could find the relative strengths of the other magnets. Then different pairs of these tested magnets could be made up and tested against each other. Provided that in all cases the poles or ends are not allowed to come very close together, it is found that the force between two poles is approximately

$$F = \frac{M_1 M_2}{d^2} K.$$

The next step in building a system of units is to create, as a *concept*, an isolated unit point pole. By *definition*, a **unit point pole** is of such strength that, when placed 1 centimeter distant from a like and equal point pole, the one repels the other with a force of 1 dyne, the medium for the magnetic field being free space. In terms of this *definition*, based on a *concept*, the actual poles of the above magnets would interact in pairs according to the law

$$F \text{ dynes} = \frac{M_1 M_2}{d^2}.$$

If another *concept* is created to explain the existence of the force, a useful addition to the theory will be made. One pole is thought of as causing an **intensity field,** or **intensity,** or **magnetizing force** in the space about it; and the other pole is thought of as being acted upon by this magnetizing force.

2. Definition of H.—Magnetizing force is denoted by the symbol H. By the definition of the unit point pole, H has unit value at a distance of 1 centimeter from a unit point pole. In the case of two like unit point poles placed 1 centimeter apart, it may be now considered that one pole was acted upon by unit value of magnetizing force H; hence, *the dynes force on a unit point pole is a measure of H at the point where the pole is placed.*

3. Electromagnetic Induction.—It is found experimentally that when conductors and magnets are moved with respect to each other, **electromotive forces** are induced in the conductors. There is, of course, no explanation ever to be found for any such phenomenon. Another *concept* is used to "explain" it: The magnet poles are thought of as having a **magnetic flux,** and the change in **flux linkage** of the flux linking with the conductor's circuit is taken to be the cause of the induced voltage.

4. Definition of Flux Density, B.—By defining either **unit amount of flux,** or of **flux density, unit electromotive force** can be defined. Consider an experiment made with a circular loop of wire and a long slim magnet. Place the loop near to one of the magnet's poles but far from the other pole. Let the axis of the loop pass through the pole itself. Now let the loop be moved outward from the pole, and let it expand in proportion to its distance from the pole. (For instance, let a wooden cone) with its apex at the pole, be surrounded by one circular turn of a flexible conductor, and let the cone stretch the loop as the loop is pulled outward.) By the process, the *solid angle at the pole subtended by the loop remains constant.* It would be found that, neglecting the effect of the other pole of the magnet, no electromotive force would be induced in the loop.

The flux is thus seen to exhibit the quality of *direction,* and the direction is radial with respect to the pole. *Unit flux density,* or *unit B,* is defined as the flux density found at a point 1 centimeter distant from a unit point pole.

The unit of flux, the **maxwell,** is defined as the amount of flux passing through 1 square centimeter of area when unit B exists normal to that area. The area of a sphere of 1-centimeter radius is 4π, and if there is a unit point pole at its center, the flux density B at the surface is unity. Then the flux through the surface of the sphere is 4π, and the "strength" of the point pole is said to be 4π. The terms, **line of flux** and **maxwell,** are synonymous.

5. Definition of the Abvolt.—Unit electromotive force, or the **abvolt,** is defined as that amount induced *by changing flux linkage at unit rate;* that is, 1 *maxwell per second rate of change of flux through one turn,* induces 1 *abvolt.* As a special case, consider a one-turn loop of circular form, having unit radius. Pass a unit point pole along the axis of the circle with a velocity of 1 centimeter per second. When the pole is passing through the center of the loop, its flux density B at the wire is everywhere unity. Flux is now passing from one side of the loop to the other at the rate of 2π lines or maxwells per second. Thus 2π abvolts are induced at that instant.

By means of the definition of the abvolt, it is possible to measure flux by observing the voltage it is allowed to induce and the time taken; when flux is measured practically, it is almost always found in terms of the voltage induced and the time. A

special device, the ballistic galvanometer (sometimes called a flux meter), combines the elements of voltage and time into one observed swing of the pointer of the instrument.

6. Definition of the Abampere.—Experiments show that magnets in some way react, with mechanical forces, upon wires carrying current. Again, there is no explanation for such a phenomenon. It is possible to *conceive* of the force on the wire as being due either to the magnet's magnetizing force H, or to its flux density B. The *concept* of an action between B and the wire's current I is the one adopted. **Unit current,** the **abampere,** is *defined as that amount flowing in a straight wire of negligible diameter, which will give a force of* 1 *dyne per centimeter length of wire, when the wire is placed normal to the lines of a field having a density of unity.* A favorite way of setting up the equivalent to this condition is to say that if a wire is curved around a 1-centimeter radius, with a unit point pole at the center of curvature, then when each centimeter of length of the wire has 1 dyne of force acting on it, the wire carries 1 abampere.

Returning to the former case of a complete circular loop of a 1-centimeter radius, when the unit point pole arrives at the center, the force on the loop would be 2π dynes, if the current is 1 abampere.

7. Energy.—Continuing with this same case wherein the unit point pole travels at unit rate along the axis of the 1-centimeter loop, it will be profitable to investigate the energies involved. Again, consider conditions at the instant when the pole reaches the center.

First, let the induced electromotive force set up a current in the loop, as it must if the loop is closed. The induced voltage is 2π, as found above. This voltage will not, except by rare chance, set up 1 abampere in the loop. The size of wire used for the loop can be selected, however, and by using the right wire, unit current can be made to result from the 2π abvolts. Then, at the instant considered, the pole's density of flux at the wire would act upon the current of the loop with a total force of 2π dynes. Action and reaction being equal and opposite, there must at this time be a force of 2π dynes acting on the unit pole. If the motion is such that the pole is moving *against* the action of the loop, then at unit velocity there is an energy transfer of 2π ergs per second in moving the pole. This energy all reappears at the loop in the form of heat. The product of abvolts times abamperes in the loop is also

2π. Furthermore, the induced electromotive force on the one hand is proportional to the speed of the point pole, and the current on the other hand is proportional to the force. Thus there is a complete argument for the conclusion that voltage times current represents power, and that in 1 abwatt there is a rate of expenditure of energy of 1 erg per second. An **abwatt** is 1 *abvolt times* 1 *abampere.*

8. Definition of the Abohm.—Resistance as a quality should first be defined. **Resistance** is that *quality* (the nature of which is still unknown) which limits the current flow due to a given electromotive force acting in a conducting circuit. Quantitatively, **resistance** is the *ratio of voltage to current.* **Unit resistance** in the absolute system is the **abohm,** the *ratio of* 1 *abvolt to* 1 *abampere.* In the closed loop of selected wire wherein 2π abvolts resulted in 1 abampere flowing, the resistance was 2π abohms. Either by experiment or by use of relations already stated, it is seen that the power loss in a conducting path varies as the square of the current times the resistance, and in the absolute system,

$$\text{Abwatts} = I^2R.$$

9. The Magnetic Circuit.—Let any source of magnetic flux be studied. Let a single loop of wire enclose a small part of the total flux. Now, by the use of a tiny compass or, in theory, by the use of a unit point pole, let the lines of flux that now pass through the *metal wire* of the loop be outlined for a distance (not the flux lines that pass through the area of the loop, but those passing the rim of the loop). The continued lines of flux would form a *sheath* or **tube of flux,** which passes through the loop. Immediately around the tube of flux but at some other place, let a second loop be placed. By definition, if the electromotive force induced in one of the loops is measured while the flux is allowed to decay uniformly in 1 second, the abvolts would be an exact measure of the maxwells in the tube. Note distinctly that nothing so far said in the above developments can make it seem evident that the *two loops* will now have induced in them the *same number of abvolts. Experiment alone* finds this result, *for the electromotive forces are found to be equal.* Thus it is found that the flux through the cross-section of a tube is a constant, no matter where the section is investigated. It follows that the total flux at one place in a magnetic circuit is the same as the total flux anywhere else.

10. Magnetic Potential Difference.—Again taking up the closed circular loop with a current of 1 abampere, and with a unit point pole passing through its center on an axial line, the force on the pole was found to be 2π dynes. Remember that this force was not found as an original quantity; it was the force on the loop that was first found to be 2π dynes. Now, force action on a pole has already been ascribed to the action of the field intensity H at the pole, upon the strength of the pole itself. Then the loop current must be causing an intensity or magnetizing force $H = 2\pi$ at its center. When the pole is moved against the force acting upon it, work is done; and in accordance with similar concepts in other similar theoretical developments, it is said that *different positions of the point pole represent different values of* **potential,** since the *work done,* hence the *potential energy,* varies with the *position* taken by the moving pole. In the direction of the lines of flux, the work done for a differential movement dl is

$$dW = Hdl.$$

If the work done is allowed to stand for the *concept* of **magnetic potential difference** along the field, then in moving from point 1 to point 2,

$$\text{Potential difference} = \int_1^2 Hdl$$

provided the motion is always in line with the direction of H; or if the angle between the motion and the direction of H is α,

$$\text{Potential difference} = \int_1^2 H \cos \alpha dll.$$

11. Magnetomotive Force.—The possibilities of the loop-and-unit-pole experiment are not yet exhausted. At points outside of the center of the loop, the force on the unit pole, and therefore the value of H due to the abampere of current, is different from 2π. Suppose the pole is carried completely through the loop and back to its starting point. Let the time in transit be 1 *second.* Then, irrespective of the shape of the path of the pole, all of its 4π lines must have changed linkage with the loop just once. Although the *instantaneous* voltage would vary considerably, yet the average value would be 4π abvolts. If in some way the current could have been maintained at 1 abampere (such as by varying the resistance of the loop in just the right way), then the average power would be 4π abwatts; so the average rate of energy expenditure in moving the pole is 4π ergs per second. But if the motion

occurred in 1 second, 4π ergs of work was done on the pole in moving it. Therefore,

$$4\pi \text{ ergs} = \int H dl$$

for the whole path if the motion of the pole was always along a line of flux. This, in terms of potential difference, means that the *total potential difference* around the magnetic path is 4π units. It can easily be shown that the pole need not follow a line of flux to achieve the same result. If the loop carries I abamperes, the result would have to be $4\pi I$; and if the loop is changed to a coil of N turns, the total ergs, or the total potential difference, is $4\pi NI$.

Thus, what is called the **total magnetomotive force** is found, for a coil, to be $4\pi NI$. The unit of magnetomotive force, or difference of magnetic potential, is called the **gilbert.** It is chosen so that a coil of NI abampere-turns has $4\pi NI$ gilberts of m.m.f. M.m.f. is the abbreviation for magnetomotive force.

M.m.f., or difference of potential, is the integral of $H \cos \alpha dl$.

12. Reluctance.—It is convenient to develop the idea of *reluctance* in magnetic circuits, as an aid to the making of practical calculations. In electric circuits, the e.m.f. drop over the resistance is thought of as sending the current through that resistance. A similar concept is now developed for magnetic circuits. A current establishes a magnetizing force H and a flux density B. Why not assume that the current is directly responsible for the appearance of H, and that H is then the cause of B? This may not be the truth of it, but as a concept, it works well. From the viewpoint of the *point function in space*, H at a point, taken in connection with the nature of the medium thereat, produces B at the point. From the *length* or *volume aspect*, difference of potential, or m.m.f., acting over some certain volume of space, is conceived of as establishing the flux through that volume. The reluctance of the volume is its quality which limits the flux which a given amount of m.m.f. may establish.

The unit of reluctance is the **oersted;** by definition, 1 *gilbert of m.m.f., acting over a reluctance of* 1 *oersted establishes* 1 *maxwell, or line, of flux.* A centimeter cube of free space, if the m.m.f. acts between opposite faces, has a reluctance of 1 oersted.

It should be kept in mind that these ideas are devised for the purpose of enabling those already accustomed to calculating problems in electric and other flow cases, to continue their trends of thought into magnetic work. It may be assumed that

current sets up m.m.f., that m.m.f. in turn sets up flux, and that the space has reluctance; but assuming does not make it so in the physical world. When one uses the m.m.f. flux-reluctance concept, a method of calculation rather than a true bit of natural philosophy is being developed. The calculation method thus built up as a sort of superstructure on the fundamental theory is a very handy thing to use. If the physical facts as found by experiment were taken, and if the barest bones of a theory were postulated to bind those facts together, the use of the skeleton theory would make practical engineering solutions take on great complexity. It would, for instance, be necessary to find the flux of a direct-current generator by handling triple integrals expressing the value of H at any point as due to the field coils; compounded with these troubles would be still greater difficulties in writing equations to allow for the presence of iron, whereas, the introduction of the concept of reluctance, while perhaps not ideal from the standpoint of theory, nevertheless vastly simplifies the work in such a problem.

13. The Unit Cube.—It was stated above that a unit cube of free space has unit reluctance. To see why this is so, let unit magnetizing force H act at all points within the cube, and normally to a face of the cube. Then the m.m.f. across the cube from face to face is easily seen (integral of Hdl) to be 1 gilbert. From the definitions of H and B as based upon the field distant 1 centimeter from the unit pole, H and B must have the same sense, direction, and magnitude in free space. Then B is everywhere of unit value in the cube. As the section area of the cube is unity, the flux Φ is unity. Then 1 gilbert, the m.m.f., divided by 1 maxwell, the flux, gives unity as the value of the reluctance R, in oersteds.

14. Equipotential Surfaces.—Let a magnetic pole be moved about in a magnetic field in free space. If its motion has a component with or against the direction of the magnetizing force H, then some energy is being transferred. But if its line of motion is always across the field normal to the flux or intensity lines, no work is being done, and no change of potential is demonstrated. Such line of motion is an **equipotential line**; and this line is only one such line in a similarly demonstrated **equipotential surface.**

15. Equal-reluctance Volumes.—Consider a tube of flux across which an equipotential surface A is established. Farther

along the tube let similar surfaces B and C be fixed, each at such positions that the m.m.f. from A to B is the same as that from B to C. The flux of the tube is constant. The m.m.f. over the two volumes just now delineated is the same; hence the two volumes have equal reluctances.

16. Reluctance Drop.—Corresponding to IR, the resistance voltage drop in the electric circuit, in the magnetic circuit there is ΦR, the reluctance drop. Flux times reluctance gives m.m.f. in gilberts.

CHAPTER II

ENERGY; PERMEABILITY; ATTRACTION

Some of the simpler energy relations are developed in this chapter. The idea of permeability is brought out, and the attraction of a magnetic field is developed in terms of energy.

17. Energy Stored in the Field.—A coil may be wound with a few turns of such fine wire that, if the turns are bundled together, practically all of the flux links with every turn. In free space, flux is proportional to the current of the coil. Upon first applying an e.m.f. to the coil, there is a rise of current; hence a rise of flux through the coil. The increase in flux linkage induces an e.m.f. opposite in sense to the current and applied e.m.f., when the current rises. The electrical energy rate of the input to the coil, aside from the resistance loss, is the induced e.m.f. times the current. Let the final current in the N turns be I abamperes, established in the time T seconds, and setting up a final flux of Φ maxwells. The induced e.m.f. at any instant is

$$-e = \frac{Nd\phi}{dt},$$

by which

$$-edt = Nd\phi.$$

Multiplying through by the instantaneous current i,

$$-iedt = Nid\phi.$$

The magnetomotive force of the coil is $4\pi Ni$ gilberts, setting up the flux ϕ at the time t when the current i flows. Let the total reluctance of the magnetic circuit be R oersteds. Then

$$4\pi Ni = R\phi,$$

or

$$\phi = \frac{4\pi Ni}{R}.$$

By substituting this value of ϕ in the $d\phi$ above,

$$-iedt = \frac{4\pi N^2}{R}idi.$$

The integral of the left-hand term over the entire time T gives abwatt-seconds, which is the electrical energy input aside from i^2R loss. The right-hand term is correspondingly integrated from zero to I abamperes, and the result is

$$\text{Abwatt} - \text{seconds} = \text{ergs} = \frac{4\pi N^2 I^2}{2R}.$$

Using the $4\pi NI = R\Phi$ equation to evaluate I, and substituting,

$$\text{Ergs} = \frac{4\pi N^2}{2R} \frac{\Phi^2 R^2}{16\pi^2 N^2} = \frac{\Phi^2 R}{8\pi}.$$

Where this energy really is stored is not known. It is usually assumed that it is stored in the magnetic field.

18. Energy per Unit Volume.—Consider a circular field, such as is set up by a straight conductor infinitely long or by a crooked conductor near to itself where a portion of it is straight. A circular tube of flux of very small cross-section as compared with its radius would have within it a very nearly constant density. Calling the length and area of the tube L and A respectively, its reluctance is $R = L/A$. If its flux is Φ, then its stored energy is

$$\text{Ergs} = \frac{\Phi^2 R}{8\pi}.$$

But $\Phi = BA$. Hence

$$\text{Ergs} = \frac{B^2 A^2 L}{8\pi A} = \frac{B^2 AL}{8\pi} = \text{volume times } \frac{B^2}{8\pi}$$

for *free space*. The energy stored per cubic centimeter is then $B^2/8\pi$.

19. The Iron Ring.—Let a symmetrical ring, having a section dimension very small compared to its radius, be so placed around a straight conductor that the conductor runs axially through the ring. The magnetomotive force due to I abamperes in the wire is $4\pi I$. The magnetizing force anywhere within the ring would by symmetry be constant, and integrating around the ring,

$$\int H dl = 4\pi I.$$

Thus the magnetizing force is just the same as if the iron were not in the space it occupies. But in the space "filled" with iron, the flux will be greatly increased over the value it would have in free space. How is this known? It is "known" in the only way possible—by measuring the flux in the ring by its voltage-inducing abilities; for instance, by a test with the ballistic galvanometer. Φ/A now gives B, which is usually many times H.

20. Permeability.—The quality by which the iron exhibits a flux capacity greater than free space is called its permeability. The permeability is defined as the ratio of B to H, and is denoted by the symbol μ.

A word here on the true nature of permeability is not out of place. Discussing from the viewpoint of the theory as developed up to this point, none of the internal characteristics of matter have been taken into account. All experiments mentioned dealt with externals. It is just as well, so far as the theory is concerned, if it is assumed that matter is solid and continuous. The electrical engineer, incidentally, usually deals with matter in the bulk, unless he happens to be working with the physical end of research on vacuum tubes, insulation, and the like. Then so far as the magnetic circuit offers the usual problem to the engineer, the problem is handled without respect to the physical constitution of matter.

From the physics standpoint, the idea of continuous matter was dropped long ago. Molecules and atoms were then discovered, and only as late as 1895, books on natural philosophy—physics—made the definite statement that the atom was the smallest particle of matter.

Recent years have disclosed surprising facts concerning matter and its makeup. It is now known that when iron apparently occupies space, it is giving a false impression; that the amount of matter is really exceedingly small; and that the free space left between the electrons of the atoms and between the atoms themselves is nearly as much as was there before the iron was introduced into the space.

It is then evident that the flux path through a piece of iron is mainly a path through free space which has unit permeability. It is absurd to think of this space in any other way. Then why so great an increase in flux? As everyone now knows, the iron atom, like all other atoms, consists of a positive nucleus surrounded at relatively great distances by a constellation of negative electrons. The electrons are whirling at tremendous speed around the nucleus. A magnetic field acts upon an electron, and to pass a flux through an iron atom is to tend to bring all orbits into a plane normal to the line of the field. Mutual interactions within the atom work to prevent such a formation. But something—exactly what, is not known—happens in the iron by which some of the electronic orbits become lined up in the direction of

the exciting field. There are several possibilities. The orbits within an atom may orient; or the atom itself may normally be polarized due to orbit positions, and the introduction of the field may open up the formerly closed chains of polarized atoms; or groups of atoms or of molecules may already be or become polarized. There are other possibilities to account for the phenomenon of the magnetic substance. The physical study is so complex that it may be years before the true state of affairs can be stated definitely.

Thus it is seen that in the light of the structure of the atom, permeability as such ceases to be a quality of continuous matter, but instead represents the amount of electronic current "induced" in the iron by the presence of an exciting field, this current itself then being responsible for the increase of flux through the free space of the magnetic material.

21. Reluctance in Iron.—In a volume of rectilinear or uniform flux in free space, the reluctance of the volume is L/A, if A is taken over an equipotential surface, and L is between equipotential end boundaries. Since iron with a permeability μ permits μ times as much flux to pass as does free space for the same value of H, the reluctance of a similar volume of flux in iron is $L/\mu A$, if μ is constant throughout the volume.

22. Energy Stored in a Field in a Magnetic Substance.—It was found above that the ergs stored per cubic centimeter are $\Phi^2 R/8\pi$. Again using the symmetrical iron ring of small cross-section (Art. 19), the energy stored in the ring is

$$\text{Ergs} = \frac{B^2 A^2 L}{8\pi\mu A} = \frac{B^2 A L}{8\pi\mu} = \text{volume} \times \frac{B^2}{8\pi\mu}.$$

23. Attraction.—Let an exciting coil establish flux in an iron ring (Fig. 1*a*). Consider a tube of flux which has a section of 1 square centimeter within a part of the volume V of the tube. Let the length of this volume V, between the equipotential surfaces which bound it, be 1 centimeter. Thus V is a cubic centimeter. Assuming proportions and selecting a tube such that B is nearly constant through V, the ergs stored in the unit volume V are

$$\text{Ergs} = \frac{B^2}{8\pi\mu}.$$

Now let a slit be made across the ring at one end of V, and let the ring be sprung open with the faces of the slit kept parallel in the operation. If the gap is widened to 1 centimeter in length,

the volume V has been changed from a volume in iron to a volume in free space.

While the operation is carried out, several things could happen, depending upon what values were held constant. Let the current in the coil be varied increasingly in exactly the right way to maintain a *constant* flux in the tube being studied. Irrespective of what other tubes of flux may do, if the flux of this one is constant, it can induce no e.m.f. in the coil. Then to maintain the flux of this tube has required no added energy input to the coil (aside from resistance loss energy).

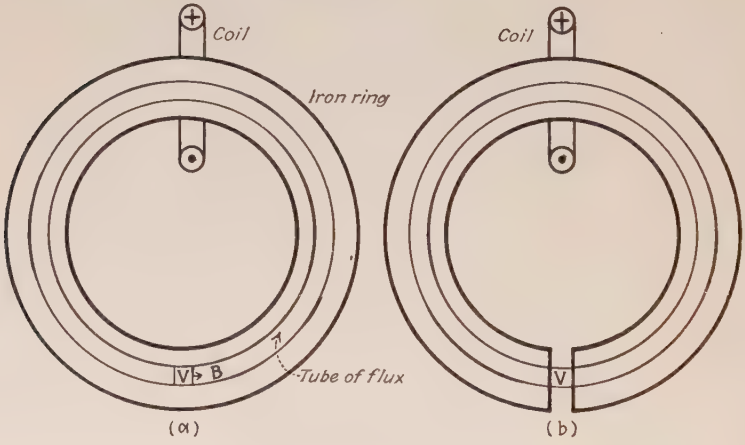

FIG. 1.—Flux in an iron ring.

The energy stored in the iron part of the tube is unchanged. But the volume V, now being in free space, has an energy storage of

$$\text{Ergs} = \frac{B^2}{8\pi}.$$

The change in energy in V is then

$$\text{Ergs} = \left(\frac{B^2}{8\pi\mu}\right) - \left(\frac{B^2}{8\pi}\right).$$

$$= \frac{B^2 - B^2/\mu}{8\pi} = \frac{B^2 - BH}{8\pi}.$$

As the coil was given no opportunity to supply the extra energy, it must have been added by virtue of mechanical work done in pulling the slit faces apart. (Forces needed to bend the ring do not come into the argument.) As the distance moved through

was 1 centimeter, and as the addition of energy was uniform, the dynes per square centimeter must be

$$\text{Dynes per square centimeter} = \frac{B^2 - B^2/\mu}{8\pi}.$$

In the usual practical problem, μ has a value varying from a minimum of 50 up to a maximum of 3,000, when approximately

$$F = \text{total force in dynes} = \frac{AB^2}{8\pi}.$$

CHAPTER III

POTENTIAL; EVALUATION OF UNITS; INTERNATIONAL STANDARDS; THE PRACTICAL SYSTEM

Students of magnetic-field problems experience so much difficulty in understanding magnetic potential that the subject deserves special consideration. The first work of this chapter simply takes the underlying potential relations and elaborates them in such a way that it is hoped the matter may be made clearer than it usually is.[1]

There then follows a word on the evaluation of the electromagnetic units which have already been defined, some notes on standards, and a tabulation of units.[2]

24. Potential Difference in a Free-space Field.—Figure 2 represents a conductor loop of N turns, whose current of I abamperes sets up a field in free space. A unit north point pole would be urged upward if placed anywhere between the conductors. If moved downward from A to B, work is done on it, the ergs of work done being already defined as the difference of potential between A and B; also, B is at the higher potential.

If the pole is started off at O and is carried along the line of flux shown through B, C, and A back to O, $4\pi NI$ ergs of work are done on it; this makes the point O begin with one value of potential and end with a different value. Yet there is only one point O, and how can it have two values of potential? This idea is found to be very confusing. To make matters worse, let the pole be carried around by the same route again. The point O now has a potential of $8\pi NI$ above the value it had originally. This multiple-valued characteristic, however, of the potential of point O can be forgotten at once, for it is never necessary to carry the pole

[1] Pender, Harold, "Electricity and Magnetism," McGraw-Hill Book Company, Inc.

[2] Pender, Harold, "Handbook for Electrical Engineers," John Wiley & Sons, Inc..

"Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc.

around the circuit more than once to find out all there is to know about the circuit.

In gravitational problems, a given mass has the same potential energy irrespective of how it may be carried to a given point. It must be distinctly understood that the magnetic case is different because of the nature of the source of the potential. In going from one point to another in the magnetic field by two different paths, the work done on the pole so carried will be the same, *provided* no sources of m.m.f. are passed through; but if such sources are different for the two paths, the work done will be different, and the potential differences will be different; that is, when a part of a path passes through a coil or a magnet, that fact must be taken into account.

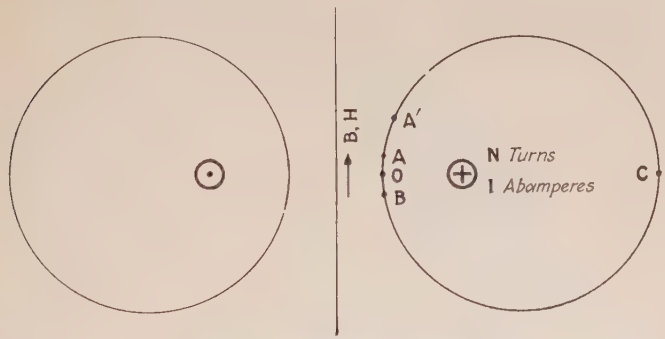

FIG. 2.— Magnetic potential.

The point O has of course the same potential as itself, if it is started from and arrived at again without the point pole having been moved therefrom. To make this sound more sensible, if A and A' are infinitely close together, they have infinitely near to the same potentials, if the pole is moved directly from A to A', to find the difference. But if the pole is moved by a wider route which passes around the coil and links with it, $4\pi NI$ ergs of work, nearly, are done. Likewise, if the pole is moved *from* O by a route that links the coil and *back* to O, the potential difference between O and O *around this path is* $4\pi NI$.

25. Statement of Potential Difference.—It is now seen that A and B have different potential differences, depending on whether path BOA is taken, or ACB. The sum of the two must be $4\pi NI$ gilberts, or the one is $4\pi NI$ minus the other. Returning to the usages prevailing in handling electric circuits, it is now

said that *the potential difference over a part of a path is equal to the total m.m.f. minus the reluctance drop over the remainder of the path.* Thus the potential difference ACB is $4\pi NI$ minus the reluctance drop BOA.

A formal statement made to include these ideas is as follows: **The m.m.f., or difference of potential, from A to B, is equal to the m.m.f. of all coils or magnets (taken algebraically) passed through in going from B to A, minus the reluctance drop from B to A.**

26. Calculation.—To help fix these ideas, a problem is introduced and partly solved. In Fig. 3 an iron core has two coils

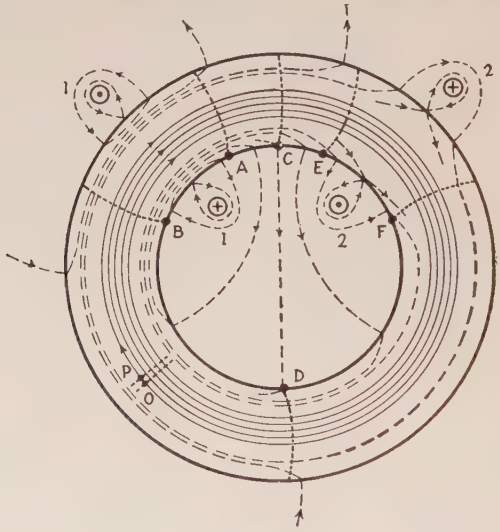

FIG. 3.—Potential, iron core.

mounted on it, the coils having opposing actions on the core. Let coil 1 have a total m.m.f. strength of 10 gilberts, and coil 2 have 8 gilberts. Assume an iron permeability of 2,000, further assumed to be constant throughout the core. Let the core section be 2 square centimeters. For length of path in the core, let lengths taken through the midpoints of the core sections be used. Let lengths then be as follows, in centimeters: $AB = 2$, $BD = 5$, $DF = 5$, $FE = 2$, $EC = 1$, $CA = 1$.

Th eflux in the air, or the flux partly in iron and partly in air, will be very small as a part of the whole. This so-called "leakage" flux is neglected temporarily. Leakage lines are shown

dotted; "main" core flux, which links both coils, are solid lines. By neglecting leakage, it is temporarily assumed that all flux stays in the core and links both coils.

Taking two points P and O located close together, to find the potential difference, $O-$ direct to $-P$, it is the m.m.f. of the coils passed in going from $P-$ around the core to $-O$, minus the reluctance drop over this same route. This amounts to all of the m.m.f., minus nearly all of the core reluctance drop. If O and P are taken together,

$$\sum \text{m.m.f.} = \sum \text{reluctance drops},$$

or

$$\text{m.m.f.} = \Phi R.$$

The reluctance of the core (length, 16; area, 2; μ, 2,000) is

$$R = \frac{16}{(2 \times 2{,}000)} = 0.004 \text{ oersted.}$$

The total m.m.f. of the two coils is

$$\text{m.m.f.} = 10 - 8 = 2 \text{ gilberts.}$$

The flux in the core is

$$\Phi = \frac{2}{0.004} = 500 \text{ maxwells.}$$

The reluctance of part BA is

$$R_{ba} = \frac{2}{(2 \times 2{,}000)} = 0.0005 \text{ oersted.}$$

Or more simply, as the section is constant, this part has two-sixteenths of the total length; hence its reluctance is one-eighth of the total, or 0.0005. The reluctance drop in part AB is

$$\Phi R = 500 \times 0.0005 = 0.25 \text{ gilbert.}$$

Bringing leakage flux back into the problem, it is at once admitted that the core values of flux and potential differences were not correct; for the flux of 500 maxwells is too low for the part of the core inside coil 1 and too high for the part inside coil 2. But the error is small, under the assumptions.

To find the m.m.f. "acting" to set up flux in the air from A to B, the m.m.f. AB is equal to the m.m.f. of the coils passed through on path BA, minus reluctance drop BA. The m.m.f. of the coils is that of only one coil in this case, coil 1. This is 10 gilberts. The reluctance drop BA is 0.25 gilbert, as found above. The AB m.m.f. is then

$$10 - 0.25 = 9.75 \text{ gilberts.}$$

The m.m.f. from C to D likewise is

$$10 - 1 = 9 \text{ gilberts.}$$

From E to F, $FDBCE$ passes likewise through coil 1 of 10 gilberts, and the drop on this part of the core is 1.75; then over EF there are

$$10 - 1.75 = 8.25 \text{ gilberts.}$$

Any of these results and, for instance, the last result should be arrived at if a circuit through coil 2 is used instead of through coil 1. Checking m.m.f. over EF, the coil m.m.f. on part FE is, with respect to the direction FE, plus 8 gilberts; the reluctance drop is minus 0.25; so m.m.f. EF is

$$8 - (-0.25) = 8.25 \text{ (which checks).}$$

Other m.m.fs. sending leakage flux through the air may likewise be found. How the remainder of the problem would be handled would depend entirely upon what is wanted. In some cases leakage flux is both small and unimportant, and is neglected entirely. In other cases it is small but very important, and an approximate solution for its amount must be made. In still other cases, leakage flux is of the same order of magnitude as the core flux; and when no iron whatever is present, the whole field might be called a leakage field, although this would be considering it thus in a specialized sense.

If the leakage flux, when calculated, was found to be large, its effect upon the main core flux would have to be determined. As a first approximation, the core flux would be solved for as it was in the first step above. Potential differences then being available, leakage is found. As a second approximation the leakage and the main fluxes would be algebraically summed up for each section or part of the core. This would give a new set of reluctance drops, leading to a recomputation of leakage flux. As many more approximations would be made as are deemed necessary.

27. Evaluation of the Abampere.—Briefly, the abampere was defined in terms of the force action upon a wire carrying it due to its being placed in a field of unit intensity. It was then found that the current also set up a field. Therefore, any actual current would set up a field which would react upon another actual current. Two coils, being connected in series, now can be built and placed near each other. For a given current in defined abamperes, the field due to one coil could be computed at every point

within the other coil; then the force action for 1 abampere could be computed. A current could then be set up and increased until the measured force action was exactly of this computed value; the current is then held steady while being passed through a silver voltameter. The silver deposited in an hour would become the referable standard.

28. International Standards.—The London Congress of 1908 settled upon the International, or "practical," standards for the ohm, the ampere, and the volt. These standards are based on the absolute system and correspond almost exactly to them, except that the factor 10 raised to some power enters in. Aside from a difference of some few parts in 100,000, there is an exact correspondence. No engineering work takes this difference into account. The definitions of the London Congress may be found in any electrical handbook or work on electricity and magnetism. The relations are:

10^{-1} abamperes = 1 ampere
10^{8} abvolts = 1 volt.
10^{9} abohms = 1 ohm.

29. Practical Units.—The practical system is no more practical, in the true sense of the word, than the absolute system. The practical system represents a compromise between the fixed habits of engineers already used to the English foot-pound-second system, and the electrical units as developed directly from the continental centimeter-gram-second system.

The practical system embraces the ampere, volt, ohm, watt, and horsepower, along with feet, pounds, and seconds. The henry is a practical unit of inductance, being the amount of inductance present when a rate of change of 1 ampere per second induces 1 volt per turn. Instead of the gilbert, another unit of m.m.f., the ampere-turn, is used; it is selected so that a coil of N turns carrying I amperes has NI ampere-turns. This takes the π out of the expression of m.m.f. and puts it somewhere else but does not get rid of it. H, the absolute unit for magnetizing force, is dropped; in its place appears ampere-turns per inch, corresponding more exactly to gilberts per centimeter. B still means maxwells per unit area, but the area is the square inch; B then means maxwells per square inch. Flux is the same in either system. Reluctance has no unit in the practical system. A table of units and their relations follows:

TABLE I.—UNITS

Quantity	Name of absolute unit	Symbol	Name of practical unit	Number of absolute units in a practical unit
Force	dyne	.....	pound	444,823
Force	gram = 980.7 dynes	.....	pound	453.6
Length	centimeter	L, l	inch	2.540
Area	square centimeter	A, a	square inch	6.452
Area	.....	M (M)	circular mil	1.273×10^{6}
Time	second	T, t	second	
Energy	erg	W, w	foot-pound	1.356×10^{7}
Electromotive force (e.m.f.)	abvolt	E, e	volt	10^{8}
Current	abampere	I, i	ampere	10^{-1}
Resistance	abohm	R, r	ohm	10^{-9}
Resistivity	abohm per centimeter cube	ρ	ohms per circular mil-foot	1.662×10^{2}
Quantity	abcoulomb	Q, q	coulomb	10^{-1}
Capacity	abfarad	C, c	farad	10^{-9}
Inductance	abhenry	L, l	henry	10^{-9}
Electric power	abwatt	P	watt	10^{7}
Power	.....	P	746 watts = 1 horsepower	
Electric energy	erg	W	joule	10^{7}
Flux	maxwell, line	Φ, ϕ	maxwell, line	
Flux density	gauss	B	maxwells per square inch	6.452
Magnetizing force	gilberts per centimeter	H	ampere-turns per inch	0.495
Permeability	permeability	μ		
Reluctance	oersted	R		

CHAPTER IV

FIELDS AND FIELD MAPS

Heat-flow fields, electrostatic fields, and magnetic fields are examples of fields in which there is a common property; the equipotential surfaces and the flux lines form an orthogonal system.

30. Mathematical Magnetic-field Treatment.—Very few magnetic fields occurring in practical work yield readily to mathematics. Some fields can be worked out mathematically, but the method may be complicated to such degree that the end does not justify the means. This does not by any means say that mathematics should be dodged on every occasion. In rectilinear fields such as that shown in the air gap (Fig. 4) only arithmetic is necessary; the simple circular field about a straight conductor (Fig. 5) yields to the use of simple calculus. Such cases occur commonly enough, and where mathematics may be applied without undue effort or without stretching the truth too far in the process of making assumptions to establish starting points, mathematics should be called in.

31. Non-mathematical Fields.—This title is meant to cover the large number of practical cases in which engineering methods other than mathematical may be applied to get sufficiently accurate solutions. Also those cases which are beyond any analytical method whatever, therefore being approachable only by experimental investigation.

32. The Assumption of Infinite Permeability for Iron.—In order most easily to develop the very useful field-mapping methods about to be outlined, it is assumed that the iron, when present, has infinite premeability. Under the assumption, the surface of the iron is an equipotential surface; and it becomes of no consequence to know about the flux within the iron itself. A great many practical engineering problems are always waiting for solution in which the iron saturation is so low and the permeability so high that the assumption, *so far as the flux formation in the air part of the path is concerned,* is nearly true.

All of the succeeding work on the mapping of magnetic fields is based on this assumption; where exception is made, the context will call attention to the fact.

33. The One-dimensional or Rectilinear Field.—Figure 4*a* shows two poles with parallel faces. The flux is uniform, being in straight lines of constant density. The equipotential surfaces, as represented by the equipotential lines of Fig. 4*b*, are flat planes parallel to the pole faces. In Fig. 4*c* both flux and equipotential lines are shown, some of each being drawn in at random.

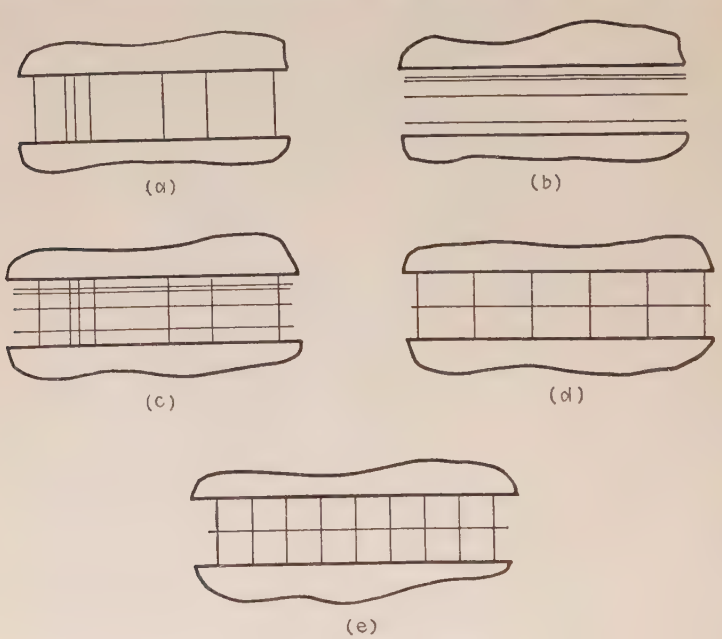

FIG. 4.—Rectilinear field.

Figure 4*d* gives a more usable representation of the field, for here the lines are equally spaced each way. But for simplest representation and for greatest ease in study and calculation, the system of Fig. 4*e* is best. Here, lines were so drawn as to divide the map into true squares. Thinking of a volume of the field as being bounded by the plane of the paper and a parallel plane axially removed by unit distance, then each square represents an equal volume of the total field. There are 16 squares shown; so 16 volumes of equal size are represented by them. The following statements can be made:

1. There is the same m.m.f. over each volume.
2. There is the same flux down each volume.
3. There is the same energy stored in each volume.
4. Where a volume ends on the iron, there is the same attraction as where any other such volume ends.

A one-dimensional, or rectilinear field, may be defined as one in which, if the usual rectangular coordinate axes are set up with one axis in line with the flux, differences of potential can be found along only one axis. In the series of Fig. 4, only along a vertical axis can changes in potential be found.

One-dimensional fields are handled by simple arithmetic.

34. Two-dimensional Fields.—In two-dimensional fields there is but one axis along which differences cannot be found; the potential varies along each of the other axes. An "axial" dimension is measured along the axis which exhibits no change of potential; it follows that all flux lines lie completely within planes placed normal to this axis. A map of the field in such a plane defines the field completely.

The circular field of Fig. 5 is an example. The flux lines are all circular, as drawn on the plane of the paper. The wire defines the axis, running normal to the paper, along which or parallel to which there is no change of potential.

35. Three-dimensional Fields.—Changes of potential occur along each of the three axes in a three-dimensional field.

36. Relations in a One-dimensional Field. I. *Reluctance.*—Consider the plane of the paper to be placed parallel to the flux lines. Let such flux and potential lines be drawn as to create a system of squares. Let the axial distance normal to the paper be 1 centimeter. If the squares happened to be centimeter squares, the volumes they represent would be centimeter cubes, which have unit reluctance, or 1 oersted. But for any other size square, the corresponding volume would also have one unit of reluctance; for length and area have been changed in proportion. A column of squares reaching from one pole to the other represents a tube of flux. The reluctance of the mapped portion of the field is then

$$R = \frac{(\text{squares per tube})}{(\text{number of tubes})}.$$

The reluctance for any axial distance D centimeters is

$$R = \frac{(\text{squares per tube})}{(\text{number of tubes} \times D)}.$$

II. *Other Absolute Relations.*

$$\Phi = \frac{\text{(m.m.f. between poles, in gilberts)}}{R}.$$

$$\text{Energy stored} = \text{(cubic centimeters)} \times \frac{B^2}{8\pi} \text{ ergs.}$$

$$\text{Attraction} = \text{(square centimeters)} \times \frac{B^2}{8\pi}.$$

III. *Practical System Relations.*

$NI = 0.313\ BL$, where L is the length of the flux path from pole to pole, in inches.

$\Phi = BA$.

P = pounds per square inch $= B^2/(72 \times 10^6)$.

The reader should verify the truth of these equations for himself. It is often convenient to use the form

$$B = 3.19 \frac{NI}{L},$$

and the constants 0.313 and 3.19 should be remembered, as well as the constant 72×10^6 in the attraction equation.

37. Relations in a Circular Field. I. *Equal-reluctance Volumes.*—The field about an infinitely long, straight conductor is circular, or the field near to the straight portion of a crooked wire is nearly cylindrical, if the wire section is small and circular. Flux lines are circular and of constant density; equipotential lines are radial. In Fig. 5*a* some random lines of each kind are drawn. Figure 5*b* gives a more useful representation, for the equipotential lines are equally spaced; hence they divide the total m.m.f. into equal portions. It can be seen offhand that the flux lines here are spaced so that the flux per tube is approximately constant, at least. It will be instructive to find out just what flux-line spacings are needed to give tubes of constant flux. This is done with reference to Fig. 5*c*, where a part of the field is outlined.

Considering an axial depth of field of 1 centimeter, the permeance of a tube element (permeance being the inverse of reluctance) is

$$dP = \frac{dr}{r\theta}.$$

Beginning with a flux line placed at radius R_1, it is desired to find the radius R_2 at which to draw another flux line so that between

the two, and ended by the equipotential lines spaced by the angle Θ, a tube of unit permeance will be defined.

$$\text{Permeance} = P = 1 = \int_{R_1}^{R_2} \frac{dr}{r\Theta}, \quad \frac{1}{\Theta} \log \frac{R_2}{R_1} = 1, \quad e^{\Theta} = \frac{R_2}{R_1}$$

The m.m.f. in Fig. 5*b* is divided into eight equal parts, or Θ is $2\pi/8 = 0.7854$.

$$e^{0.7854} = 2.26 = \frac{R_2}{R_1}.$$

Then in this special case, if eight equipotential lines equally spaced are used, and if flux lines are placed so that the radius of one is 2.26 times the radius of the next inner line, the field is marked off into areas each of which represents a volume of unit permeance or reluctance.

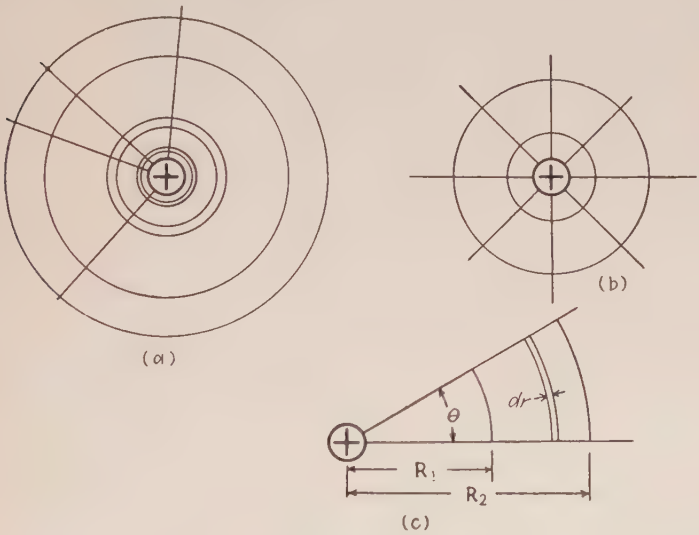

FIG. 5.—Field of a straight conductor.

II. *Absolute Relations.*—Using D for axial depth of field considered, let the field between two plane pole faces placed at the angle Θ, and with an m.m.f. of M gilberts, be studied. Radial distances are taken from where the plane faces would intersect, if extended. If the radial dimension of the pole faces is great, then a portion of the field not near the inner or outer ends will be nearly circular.

$$B = \frac{M}{r\Theta}.$$

$$d\phi = BdA = \left(\frac{M}{r\Theta}\right)Ddr.$$

$$\phi = \int_{R_1}^{R_2} \frac{MD}{\Theta}\frac{dr}{r} = \frac{MD}{\Theta}\log_e \frac{R_2}{R_1}.$$

Attraction for a pole face is

$$dF = \left(\frac{B^2}{8\pi}\right)dA = \frac{M^2}{r\Theta^2 8\pi}Ddr.$$

$$F = \int_{R_1}^{R_2} \frac{M^2D}{8\pi\Theta^2}\frac{dr}{r^2} = \frac{M^2D}{8\pi\Theta^2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \text{dynes.}$$

III. *Practical Relations.*—In the practical system,

$$B = 3.19\frac{NI}{r\Theta}.$$

$$d\phi = BdA = \left(\frac{3.19NI}{r\Theta}\right)Ddr.$$

$$\Phi = \int_{R_1}^{R_2} \frac{3.19NID}{\Theta}\frac{dr}{r} = \frac{3.19NID}{\Theta}\log_e \frac{R_2}{R_1}.$$

Attraction at a pole face is

$$dF = \frac{B^2}{72 \times 10^6}dA = \left(\frac{3.19NI}{\Theta}\right)^2 \frac{1}{72 \times 10^6}D\frac{dr}{r^2}.$$

$$F = \left(\frac{3.19NI}{\Theta}\right)^2 \frac{D}{72 \times 10^6}\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

38. Curvilinear Squares.—In Fig. 5*b*, assume that the lines have been so drawn as to produce areas each of which stands for a volume of unit reluctance, each volume being 1 centimeter deep axially. One of these areas may now be subdivided. Let a new flux line be put across it, so as to divide the area into two areas, each of which represents twice the original reluctance. Now let a midpotential line be put in, giving four areas, each now representing unit reluctance again. By the further proper addition of two more flux and two equipotential lines, the four areas can be split up into sixteen, each of unit reluctance. The process can be continued indefinitely.

It has already been shown (Art. 36) that if a volume has unit axial dimension and has a square face of any size, its reluctance is unity. If the above process of subdivision is carried to the limit, each infinitesimal area, which inherently represents unit reluc-

tance, becomes rectangular, and by Art. 36, the rectangle must also be a square. There is the same number of these little squares on the two sides of the original large area. The original area, which also had unit reluctance, being built up regularly of infinitesimal squares, may then be called a **curvilinear square.**

39. Subdivision.—There are any number of lines of either kind in a field. Given an area bounded by two flux and two equipotential lines, this area may be subdivided into any number of areas, in any number of ways, depending on the will of the operator. The process of subdivision carried out in the preceding article was a *regular* process; equal numbers of each kind of line were always added, and they were always added so that the flux and m.m.f. of every area were divided into halves. If the original area was a curvilinear square, this regular subdivision, in the limit, will divide the area into an infinite number of true squares, there being the same number of these to a side of the original area. This particular regular subdivision is the kind meant hereafter when it is said that a field is subdivided. For special solutions it is sometimes profitable to depart from the regular process.

40. Curvilinear Squares in a Warped Two-dimensional Field. Figure 6 shows a volumetric aspect of a non-mathematical two-

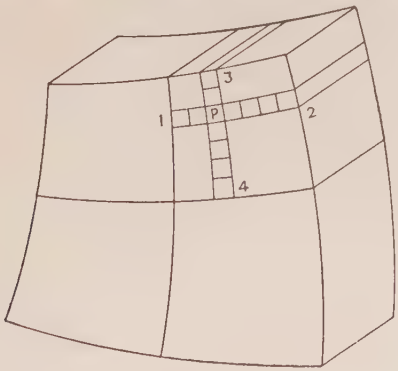

FIG. 6.—Part of a two-dimensional curvilinear field.

dimensional field. At the point P let two flux lines infinitely close together be selected. These are lines 3-4. Also let two equipotential lines, separated here by the same distance as are the flux lines, be chosen. These are lines 1-2. The four lines then bound an infinitesimal square. A great number N of such squares can be marked off each way from P between each of the

pairs of lines. Between 1 and 2 is a large number N of such squares, which vary in size as the two equipotential lines vary in separation, due to the warp in the field. There are N squares from 3 to 4. Now the four flux and equipotential lines through the ends 1, 2, 3, and 4 can be drawn in, thus defining a large area. Each infinitesimal square represents the same reluctance, for a given axial depth of field. If that depth is unity, each represents unit reluctance. Therefore, the large area represents unit reluctance. The large area is a curvilinear square.

41. Properties of the Curvilinear Square.—For a given axial depth of field, it is easy to see that for each curvilinear square in a subdivided field, the following statements hold true:

1. The volumes back of the squares have the same reluctance.
2. There is the same m.m.f. over each volume.
3. The volumes have the same flux.
4. The energy stored in each volume is the same.
5. If the subdivision is carried out to such a degree that those "squares" ending on a pole face are nearly true squares, the volume represented by a square has an attraction for the pole face that is nearly inversely proportional to the width of the square.

The reason for using a subdivision of squares, instead of rectangles, is twofold: The calculations are made a little easier, and it is much easier to look at a mapped field and check up inaccuracies in squares than if rectangles were used.

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CHAPTER V

THE TECHNIQUE OF FIELD MAPPING[1]

The art of field mapping is a highly useful one. It happens to have been strangely neglected in the literature of electrical engineering. Applying as it does to the electrostatic and magnetic fields, as well as to heat flow, many important problems beyond the reach of mathematics may be brought to reasonably close solutions.

The development of a technique depends on three things: (*a*) a knowledge of the properties of the orthogonal system; (*b*) the development of facts concerning special tendencies of line formations in curvilinear fields, and especially the development of special cases; (*c*) study of fields already mapped and, particularly, practice in the art itself.

The properties of the orthogonal system and of the curvilinear square have already been brought out. The technique of field mapping is treated in this chapter.[2]

42. Influence of the Boundaries.—In physical problems, given a set of boundary conditions operating on a known medium, there can be but one internal result. In heat, given the boundary temperatures and the qualities of the medium and its dimensions, the steady-state flow can have but one conformation. In electrostatics, fixing the potentials, the dimensions, and the dielectric constants fixes the resulting field. In magnetics, the same is true.

[1] The author developed his technique of field mapping without knowledge of Lehmann's works. These are:

Lehmann, Th., "Graphical Method for Determining the Tracings of Lines of Force in Air," *La Lumière Électrique*, vol. 8, 1909.

"The Graphical Determination of Two-dimensional Magnetic Fields in Regions Where Laplace's Equation Is Satisfied, and Also in Regions Where the Curl of the Field Is Not Zero," *La Revue Générale de l'Électricité*, vol.14, 1923.

"Sketches of Magnetic Fields in Iron," *La Revue Générale de l'Électricité*, 1926.

[2] Moore, A. D., "Mapping Magnetic and Electrostatic Fields," *Elec. Jour.*, July, 1926.

If a magnetic field could be persuaded to put itself on accurate record, a certain conformation of lines would be obtained, or, by use of the laws of the field, a mathematician of sufficient ability could find the equations for and plot the curved-line duplicates of the record the actual field would make. The ideal of the field mapper is to achieve, by cut-and-try methods, a conformation of lines such that the resulting map is as nearly as need be a duplicate of either of the other results. Given a set of boundaries and a homogeneous medium, all three methods, in the limit, would give results which would be identical.

43. Mapping a Simple Case.—Figure 7*a* shows parts of two equipotential boundaries, *AB* and *CD*, between which a field in free space is set up. These lines might coincide with the faces of opposite poles.

A guess at the midpotential line was made by drawing the smooth curve *KE*, allowing it to hug the inner boundary somewhat. Flux line *AKC* was then drawn; then *GII* was drawn, placing it so that an approximate square was obtained in area 1. The remaining flux lines, in order, were then added, making areas 1, 2, 3, etc. approximate squares and keeping all lines smoothly curved and all intersections approximately right angular. It is at once evident that areas 5, 6, 7, etc. are all curvilinear rectangles, with the possible exception of area 6. Further note that the intersection at the common corner of areas 1, 2, 5, and 6 is obviously not right angular. To correct the map, the midpotential must be moved to a new position somewhere in the vicinity of the dotted line *KF*.

Figure 7*b* shows the next attempt, with the midpotential moved as suggested. Flux line *AC* was again placed, then *GH*, and so on. The most evident errors now are that areas 1 and 5 are too narrow (due to placing *GH* too near *AC*), and area 9 is narrow, calling for the moving of the midpotential still further out in this region.

Figure 7*c* was the next improvement. Various minor errors can be pointed out here, but when the map has reached this *stage of relative perfection with this degree of subdivision*, it is a waste of time to try for improvement without resorting to the more searching test of greater subdivision; errors will then be much more evident. The next step in subdivision was taken (Fig. 7*d*), the map being first done in pencil so that erasures and changes could easily be made.

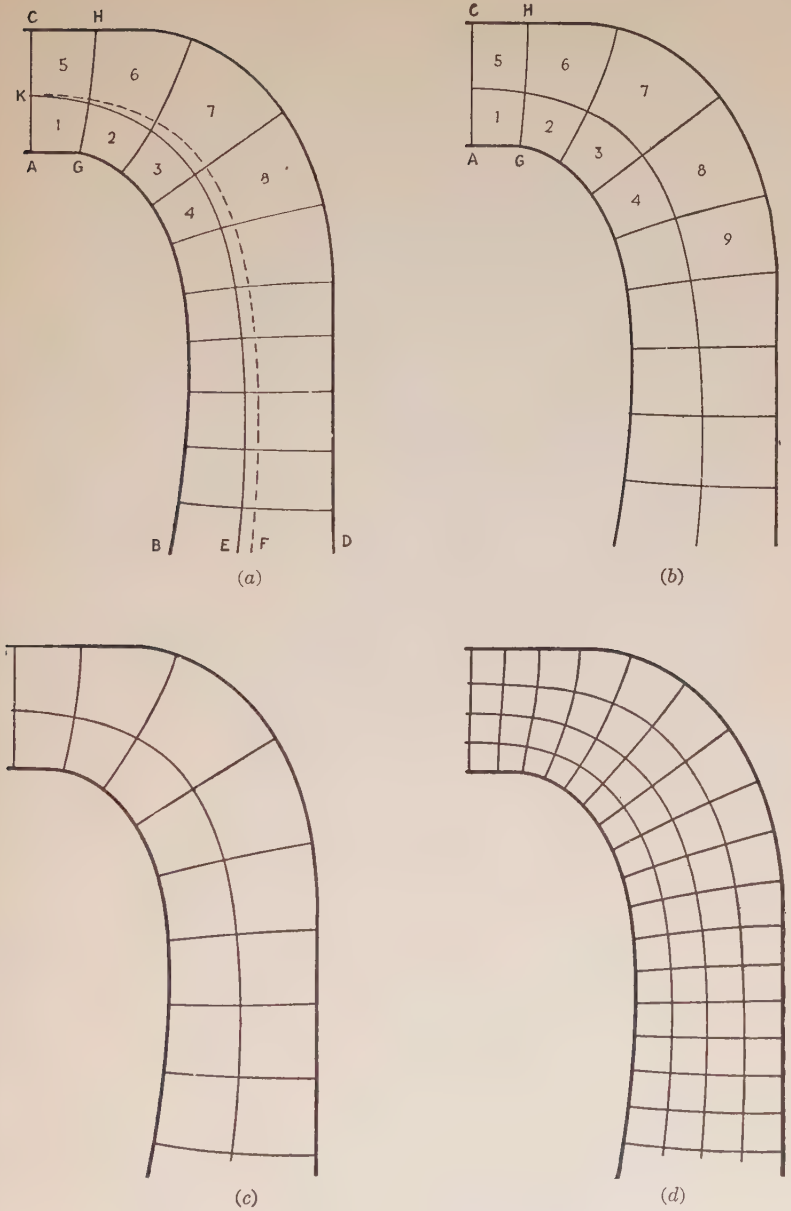

FIG. 7.—Mapping a simple case—two-dimensional.

This final map checks out to a reasonably high state of accuracy; tests for normal intersections and for equality of average widths of squares can now be made with instruments, if it is thought necessary, whereas before this subdivision, such tests would have been wasted effort. Flux valves found by use of this map would probably be correct to within 3 per cent.

44. Intersections.—Some of the field pictures in this book are given only for showing the general trend of the field and are not meant to be accurate; but in all of the maps that are properly subdivided, even including the series of Fig. 7, the intersections are made nearly right angular. In the early steps of the series of Fig. 7, the defect of "long" squares was shown, but even here the defect of very bad intersections was carefully avoided. *The student of the subject of field mapping should make it an unbreakable rule not to tolerate a bad intersection, no matter how his preliminary efforts may suffer by the application of the rule.* If a preliminary map has true intersections, it at least shows an orthogonal system and therefore has the merit of representing some possible field; with bad intersections, no field whatever is represented. Furthermore, if intersections are good, the long and the short areas indicate very plainly how to move the various lines to secure betterment; whereas, if bad intersections are present, this defect, combined with the inevitable rectangularity of desired squares, usually so confuses the visual sense that it is difficult to see how next to move.

CHAPTER VI

CORNER-FLUX PRINCIPLES

The author has developed a series of principles governing flux fields adjacent to corners of equipotential bodies, that are indispensable in the mapping of most practical cases.

45. Right-angle Exterior Corner.—Let a pole, having an equipotential surface, be bounded on two sides by plane faces at right angles. See faces *BO* and *ON* (Fig. 8*a*). If these faces extend for a considerable distance, and if the opposite pole is far away, the field near the corner will be governed primarily by its own laws. If the faces extend to infinity, and if the opposite pole is at an infinite distance, then the whole field extends to infinity. This case has of course been handled in mathematics; it will be handled here on its own merits from the field mapping standpoint.

Because the boundary is symmetrical, a flux line must leave the corner and extend straight to infinity, making equal angles with the two faces. This is line *OM*. *BO*, *OM*, and infinity bound a portion of the field. In this portion, all equipotential lines arrive normally to the straight flux line *OM*, and all flux lines arrive normally to the straight equipotential line *BO*; moreover, the family of flux lines and the family of equipotentials everywhere produce normal intersections. By virtue of these features of symmetry, the field must be symmetrical about *AO*, the bisector of angle *BOM*. Let *AO* be called a *construction bisector*. Also, by symmetry, a flux line crossing *AO* must be identical in shape with the equipotential line crossing *AO* at the same point.

Let any point *A* on *AO* be selected. *PA* and *PQ* are drawn normal to each other and at equal angles with *AO*. The flux and the equipotential lines through *A* must be tangent to *PA* and *PQ*, respectively, at *A*.

The flux line through *A* cannot be straight, for it must, when prolonged, arrive normally on *BO*. For the same reason, it cannot curve to the left when approaching *BO*. It must then curve to the right. Knowing that the corner is a region of high flux density, it is also evident that flux lines must continue to

spread as they leave the corner; hence, as the flux line leaves the pole, it must continue to decrease in curvature.

Any smooth curve BA is now drawn so that it is normal to BO, has its greatest curvature there, decreases in curvature while curving to the right, and is tangent to PA at A. As any number

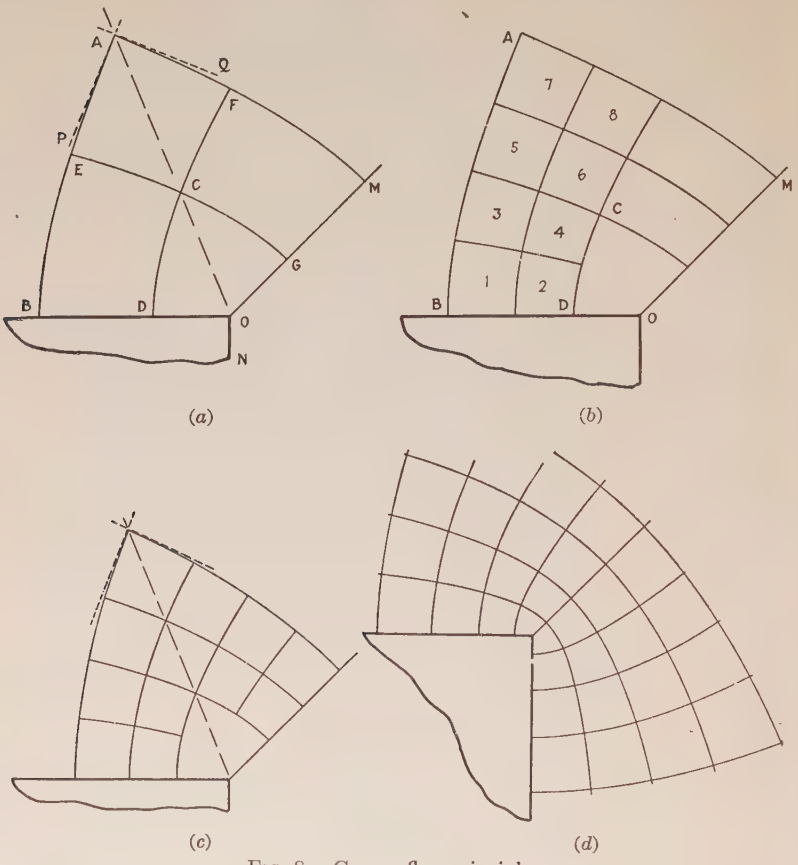

Fig. 8.—Corner flux principles.

of curves will satisfy these requirements, starting from any number of positions around point B, the chances are infinity to one that the curve is wrong, but the question is, How far wrong?

AM is drawn as an exact duplicate of BA.

At this time, it is necessary to make the assumption that every line in the field is *similar* to every other line. There is every

reason of regularity and symmetry for believing in advance that this is true, and the sequel will prove it.

Now let another point C on AO be selected, with the hope that a pair of lines through C will divide $BAMO$ into curvilinear squares. On the assumption of the preceding paragraph, CD and CG are drawn as small-scale duplicates of AB and AM. CD and CG are prolonged smoothly to F and E, making good intersections at these points.

Area $CEAF$ looks like a good curvilinear square. But area $BECD$ is too long in the BE direction. To correct, C is moved a little and the process is repeated (Fig. 8*b*). To insure that CD is closely similar to AB, OD was measured off so that $OC:OA::OD:OB$. By using a correct intersection at C, a normal arrival at D, and a smooth curve of the right tendency for CD, this similarity is reasonably well attained for graphical purposes. The four areas in Fig. 8*b* appear to be good enough to warrant subdivision, which was carefully done. A series of smaller areas is obtained, most of them being fairly satisfactory; but a check with instruments shows that area 4, at least, cannot be called a curvilinear square.

No material improvement from a further shifting of C can be attained; the map of Fig. 8*b* indicates that the initial curve AB is incorrect in that it falls too far out at B, making the lower areas too wide. In the next trial (Fig. 8*c*), B is moved in a trifle, giving the desired improvement.

Figure 8*c* could be corrected only to a minor extent, no matter how painstaking the work, or how prolonged. Figure 8*d* duplicates it, except that both halves of the field are shown.

To investigate the validity of the process, remember that this final map was attained by methods based directly on the simple properties of a flux field and upon the one assumption that all lines are similar. By the use of the assumption a good result was attained; it is evident that with unlimited time and patience, a perfect field map could thus be achieved, which is all that could be obtained by any other method. There is only one possible conformation attainable; hence this possible ideal gotten by field mapping methods would be that conformation.

To show that all parts of the field are subject to the analysis, it is only necessary to point out that A could have been taken as near to O, or as far from O, as desired. The conformation shown then can be expanded or contracted to represent as much or as little of the field as is desired.

The small area *DCGO* is undivided; but this area is exactly similar to the large area *BAMO* and will therefore subdivide just as it did. Repeated subdivision of the remaining area next to the corner would eventually leave an infinitely small area at the corner of no consequence; it is only of mathematical interest in that it refuses to become a square, for the angle at the corner is greater than 90 degrees.

46. Right-angle Interior Corner.—Figure 9*a* shows an interior corner formed by two plane faces of an equipotential body, the field extending from these faces to infinity. The problem is similar in all vital respects to the preceding one. A straight flux line *OM* bisects the corner. Half of the field is symmetrical

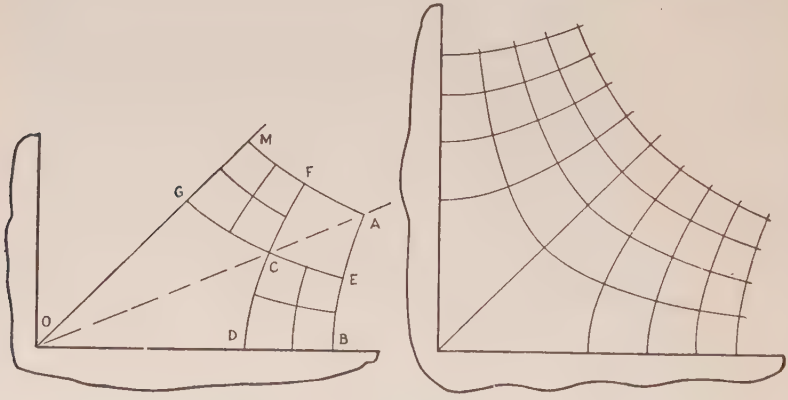

FIG. 9.—Interior corner.

about the construction bisector *AO*. The point *A* is selected on *AO* at random. The two lines through *A* are now to be drawn convex inward. *AB* and *AM* are drawn as duplicate trial lines, point *C* is guessed at, *CD* and *CG* are made as small-scale duplicates of the first lines, and the map is inspected for error. Figure 9*b* shows the finished map.

47. Field Near Any Corner.—The method developed is applicable to any interior or exterior corner in two-dimensional cases.

48. Slot of Infinite Depth.—Thinking of an interior angle in the act of being closed up, as the angle approaches zero, the sides become parallel. In the limit, the field in such a narrow opening approaches the field of a slot of infinite depth in which all of the flux in the slot emerges from the top of the slot at infinity.

Referring to Fig. 10*a*, the flux line bisecting the angle *OM* is now parallel to the sides of the slot. The right-hand half of the field is symmetrical about the construction bisector *OA*. The four solid lines *AM*, *AB*, *GC*, and *DC* were obtained by exactly the same methods as used above for corner cases. Parts of other lines are shown dotted.

In mathematics, when a special case is treated, it is usually found that the degenerating of the general problem into a special one gives simpler relations. A special case of corner-flux fields is here handled in graphical style. Whereas up to now, it could

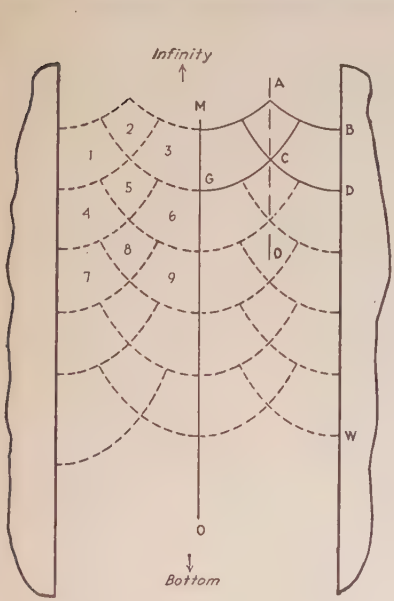

Fig. 10*a*.—Slot of infinite depth.

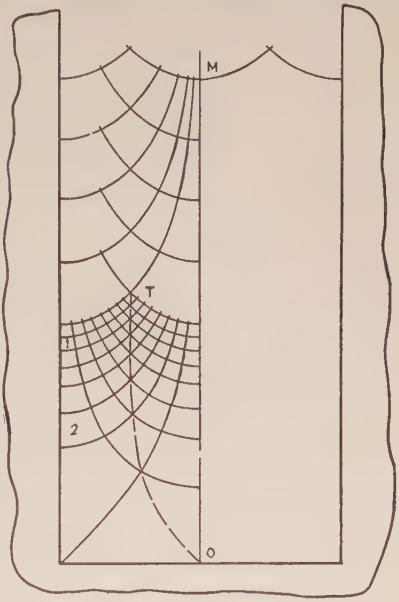

Fig. 10*b*.—Slot of finite depth.

be said that all lines of the field are similar, now it can be said of the lines in the slot that they are all exact duplicates. Refer to Fig. 10*a*.

49. Slot of Finite Depth.—Figure 10*b* shows the case of a slot with a bottom, the depth being great compared to the width of the slot. Midway down the slot the field would be governed almost entirely by the laws holding for a slot infinitely deep. This field would be modified, or warped, as the bottom is approached. It will be observed in this drawing that the symmetrical field remains practically unwarped down as far as the

level of the point T, which is about as far as the top of the largest square that could be fitted into the slot bottom space. The warped field below T is shown.

The behavior of the construction bisector is interesting. It curves from T to O, entering the angle at O so as to bisect it. The field close to O is governed mainly by the laws applying for the field from an interior right-angle corner, which demands that TO in this corner shall be a bisector.

50. Angle of Less than Zero.—Some little imagination is necessary in following the development of this case. The

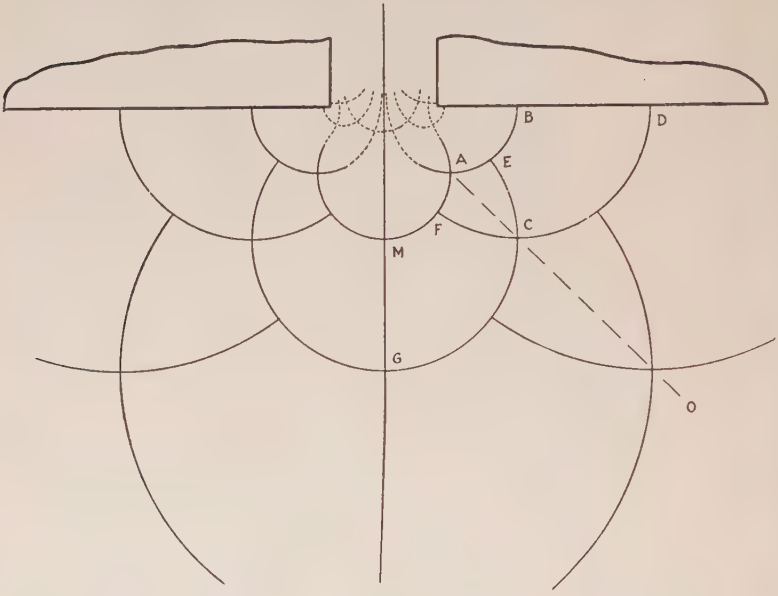

FIG. 11.—Field below bottom of slot that opens at bottom.

problem looks like one of strictly academic interest, but it nevertheless has a practical application. It is already seen that an interior angle, when closed up, becomes equivalent to a slot of infinite depth. What if it now opened at the bottom, spreading out there into a cavity of infinite size? The angle has passed through zero and has become "less than zero" in opening.

Figure 11 represents the boundary conditions, where the bottom of the slot has been opened out into a cavity of infinite extent, with the surfaces next to the slot bottom made normal to these sides. All of the equipotential lines in the cavity must

close up, hairpin style, and squeeze into the slot; none of them can enter the boundaries, for the boundaries are really one equipotential surface.

All of the flux from the walls of the cavity must also squeeze into the slot. Near the slot bottom there will be a warped field where the symmetrical field within the slot opens out to join the symmetrical field at some distance out in the cavity. Out in the cavity, half the field will be symmetrical about the construction bisector *OA*. All of the lettering on this drawing corresponds to that on the other corner-flux maps shown; so the construction methods can easily be followed. In this special case, the lines are arcs of circles.

The warped field, where the two symmetrical field portions join, is shown dotted.

51. Extreme Curvilinear Squares. I. *Possibility of Subdivision.*—In his earlier work, the field mapper sometimes runs into some odd situations. He may find areas in his preliminary maps that give no hope of subdividing properly; but as experience adds to his judgment of shapes of areas, the peculiarities begin to vanish. Without some prior knowledge, one would never guess that in Fig. 10*a* area *OGMABDW*, which reaches downward to the infinitely remote bottom, is a curvilinear square. But subdivision shows that that is just what it is. Referring to the similar area on the left side of the slot space, the first subdivision gives areas 1, 2, and 3, each of which is a curvilinear square. The remainder area below is the "area of doubt"; if it will subdivide, the case is proved. It in turn is subdivided into areas 4, 5, and 6, each being acceptable, leaving a new area of doubt; the process is continued indefinitely.

II. *Variation of Flux Density.*—Before completing the argument of case I, another important characteristic of all these fields is profitably brought out. The total flux in the *mapped* part of the left half of the field is obviously equally divided between the volumes back of areas 2 and 3. The first remainder volume, represented by the first area of doubt, then carries only half of the flux under consideration. Similarly, this half flux is equally divided between 5 and 6; so the second area of doubt, below the 4-5-6 areas, has only one-fourth the flux; the next area of doubt has one-eighth, and so on. Thus the area of doubt soon becomes of no consequence, no matter what its shape may be. This amounts to saying that, in a field of both great

and small densities, the small densities are of little relative importance.

The original area in question thus is shown to be a curvilinear square.

52. Corner-flux Principles in General Mapping.—Some of the usefulness of preceding developments is indicated in the problems of Fig. 12. Figure 12*a* shows a square-cornered air gap between poles. The two wings of the gap are equal; hence there will be a corner-to-corner flux line as shown. Two other flux lines and sections of the midpotential line are put in where the field is far enough from the corner vicinity to be nearly rectilinear. Figure

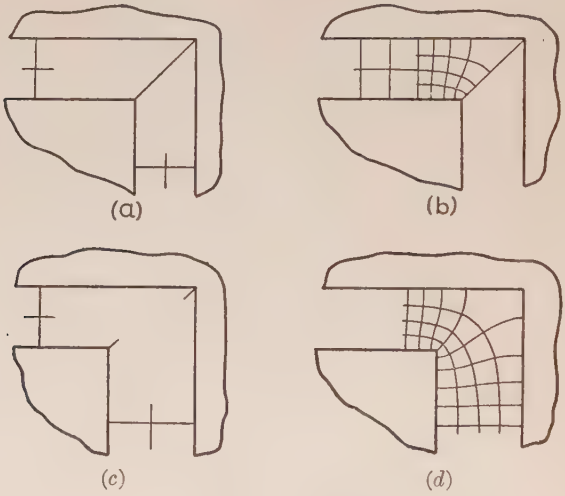

Fig. 12.—Field-mapping cases.

12*b* shows half of the completed job. Some important characteristics, all more or less obvious, should be noted.

First, the equipotentials regularly increase in spacing from the inner to the outer corner. Second, all flux lines between the corner line and the beginnings of the rectilinear field *are reverse curves*. Moreover, they are *smoothly* curved; beginners too often try to put most of the bending into one short arc near the point of inflection.

In Fig. 12*c* there is a similar, but unsymmetrical, problem. The variations in line spacings, changes in curvature, and smoothness of curves are points to be carefully noted in the study of this case.

These particular maps are by no means good examples of field mapping. They cannot compare in accuracy with the maps given in connection with corner-flux developments, for instance. The reason is that the originals of the corner-flux maps were done on a scale about twice as large as were those of Fig. 12. The average person has little sketching ability and therefore should never attempt to do good mapping on a cramped scale; but granted that a beginning is made with sufficiently large boundaries, anyone still having the use of one eye and one hand can do a good job of mapping, provided he has managed to stay in an engineering school long enough to be studying this book.

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CHAPTER VII

FIELD MAPPING: CONDUCTORS PRESENT

Two-dimensional fields between two poles are relatively simple because one cannot lose track of the lines in the first or preliminary analysis. All flux lines pass from one pole to the other, and all equipotential lines lie between the poles. But the addition of the electric circuit badly complicates the field. Equipotential lines may all begin and end on conductors, or some may go to infinity; the **singular point** is added to the case. The average student of flux fields cannot hope to analyze these problems unless he first trains himself to analyze the simpler academic cases. And unless one can analyze a problem "in the rough," no start of any worth-while kind can be made towards mapping the field into curvilinear squares.

53. Two Straight Parallel Conductors. I. *Like and Equal Currents.*—This case (Fig. 13*a*) can of course be constructed by the use of well-known graphical rules. It will be instructive to attack it wholly by the use of principles developed heretofore in this book.

A feature assisting greatly in mapping this field is one the student should get into the habit of looking for and using. The dotted-line asymptotes are referred to. The three dotted lines are spaced so as to divide their quadrant into equal angles, all being drawn from the center O, for, in the outer space far removed from the local region of the two wires, the field must become circular, with radial equipotential lines. With equal currents, each current is responsible for the same m.m.f., and all of the equipotential lines of the left-hand conductor must spread out over the left half of the total field. Then if it is desired to locate the three equipotential lines D, E, and F which will divide the quadrant into equal m.m.f. regions, these three lines must become asymptotic to the dotted lines at infinity.

The angles at O are all equal. This is evident, once corner-flux characteristics are remembered. AO, by symmetry, is a straight equipotential line; OG, for the same reason, is a straight

equipotential line. The interior angle AOG is bound to have symmetrical flux near the corner of the angle.

II. *Unlike and Equal Currents.*—As shown in Fig. 13*b*, all equipotential lines begin on one conductor and end on the other. The local field in the vicinity of the wires is fairly easy to

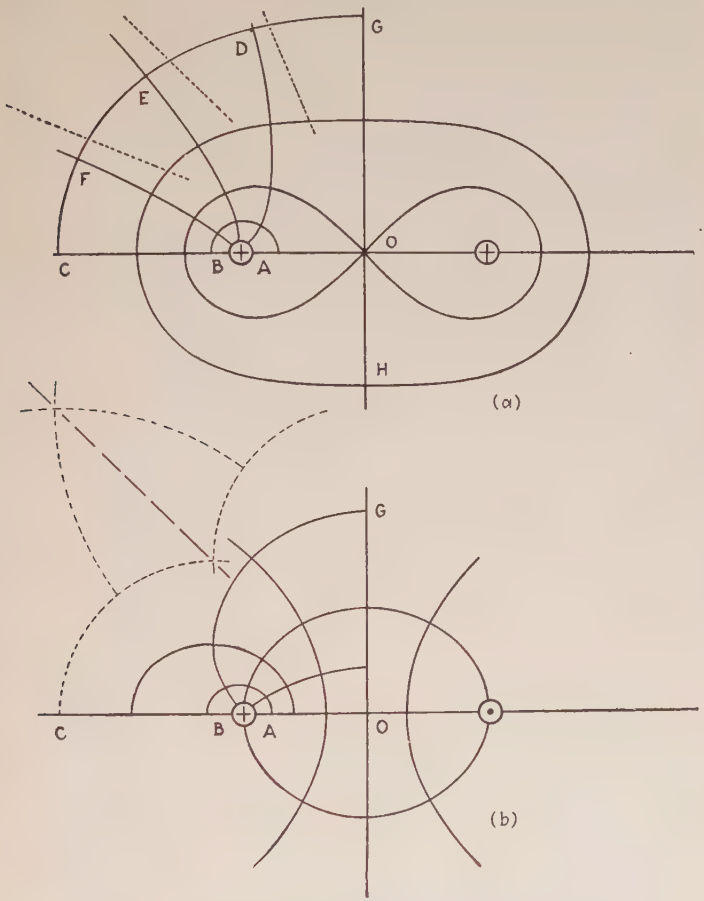


Fig. 13.—Fields of two parallel wires, equal currents.

sketch in an approximate way. But in getting the field in the outer region, a separate development is necessary. Note that CB is a straight equipotential line, that OG is a straight flux line, and that these two are at right angles. Moreover, all of the flux lines must enter the upper left quadrant through AO, and all of the equipotentials in this region cross OG. Turning back to Fig.

11*a*, the solution for the outer space is found already worked out. The shapes of the dotted lines in Fig. 13*b* were copied from the other map, and as the local field is extended outward, it ultimately must conform to the shapes of the ideal dotted-line field. That it is beginning to conform is easily seen from the map.

54. The Singular Point.—The point *O* (Fig. 13*a*) is a **singular point** in the field. A singular point may be defined or described in terms of the properties of the field thereat. Where an equipotential line makes a sharp bend, if the flux density at the bend is zero, then there is a singular point at the tip of the bend; or, when an equipotential line meets an equipotential surface, being thereby divided so that the line apparently follows the surface each way from the meeting point, that point is a singular point.

55. Conductors and Magnetic Bodies.—When a magnetic body having an equipotential surface is introduced into a field caused by a current, it may be thought of as warping the field so that it conforms to the new equipotential surface so introduced.

56. Body with Plane Surface; Straight Conductor.—To have a two-dimensional field, the conductor must run parallel to the plane face of the body. This case is already worked out; for in Fig. 13*a*, consider that the entire left half of the field, with its conductor, is wiped out, and that all this space is filled by an equipotential body. *HG* in the previous development was an equipotential line; it still is; therefore, the field due to conductor *BA* remains just as it was.

57. Conductor in a Slot.—A wire is placed centrally and near to the bottom of a slot (Fig. 14*a*). By symmetry, the equipotential lines *O* and 4 are vertical. The corners *A* and *B* are interior right-angle corners, the field formations in such a region already being known. The equipotentials 1, 2, and 3, which divide the m.m.f. between *O* and 4 into four equal parts, are bound to straighten up, become parallel, and occur at equal spacings, far above the vicinity of the conductor. It is seen from the map, which is done in only fairly good style, that the field becomes nearly rectilinear at a point about as far above the conductor as the slot is wide. Some further subdivision in the warped bottom region is shown by the dotted lines.

Figure 14*b* places the wire higher up in the slot. A problem of this nature is a bad one for the beginner to try, unless he is well versed in principles already developed; it then becomes relatively easy. Note that *ABCD* is one broken equipotential line, which

establishes a boundary exactly like the inside slot surface of Fig. 10*b*. Then in the region *ABCD*, and extending with fair accuracy nearly as far up as the point *A*, the field will be a duplicate of the field near the bottom of a slot (Fig. 10*b*); that is, *each half* of the present case will, near the bottom, have such a field. This fact gives the cue for the behavior of lines coming down past the wire and entering this region.

It is evident from either case that the flux below the conductor does not amount to very much. More will be said about this feature in the treatment of Commutation.

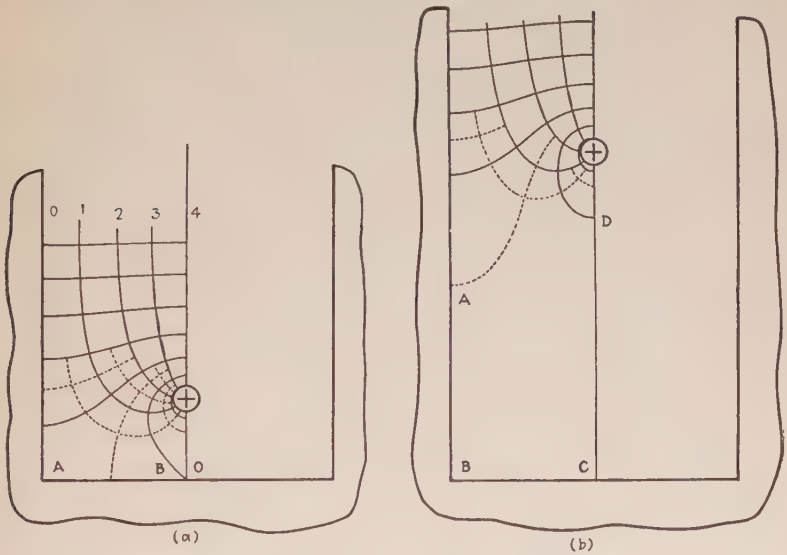


FIG. 14.—Field of conductor in a slot.

58. Division of M.M.F.—This important subject can be opened up by considering the simple case of the field about a long straight conductor. If no iron is present, the m.m.f. is symmetrically divided; half of it acts over each semicircle. With two parallel straight conductors carrying like and equal currents, the m.m.f. of each conductor is "thrown" to its half of the field; and in the upper left quadrant (Fig. 13*a*) one-eighth of the m.m.f. of the left conductor covers nearly 45 degrees in region *CEF*. In Fig. 14*a*, half of the m.m.f. is given to each side of the slot volume. Now consider Fig. 15*a*, a symmetrical case of a conductor in a slot in one equipotential body, and a nearby equipotential body located opposite. The air gap has equal right and left wings. If

these wings extend to infinity, the field will be symmetrical, with half of the m.m.f. given to each wing. By dividing the m.m.f. of the current into four equal portions, as shown, two of the equipotential lines become midpotential lines in the two wings of the gap.

But suppose that the wings do not extend to infinity, and that somewhere beyond the limits of the picture, they become unequal; what of the division of m.m.f. between them? A fundamental part of the theory demands that the flux set up by the current of the wire shall be a constant around the magnetic circuit. If the

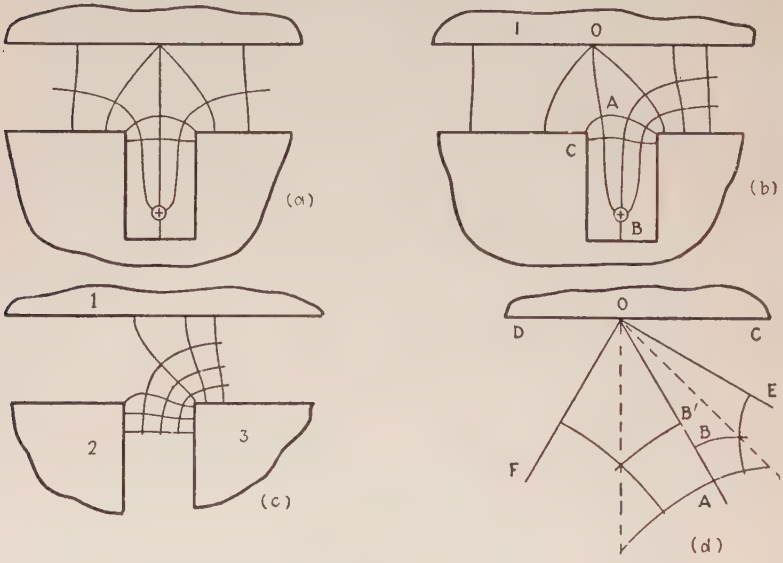


Fig. 15.—Slot and gap, with conductor.

flux in the slot and immediately above it is negligibly small compared with that in the wings of the gap, then the upward flux in one wing must equal the downward flux in the other. The wing flux is mostly rectilinear. If the left-wing reluctance is, for instance, one-third that of the right wing, to have equal fluxes, the left-wing m.m.f. must be one-third that of the right. Such a division of m.m.f. is indicated in Fig. 15*b*, where, with the m.m.f. of the wire divided into four equal parts, three parts are thrown to the right.

By carrying the preceding idea to an extreme another useful conformation is secured. Suppose the reluctance of the left wing

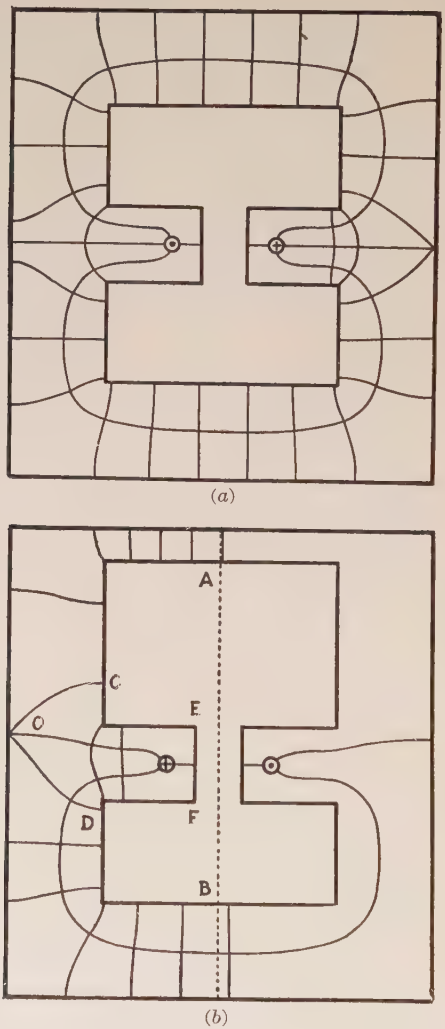
becomes a very small quantity compared with that of the right. Nearly all of the m.m.f. would be thrown to the right. The equipotential line to the singular point O then has nearly the same potential as has equipotential B. But B goes to the lower (slot-bottom) singular point; so B and C are of one potential. Hence, equipotential OA will hug the surface C very closely for a long distance to the left. Singular point O will carry far to the left; and the body 1 will have nearly the same potential as has surface C. In the limit, as when the upper body joins the lower one somewhere to the left, the statements just made that were nearly true become exact statements. The case degenerates into that of Fig. 15*c*, so far as the local slot region is concerned. Figure 15*c* was drawn as if bodies 1 and 2 had equal potentials, with body 3 at a different potential. The unmapped region in the left wing has a field that is already developed; it is simply the field in a slot of finite depth. The unmapped right-wing field becomes rectilinear.

In studying this general problem, the writer was confronted with the specific question, What angles must prevail at the upper singular point? That is, in two-dimensional fields, when one current causes one singular point to form on a nearby equipotential surface, what are the angles?

None of the principles brought out so far seemed to be usable in finding the answer. It is instructive, however, to know that in order to make progress in mapping this field, it was found by trying it that AO had to fall about normal to the upper pole face. To show that this is truly a right angle, consider Fig. 15*d*. This map represents the field in a small area infinitely close to the point O. The short flux line arc AO would be nearly straight. Assume that AO makes the acute angle AOC with the upper surface. Then two interior angle fields appear locally, in angles AOC and AOD. These fields are definitely fixed by the sides of the angles, which are their boundaries. Proceeding with the usual attack, flux line EO bisects its corner angle, and flux line FO does likewise to its corner angle. The dotted lines are the construction bisectors.

With the usual corner-flux construction as partially represented here, in the AOD region, half of the flux crossing AO crosses AB'. In the AOC region, half of the flux crossing AO crosses AB. But AB and AB' cannot be identical unless AOC is a right angle. This proves the proposition.

59. Unequal Division of M.M.F.—In Fig. 16*a* an outer equipotential shell surrounds an inner equipotential body having two slots; a conductor is in each slot, each acting as the return for the other's current. The case is symmetrical about a vertical and a horizontal axis. This insures without further inquiry that the



(*a*)

(*b*)

FIG. 16.—Division of m.m.f.

m.m.f. division will be symmetrical. The map shows as much; the m.m.f. of each wire is divided into four equal parts, one-half,

or two parts, going to each part of the gap. The inward flux in the upper gap equals the outward flux in the lower gap; the number of tubes in the map can be counted to check this statement. Thus the requirement that there must be as much flux leaving the inner body as enters it is fulfilled; that is, the flux in the magnetic circuit is a constant.

But in Fig. 16*b*, only vertical-axis symmetry obtains. The inner body has been expanded upwards so that the upper air gap is radically shortened. If the m.m.f. should still happen to divide equally, the flux crossing the upper gap would be much greater than that crossing the lower gap; this is impossible, and therefore the upper gap must be allotted less than half of the total m.m.f.

As a preliminary trial value, it was assumed that a 2:1 division of m.m.f. took place. This time, *three* equipotential lines are shown leaving the conductor to indicate that its m.m.f. is now arbitrarily divided equally into three parts. Two parts of m.m.f. are given to the lower and longer gap and one part to the upper gap. A tube of flux in the upper gap should now contain one square, as shown, while below each tube gets two squares. With this arrangement of the map, an upper tube must carry the same flux as a lower tube.

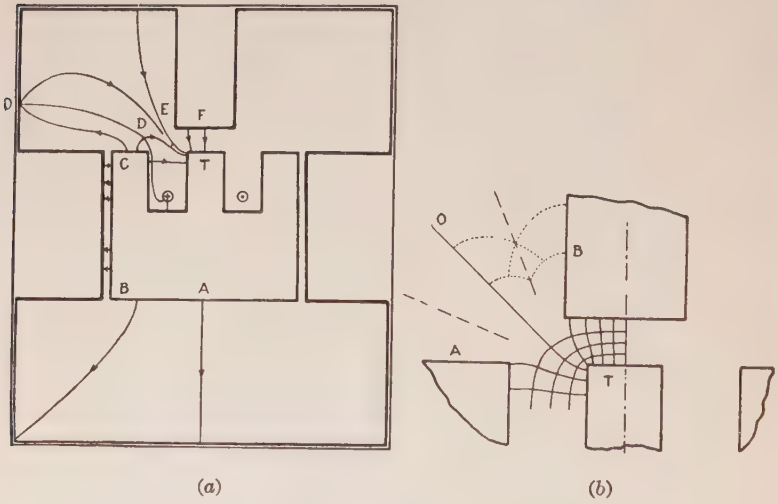
To check up on the 2:1 assumption, the flux in tubes between *C* and *D* can be left out, for the tubes in the region *OCEFDO* evidently have constant flux around their part of the circuit. Counting the upper tubes, there are about 5.9. The lower tubes between *D* and *B* number about 5.7. This is not an exact check, but it shows that the division of 2:1 of m.m.f. is nearly right; the next assumption would be in favor of slightly increasing this ratio to improve the guess.

60. Extremely Unequal Division of M.m.f.—Coming now to a practical case, Fig. 17*a* represents roughly the magnetic circuit of a direct-current machine. The two slots shown carry equal but opposite currents, and these slots are separated by a tooth which is directly under the interpole, or commutating pole. It is desired to evaluate the flux in the vicinity of the base of the interpole caused by the slot currents, for use in the treatment of commutation.

As a rough preliminary analysis, there will be outward flux from surface *ABC*, which encounters very low reluctance; whereas, the returning flux will be nearly all crowded into the tooth top *T* through a relatively narrow space in the air gap and thereabouts.

The inward flux then encounters a high reluctance. Hence, nearly all of the m.m.f. will be thrown across this inward path. A singular point in the left half of the case, which corresponds to the singular point O in Fig. 15*b*, in the present case is carried somewhere far to the left to some point O (Fig. 17*a*). This drawing represents the first rough analysis.

It is now possible to make the assumption, which holds good for all usual direct-current machines with about 10 per cent error or less, that *all* of the m.m.f. is given to the inward flux region near T. This brings the problem to the stage represented in Fig. 15*d*. Referring now to Fig. 17*b*, the tooth T is taken at one potential, and the two adjacent teeth and the base of the interpole are taken as at one other potential. The map shows the resulting field. The region AOB is of interest as



(*a*) (*b*)

FIG. 17.—Field of a slot current pair.

presenting the problem of an angle "less than zero" and the field therein.

61. The Two-conductor Circuit.—The general case of two parallel wires carrying equal but opposite currents is of frequent occurrence in analyses of all kinds of apparatus. The series of Fig. 18 shows several developments. In Fig. 18*a* a body of infinite permeability is inserted symmetrically between the conductors. In Fig. 18*b* the body is moved downwards. Note the change in positions of the singular points. In Fig. 18*c* the

body is expanded laterally. In Fig. 18*d*, three humps have been added to the body so as to place the conductors in slots. The body is changed to an open box in Fig. 18*e*. Finally, the box is

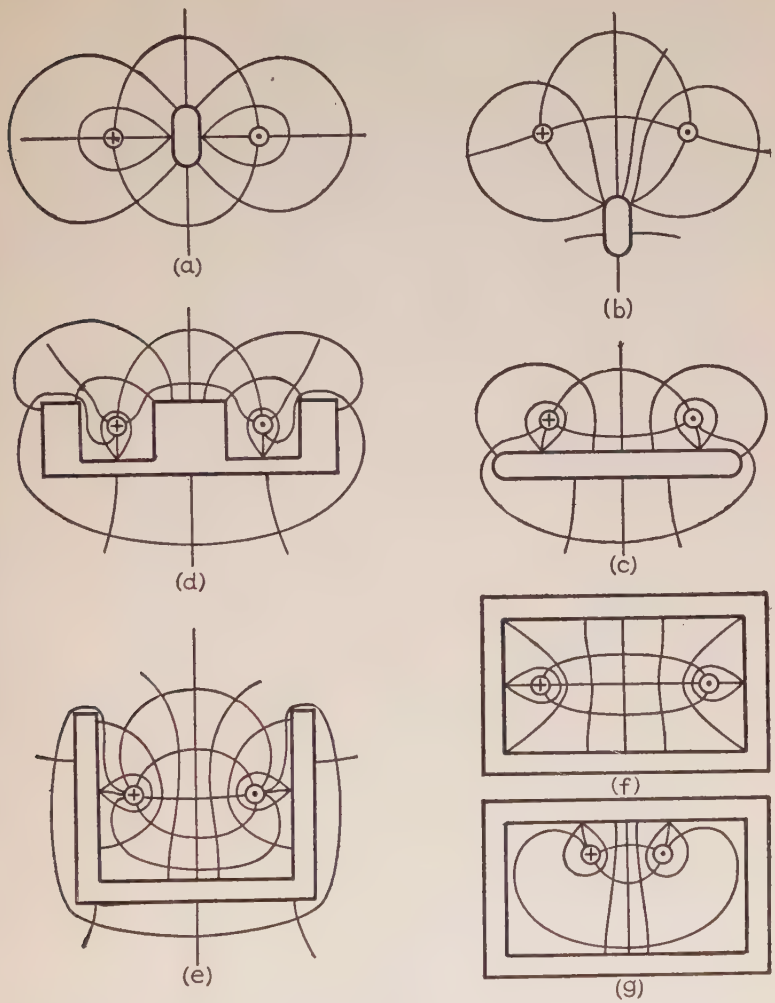


FIG. 18.—Preliminary field analysis.

closed in Fig. 18*f*, with the wires located symmetrically within it; in Fig. 18*g*, the wires are moved so that only vertical-axis symmetry remains. These drawings are not meant to be good field

maps; they show only enough essential lines to enable the mapper, first, to know that his preliminary analysis is correct and, second, to go ahead with the real job of mapping the field into curvilinear squares.

62. Preliminary Field Analysis.—It is often quite a problem to analyze the problem. In a complex field there are two distinct steps to be taken, as is indicated above. First, a mental or graphical analysis is made. If graphical, samples of every essential line of the field are sketched in. Many times, the essential-line analysis points out important conformation characteristics that become of high value in aiding the later attempts at real mapping. Second, the real job of mapping is attacked.

CHAPTER VIII

ATTRACTION

Certain characteristics of a two-dimensional field pertaining to the attraction exhibited for a pole face have been mentioned. The special problems arising in the handling of poles with sharp corner require that special developments of attraction cases be made.

63. Variation of Unit Force.—It has been proved that, for fields adjacent to iron of infinite permeability, the unit attraction is $B^2/8\pi$ dynes per square centimeter in the absolute system, or $B^2/(72 \times 10^6)$ pounds per square inch in the practical system. Thus force per unit area varies with the square of the density. In a subdivided two-dimensional field, the tubes of flux may end at a pole face, terminating there in a series or row of curvilinear squares; if the field is well enough subdivided so that each of these areas is nearly a true square, the attraction of a tube varies inversely as the width of its base at the pole. For the flux per tube is a constant; the average density then varies inversely as the width of the tube base; the average density squared, times the width, then varies inversely as the width.

As an example, in Fig. 10*b* there are two squares at the left wall of the slot marked 1 and 2. The width of 2 is about twice the width of 1. Then the attraction due to 2 is about half that due to 1.

It is thus seen that, once a field is mapped, if the attraction of one tube for its share of a pole face is found, all the attractions of the other tube are easily obtained. The exception to this statement arises when corner flux is present.

64. Attraction at a Corner. I. *Densities.*—Referring to Fig. 19*a*, tubes 1, 2, 3, and 4 carry equal fluxes. These tubes have finite base widths; therefore the *average* density is finite in each. But the actual density near the corner is very high, and at the corner is infinite. This is easily proved. *BD*, which carries the flux of tubes 1 and 2, is wider than *DO*, carrying the flux of tubes 3 and 4. In another way, of all the flux

between B and O, BD gets half, and DO the other half; and BD is the wider of the two parts. Then the average density over BD is less than that of DO. Likewise, DH gets half the flux between D and O, and DH is wider than HO; then the average

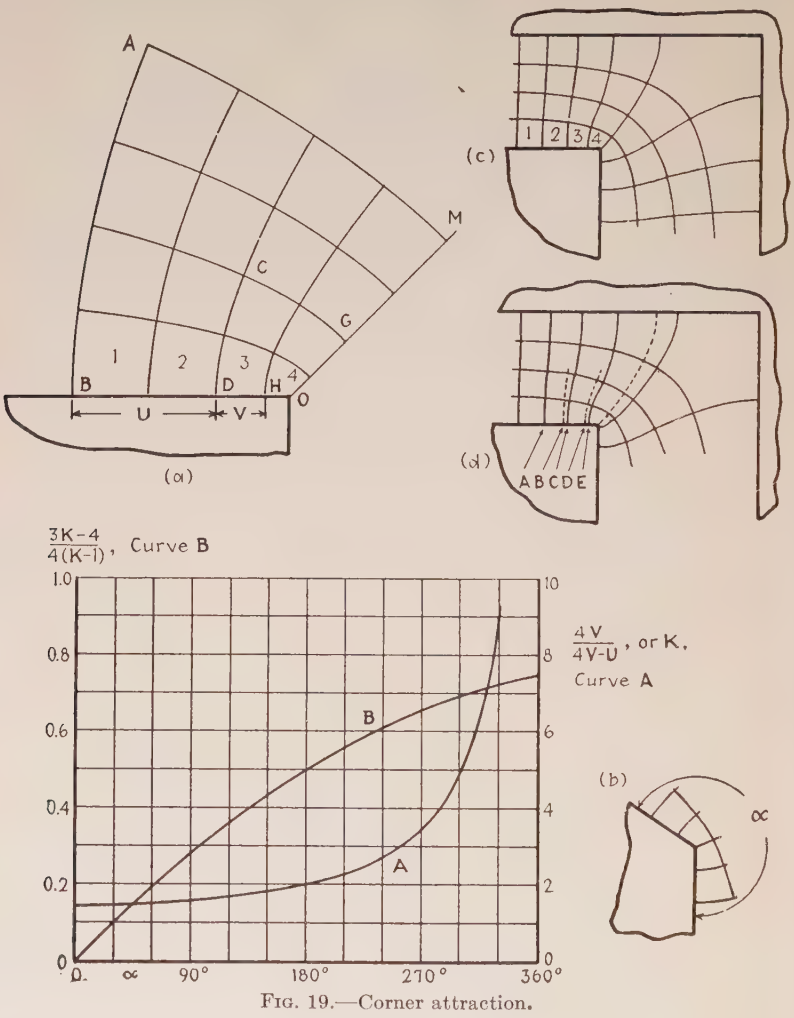


FIG. 19.—Corner attraction.

density over DH is less than that over HO. This process of splitting the flux between B and O into halves and then splitting the remaining half into halves may be continued indefinitely, and the remaining half of flux always exhibits an increasingly higher density average. The corner density must then be infinite.

But even so, this infinite density covers an infinitely small width; its attraction is then likely to be finite, but it must be evaluated.

II. *Similarity of Boundaries.*—Recalling the characteristics of this field formation, it is seen that boundary *BAMO* is similar to boundary *DCGO*. As similar boundaries can set up only similar fields in the same medium, the field in the smaller area is similar to that in the larger. In a graphical sense, if the lines of the smaller field could be photographed and then enlarged to a proper scale, they would coincide exactly with the lines of the larger field section. Interpreting this fact in terms of densities along *BO*, if a curve were plotted for the densities along *DO*, and if both scales of this curve were changed by the proper amounts, it would then exactly represent the densities along *BO*.

The tube ending on *DH* in the smaller area corresponds in every way to the tube (or double tube) ending on *BD* in the larger area. Let *BD* be divided into a certain large number of equal increments, and let *DH* likewise be divided into the same number of equal increments. Let U stand for *BD*, and V for *DH*. Comparing homologous increments,

dU is to dV as U is to V.
dV flux is one-half of dU flux.
dV density is $U/2V$ times dU density.
dV attraction is $(V/U)(U/2V)^2$ times dU attraction, or $U/4V$ times dU attraction.

Then total V-attraction is $U/4V$ times total U-attraction.

That is, *of all the flux from B to the corner, if the first half of it gives an attraction of* F_u, *the first half of the remaining flux has an attraction of* $F_u(U/4V)$.

If the constant $U/4V$ be called X, and if the succession of first halves of the remaining flux be continued indefinitely, the total attraction is the sum of the series.

$$\begin{aligned}\text{Force} &= F_u + F_uX + F_uX^2 + F_uX^3 + \cdots \\ &= F_u\frac{1}{1-X}. \\ &= F_u\frac{4V}{4V-U}.\end{aligned}$$

Values of $4V/(4V - U)$ are plotted in Fig. 19*b*, against α, the angle of opening between the faces of the equipotential body. The values for plotting this curve could have been obtained by

mapping the fields for several angles. Thinking that this curve might find some application, the author has taken values obtained mathematically, so that the curve is as good as is necessary for any practical case.

65. Variation of Tube Widths.—When a corner appears in a practical field map, the curvilinear squares near the corner, where densities are highest and therefore of greatest importance, unfortunately become so small that accurate mapping and measurement of widths become difficult. This trouble may be gotten around by making a separate large-scale map of the corner region, or by the use of the following development, accurate widths of tubes near the corner may be found in an easier manner.

Suppose (Fig. 19*a*) that a field is mapped up to flux line B, and that from B on it is assumed that the field becomes a true corner-flux field. It is desired to compute the tube widths from B to the corner, to obviate the necessity of measuring the widths of these increasingly smaller tubes. What does U, which has the first half of the flux, amount to in terms of the total distance BO?

Let K stand for the constant $4V/(4V - U)$.

Then

$$\frac{4V}{4V - U} = K.$$

$$4V = 4KV - KU.$$

$$V = \frac{KU}{(4K - 4)}.$$

Let Y stand for $K/(4K - 4)$.

Then

$$V = YU.$$

The distance BO is made up of a series, $U + YU + Y^2U + Y^3U + \cdots$

or

$$BO = U\frac{1}{1 - Y} = U\frac{1}{1 - \dfrac{K}{4K - 4}} = U\frac{4K - 4}{3K - 4}$$

or

$$U = BO\frac{3K - 4}{4K - 4}.$$

The values of $(3K - 4)/4(K - 1)$ are plotted in Fig. 19*b*. It then becomes an easy matter to subdivide corner fields accu-

rately; or the desired widths can be computed, for the purpose of finding attraction.

66. Solution of a Corner-attraction Case. I. *Corner-flux Line a Part of the Map.*—Referring to Fig. 19*c*, it is desired to find the upward component of force on the inner body, which has a corner angle of 270 degrees. The mapping was intentionally done so that the corner-flux line appears as part of the map.

By using the length of air gap to the left where the field becomes rectilinear and by knowing the ampere-turns acting between the pole faces, the density in the rectilinear field is computed. The attraction due to a rectilinear tube is found. The width of a rectilinear tube is, with this subdivision shown, one-fourth the length of the gap. The widths of tubes 1, 2, and 3 are measured at the inner pole face, and the attraction due to these tubes is computed.

Considering from the appearance of the field that tubes 3 and 4, where they end at the pole face, conform approximately to ideal corner flux, the width of tube 3 becomes U, in accordance with the previous treatment. As a check on the accuracy of the map or as an aid in the job of mapping the field initially, the value of $(3K - 4)/(4K - 4)$ is found from the curve (Fig. 19*b*) to be 0.65. This means that the width of tube base 3 should be 65 per cent of the widths of tubes 3 and 4 combined. This cannot be strictly true, for the field of the corner is warped from its ideal shape.

From the curve of K, or $4V/(4V - U)$, the value 2.7 is obtained. This means that if the attraction due to tube 3 is F_u, the attraction due to tubes 3 and 4 combined is $2.7F_u$.

II. *Corner-flux Line Not a Part of the Map.*—Figure 19*d* shows a case in which the treatment is not so direct as the preceding one. Here, the mapping was for some reason started somewhere to the left, and in the process of delineating the field around the corner, the corner-flux line did not appear.

The work of finding the attraction of each tube is carried out as before, up to the point E. Then beginning at the corner and working back, the dotted lines are drawn in, giving a supplementary map of the kind already dealt with so often in this book. These new lines are drawn with respect to the equipotential lines already present.

As BD is nearly equal to CE, and as the tubes ending on these two widths carry equal fluxes, the attractions for these two widths

is nearly the same. The attraction from B to O is now obtained. The total attraction is then that due to all tubes to the left of A, plus that due to a part of a tube AB, plus that from B to the corner O.

III. *Errors Due to Iron Saturation.*—In most usual cases, attraction for an iron surface can be computed on the assumption that the surface is an equipotential one. But near a corner, there is bound always to be high saturation, whereby the m.m.f. drop in the corner iron cannot with strict accuracy be neglected.

In an ordinary case, such as Fig. 19*a* might illustrate, the density in area 1 might be 80,000 maxwells per square inch. If the "iron" is good electrical steel, the ampere-turns per inch would here be far less than that for the air adjacent. Area 3, which is about two-thirds as wide as area 1, has about 120,000 for density. Even at this density, the m.m.f. gradient in the iron is very low; that is, iron surface BD will be very nearly a true equipotential surface. Taking a true equipotential line passing through the point H, as the line is followed to the right it would submerge more and more into the iron as the densities toward the corner become greater and greater. This line would curve around inside the iron at the corner on a small radius. Thus the actual density at the corner can never become infinite.

In spite of the actual difference between the real field conformation at the corner and the theoretical conformation, the map in general would not be much altered by corner saturation. As to the attraction, theory has it that, if attraction over BD is taken as 1, that for BO is 2.7. Of this total, DH has, roughly, 0.7 and DO has 1.7. This leaves something like 1.0 for HO, which is the part seriously affected by corner saturation. Thus 1 out of 2.7, or about 37 per cent, is in doubt. But in most cases, with normal densities and good steels, the error due to saturation would probably be less than 15 per cent. Whenever there is doubt, and if the case warrants it, the corner should be investigated in detail.

CHAPTER IX

THREE-DIMENSIONAL FIELDS WITH AN AXIS OF SYMMETRY

In a field having an axis of symmetry, a plane that includes the axis will display every typical flux and equipotential line; that is, a flux line starting at one pole face and lying in such a plane will stay within the plane and can therefore be mapped in it. Whereas, a twisted non-mathematical field has flux lines that twist through space, and to map them, a curved surface is required.

67. Characteristics of the Field.—As the field now dealt with has an axis of symmetry, it is unnecessary to make a complete map in an axial plane, for the two halves of the map would be identical. Figure 20 shows the case of an outer pole that is bored out to have a cylindrical cavity with a flat bottom; within it and extending down the cavity is an inner pole having the form of a right cylinder, and placed coaxial with the outer body.

It is now convenient to define a *tube of flux* in a special way. It may be said that the volume beginning on one pole in an annular or cylindrical or conical area and reaching to the other pole in such a way as to enclose always the same flux lines is a tube of flux. Thus, as in Fig. 20, if the upper end of the curved column CC is swept around the axis, it generates an annular or ring area; and the two flux lines bounding CC, when carried around with it, cut out or define the whole of the tube CC. The map then cannot show the whole tube, but only one of its cross-sections.

The map of such a field must of course consist of two sets of curved lines, crossing normally at their intersections.

If a field is sufficiently subdivided, then part of a tube, such as part 2, tube BB, has a length approximately equal to L_2 and an approximate area of $2\pi R_2 W_2$. As the 2π factor would apply to all such parts, the work can be simplified by saying that the reluctance of such a part is *proportional* to L/RW.

It is now easy to see that it is impossible to subdivide this field into curvilinear squares. If the tube BB had a section consisting of areas 1, 2, 3, and 4, each of which was a curvilinear square, then

the ratio L/W for each would be the same. But they all have different values of R; hence, to have equal reluctances in the four parts of a tube and, further, to have equal reluctances for the four parts of any other tube, L/W must change with R.

An interesting and sometimes useful characteristic also can be seen from the relations of R, L, and W. Suppose the field is correctly mapped into tubes of equal flux, and with equipotentials so placed as to divide the m.m.f. into equal parts. (So far as flux and potential are concerned, this is exactly what was always done in two-dimensional mapping.) Since each area in the map represents a volumetric part of a tube, each area represents the same amount of flux and the same amount of m.m.f. Then each represents the same reluctance, or L/RW is a constant over the whole map. Now, if the map be studied all along a line of fixed radius, R is constant, and so L/W is constant. All areas having their approximate centers at this radius would then approximately be similar rectangles. A field that is carelessly mapped gives itself away at once before such an inspection.

68. Method of Mapping.—Consider the pole-face boundaries of Fig. 20. The reader should realize that it is possible to put *any number* of orthogonal systems into these boundaries. From the magnetic standpoint, imagine that these boundaries, when leaving the plane of the paper in one direction, are allowed to curve in some peculiar way, but that they also curve away in the other direction in an identical way; that is, the boundaries merely are required to be symmetrical with respect to the plane of the paper. By symmetry, the field would be curved in only two dimensions in this plane and therefore could be mapped in this plane. As any number of ways of curving the boundaries in space are possible, any number of field maps in the plane are possible.

The problem presented here is to get the particular map resulting when the boundaries are cylindrical.

Either flux or equipotential lines may be sketched in first, in the preliminary trial. After sketching what seems to be a pretty good set of lines of each kind, lines can be added or shifted until about the degree of subdivision shown in the drawing is reached. The intersections deserve particular attention to see that this part of the job, at least, is done with a relative degree of perfection.

Having obtained what is so far simply an orthogonal system of curves that fit the boundaries, it remains to check them in some

manner that either will prove them to be right, or if wrong, will suggest how to correct them. In the first place, the tubes of the preliminary map do not have equal flux values, except by sheer

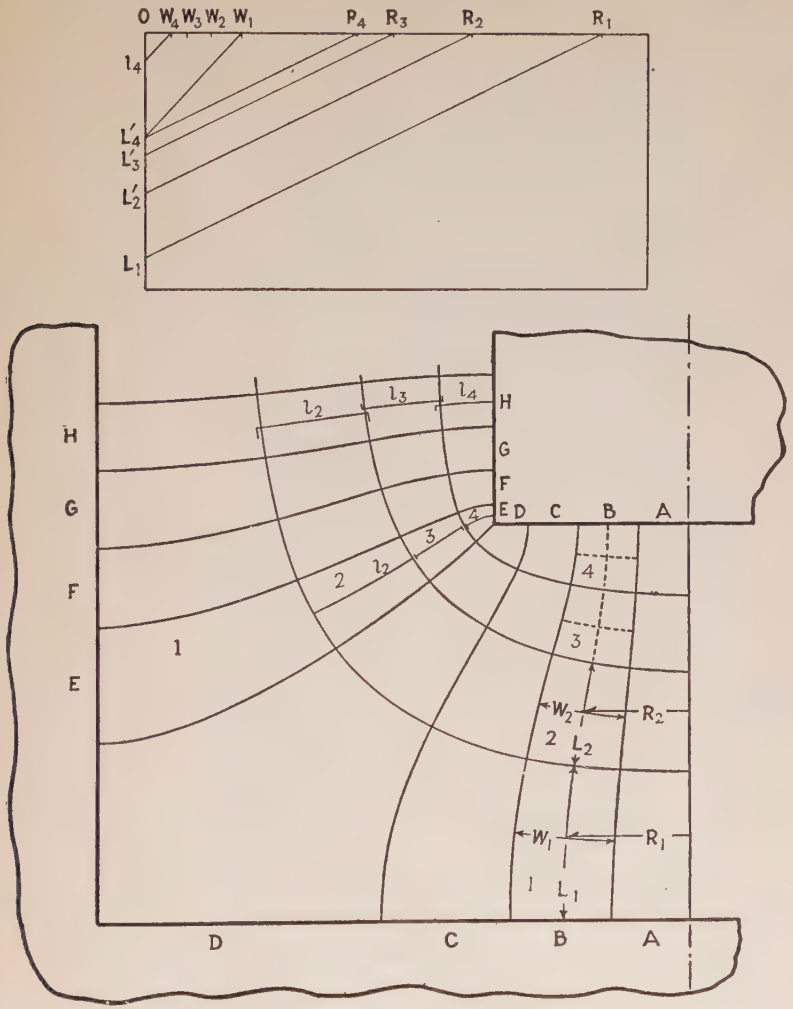


FIG. 20.—Three-dimensional field.

accident. It is not at all necessary, however, that tubes of equal flux be gotten now or at any time. In the second place, it is not known whether the equipotentials divide the m.m.f. into equal parts, which is a thing to be attained. If now the reluctances for

the parts (in this case, four) of a tube are obtained, and if the several values are all equal, then at least in this tube, the m.m.f. is correctly and equally divided. If further every tube is so investigated, and in every case, the several values of reluctance within a tube are equal, then the map is correct in every way. Note that, as the flux lines were chosen at random, it cannot be expected that the reluctance values of one tube should equal those of any other tube.

To obtain reluctance values, or rather, that figure which is just as useful, L/RW, it is in theory necessary to find what might be termed the *effective center* of each part tube. Any such area has curved boundaries; so it really has no one radius, no one length, and no one width. But there is no question but that it has a definite reluctance and that there is some point within its boundaries—its effective center—so placed that if R is taken there, if L is measured along the flux line through it, and if W is meaured along the equipotential through it, then L/RW so obtained would exactly represent the reluctance.

Now suppose that exact subdivider lines, such as the dotted lines in $B3$ and $B4$, are put in; that is, within such an area, the exact midpotential and the flux line that divides the flux in two are put in. Their intersection point *is not*, except by accident, the effective center. But as can be shown, if the area is not far from being a rectangle, this intersection is very close to the effective center and may be so used.

Thus the tube BB may be investigated, first by running in the subdividers and then measuring L, R, and W for each part of the tube. L/RW is now calculated. In Fig. 20, if the four values came to 4.00, 4.10, 4.15, and 3.95, it would indicate that this tube is nearly correct, *provided* that in making changes in other parts of the field, this tube does not also come in for some changes.

The present case has eight tubes shown, each with four parts. This means that 3×32, or 96, measurements, must be made and tabulated; and that 32 calculations of L/RW are ahead. When the work is finished, the mapper is faced with a lot of values which, if studied intensely, will really indicate to him how to shift lines to better positions, after which the previous work is done all over again. At best, the job is very laborious and without inspirational value as a saving factor.

69. The Graphical Field Check.—To avoid this labor and to speed up the work, the author has devised a simple graphical

method by which no measurements are read and recorded, and no calculations are made. All that is needed is a pair of triangles and a piece of paper, or preferably a stiff card such as is outlined by the rectangle at the top of Fig. 20. The example shown on the card is for tube *EE*.

The card is placed with the corner O at the centers of the four areas in succession, the tube previously having been subdivided, with the upper edge running horizontally and crossing the axis. Thus the points R_1, R_2, etc. are marked on the upper edge, so that the radius R_1 lies between the corner O and R_1, etc. The card is then rapidly shifted across the parts of the tube so that the tube part widths are laid off from O on the upper card edge to the points W_1, W_2, etc. The length L_1 is likewise laid off along the left-hand edge from O. The line R_1L_1 is drawn, and the three lines parallel to it are drawn from the other three R points, as shown.

The assumption is now made that tube part $B1$ is perfect, so as to criticise the other three parts with respect to it. Part $B2$, for instance, may be too wide, or too long, or both; or too small either way, or both; or a combination of these evils may exist. Little can be accomplished by trying to think of all these things at one and the same time. So the next temporary assumption is that the *widths* of the other three tube parts are also perfect.

To investigate and criticise part $B4$, draw line W_1L_4'. If W_4 happened to equal W_1, then since L_1/R_1W_1 should equal L_4/R_4W_4, L_4' by the parallel-line construction is the proper length for part $B4$. But $B4$ has really its own width, W_4. To keep the above equality of reluctances, its length must be to L_4' as W_4 is to W_1. The line W_4l_4 is drawn from W_4, parallel to W_1L_4'. l_4 is thus found, as is transferred to part $B4$ and marked thereon. A method of marking is better shown in tube *III*, where the spaces are more open to the eye.

Discussing the results with respect to tube *III*, the part $II1$, was used in the above manner as a reference standard, and all of its widths were assumed to be correct. The lengths 1_2, 1_3, and 1_4 were marked on as shown. They indicate that each of the parts 2, 3, and 4 are too short; that parts 2 and 3 are about all right *with respect to each other;* and that part 4 is short compared to either part 2 or 3. All these remarks may be inverted, for any one of these parts might have been used as a standard against which to criticise the others.

If parts 2, 3, and 4 are given all of their apparently desired lengths, by moving the equipotentials outward to that extent part 1 will be shortened entirely too much; and remember that there is a compound effect; that moving the equipotentials requires that the flux lines be moved also, and it is very easy to overcorrect the field.

Tube *EE* has been marked similarly, finding that, with respect to part 1 as reference, part 2 is correct, part 3 is long, and so is part 4.

In tube *DD* it would be unwise to use the outer part as a reference, because of its shape. An inner and more rectangular part should be used, and to check the warped outer area, it should first be broken up into four smaller areas, provided the weak field here is of much importance.

As a general comment, the map in Fig. 20 is already well towards a good stage of perfection. It can of course still be improved upon, but it would require careful work to do it. One needs only to attempt to map his first three-dimensional field to acquire a vast amount of respect for the undertaking.

70. Corner Flux.—What angle will the corner flux line make with the adjacent surfaces in this case? If a small volume be considered which included the corner, the smaller the volume, the more nearly flat the surfaces of the pole inclosed in the volume will appear. In the limit, the surfaces become flat. Therefore, infinitely close to the corner, the corner-flux line would, if extended, bisect the corner angle. In Fig. 20 the line emerges at 45 degrees with the horizontal or vertical.

Another important conclusion due to the same reasoning is this: the three-dimensional field degenerates into a two-dimensional field near the corner.

71. Flux Computations.—Once the field is mapped and checked to the desired degree of accuracy, calculations are easily made. The m.m.f. over each tube part becomes known, and the approximate length of the part being measured, the approximate density within is calculated.

72. Attraction.—The inner pole in Fig. 20 might be the plunger in an electromagnet, in which case the vertical attraction on its lower surface must be computed. Knowing the average density at the base of a tube and the annular area of that base, the force of the tube is found. This process is carried out for all tubes from the axis to a point near enough to the corner to enable the

remainder from there to the corner to be considered as a two-dimensional field. In Fig. 20, the upper end of tube *DD* gives approximately a two-dimensional field. The width of the upper end of *DD* must of course be reasonably small compared to its distance from the axis, for this to be true. Then, speaking in terms of two-dimensional development, the total attraction of the corner tube is figured for an axial inch. There is actually no such thing here as an axial inch; there are various radii at which a perimetral inch may be taken. But if the attraction for an axial inch (or centimeter) is gotten, that figure times the average perimeter of the tube surface at the pole gives approximately the corner tube attraction.

CHAPTER X

MAGNETIC MATÈRIALS

This book is not supposed to be a text on electrical machine design; instead, it is written with the intention of developing much of the theory underlying the designer's work and of presenting some of the types of problems the designer is called upon to solve. It would therefore be out of place here to insert a comprehensive survey of magnetic materials. Only enough basic facts and data are given to enable the further work of the book to be carried out. The student wishing for more complete data can easily refer to any recent edition of one of the electrical handbooks. The practicing designer who may read this book will have at his elbow much more complete, new, and authentic data than could be printed here.

73. Paramagnetic Materials.—These include all materials having a permeability greater than that of free space, but the term paramagnetic usually is restricted to apply to that great number of materials in which the permeability is only slightly greater than unity. For all ordinary purposes, all materials except those called ferromagnetic may be considered as having unit permeability.

74. Ferromagnetic Materials.—These include all distinctly magnetic substances, as nickel, cobalt, and certain of their alloys; the non-ferrous Heusler alloys; and iron and its alloys. The only materials of industrial importance, because of the cost consideration, are various iron alloys.

75. Diamagnetic Materials.—Primarily bismuth and some few other materials have permeabilities slightly less than unity. They are of no engineering consequence.

76. Retentive and Non-retentive Materials.—Commercial materials may be classed in one or the other of two groups. The retentive materials, notably tungsten, chromium, and molybdenum steel alloys, have high retentivity, being used only in such applications as permanent magnets. The non-retentive materials, such as ordinary cast iron, malleable cast iron, ordinary

electrical steel in cast, rolled, or sheet form, etc., are of low retentivity.

77. Electrical Sheet Steel.—The various commercial electrical sheets, rolled from different steel alloys, go by many trade names. An indication of the different grades available, and their characteristics, is given in Table II. The makeup and properties of these listed sheets are as follows:

I. *Swedish Charcoal Iron.*—Carbon content, 0.163 per cent. Given in the table as a standard with which to compare the other materials.

II. *Medium-grade Sheet.*—Carbon less than 0.1 per cent; manganese less than 0.3 per cent; silicon and other impurities low. Is a slightly aging sheet, suitable for laminated fields.

III. *Armature Sheet.*—A medium silicon alloy, not enough silicon to make it too brittle for armatures and rotors. Suitable for all such applications.

IV. *Four Per Cent Silicon Steel.*—Too brittle for armatures. Used for transformer cores.

TABLE II.—ELECTRICAL SHEET

Material	Approximate resistivity at 20°C., microm-centimeters	Hysteresis coefficient
Swedish charcoal iron	11	
Medium-grade sheet	10	0.001 61
Armature sheet	17	0.001 33
Four per cent silicon steel	45	0.000 71

78. Hysteresis Loss.—Hysteresis loss depends upon maximum density, frequency, and a constant expressing the hysteretic quality of the material. This constant, the hysteresis coefficient, is given in Table II to indicate the relative qualities of usual electrical sheets. Hysteresis loss, for practical work, is best found by use of data furnished by the manufacturer of the sheet used. Furthermore, this loss is usually not separated from the eddy-current loss; the two are lumped together by means of such a curve as that of Fig. 21, which stands for the iron loss of armature sheet operating under usual conditions.

79. Eddy-current Loss.—In perfectly laminated magnetic paths, the eddy currents are confined to the individual sheets; the loss then depends on the resistivity, density, and frequency.

To study the eddy-current problem in a single lamination, imagine a lamination 0.014 inch thick and 0.14 inch wide, so that its section is a 10:1 rectangle. Assume that the flux density is spread uniformly over the section. Divide the section into a large number of small squares. If the lamination forms, for instance, part of a transformer core, it may be assumed for purposes of present study that the flux variation is sinusoidal. It is now possible to treat the *component* eddy current set up by *one* of the small squares in the section.

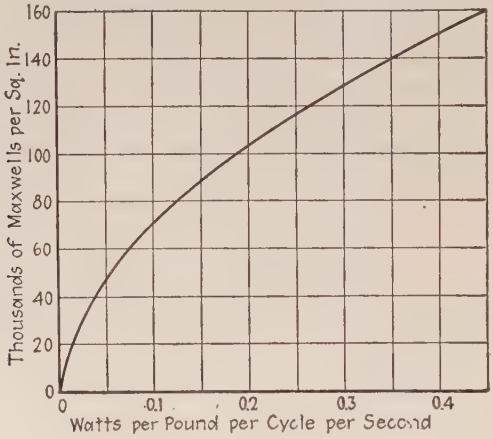


FIG. 21.—Iron loss, rotating machinery.

The small square represents a tube of varying flux, and the flux variation would induce around it some definite voltage which would set up a current in the iron which could be represented in the section by closed streamlines linking with the tube. Now, assuming constant resistivity, the current and equivoltage line system would conform with the flux-equipotential lines that would be set up in a certain substitute system, namely, the case of a square filament of current placed in a medium of constant permeability, the medium having a rectangular section like that of the lamination, and with zero permeability outside the medium. The latter substitute system can be mapped in the usual way as a two-dimensional field. Thus the eddy currents could be mapped for a single square tube of flux. Those due to

other squares could then be found and superimposed upon the first, and the current densities at each point could be added vectorially.

Such a study as this indicates clearly that in a given neighborhood in a lamination, the eddy-current strengths depend to a very large extent only upon the flux densities found in any direction within five times the thickness. In other words, the eddy-current loss in a small piece of lamination is pretty well determined by the flux density at and near to the small piece.

The usual eddy-current theory presented for the lamination case is built up for a lamination infinitely wide. This special case becomes very easy. Imagine the section before us, with an axis located centrally and running to infinity either way, horizontally. The thickness of the lamination is then placed vertically. Let two horizontal, parallel lines be placed at equal distances above and below the axis in the section. By symmetry, current set up to the right in one line would have to return in equal quantity along the other. The voltage setting up such a correct filament would be proportional to the flux found between the two lines, which in turn is proportional to the spacing between them. Thus current density in the section is proportional to the distance from the axis.

If the thickness is doubled, the current density at the edge is doubled, but the losses per unit volume there will be quadrupled; and the average loss per unit volume is quadrupled. The loss per lamination is eight times as great. Thus, in a given core, if the laminations are doubled in thickness, but the core section is retained, the loss per lamination is eight times as great; but, there being half as many laminations, the net loss is only quadrupled. Eddy-current loss then varies with the square of the thickness of lamination used. It is easy to see that it also depends on the square of the frequency and the square of the density.

The above statements depend on the assumption that the eddy currents do not cause a material redistribution of flux density; this assumption is usually fairly well realized in low-frequency apparatus.

80. Notes on Iron Loss.—Volumes have been written on this subject. The facts concerning iron loss may be summed up in the statement that if it is important for the iron loss to be predicted with accuracy, the individual problem must be given

detailed and individual treatment. In manufacturing the electrical sheet, variations occur that ideally could be controlled, but narrow control limits mean high cost. The sheet is taken by the apparatus manufacturer, who inevitably must handle it; handling means some bending, at least, and bending alone raises the loss. The sheet must then be punched, which adds stresses that are removed only by subsequent annealing. After annealing, all sheets operating continuously at usual frequencies are given a coating of varnish to add to the surface insulation already furnished to a fair degree by the oxide acquired in the annealing ovens. In assembly, more bending occurs, which never lowers the loss and often raises it. The final act of assembling laminations is to rivet or clamp them tightly together. The tighter the assembly, the greater will be the leakage of eddy currents from one sheet to the next; the looser the packing, the greater the vibration in service, which is undesirable from every standpoint. Armatures are normally held together with about a hundred pounds to the square inch. If the sheet goes into a stator or rotor with coils to be wound into slots, or if the air gap is small, filing the slots or grinding the rotor surface may be necessary, either of which operations will create edge contacts between punchings and will raise the eddy-current loss. Dull dies leave burrs on the edges of the punchings to cause the same trouble. It is no uncommon event for iron losses to vary from predicted values by as much as 30 or 50 per cent.

81. Notes on Electrical Sheet.—Armature and transformer cores are built up of punchings 0.014 inch thick; low-loss transformers sometimes use a thickness of 0.017. Pole pieces are built of sheets of varying thickness, ranging from 0.025 to 0.06 inch.

The *stacking factor* expresses the amount of active iron present in a laminated structure. For 0.014-inch laminations, annealed and varnished, the stacking factor is about 0.93.

The weight of steel, not including oxide or varnish, may be taken as 0.283 pound per cubic inch. The iron loss curve of Fig. 21 is given in terms of loss per pound of laminations; the weight of a cubic inch of laminations may be taken as 0.28 with sufficient accuracy.

82. Saturation Curves.—Figure 22 gives typical saturation curves in the practical system. These curves are merely typical of usual materials; those wishing more exact information con-

cerning a given grade or make of magnetic material should refer to a handbook, or to the manufacturer.

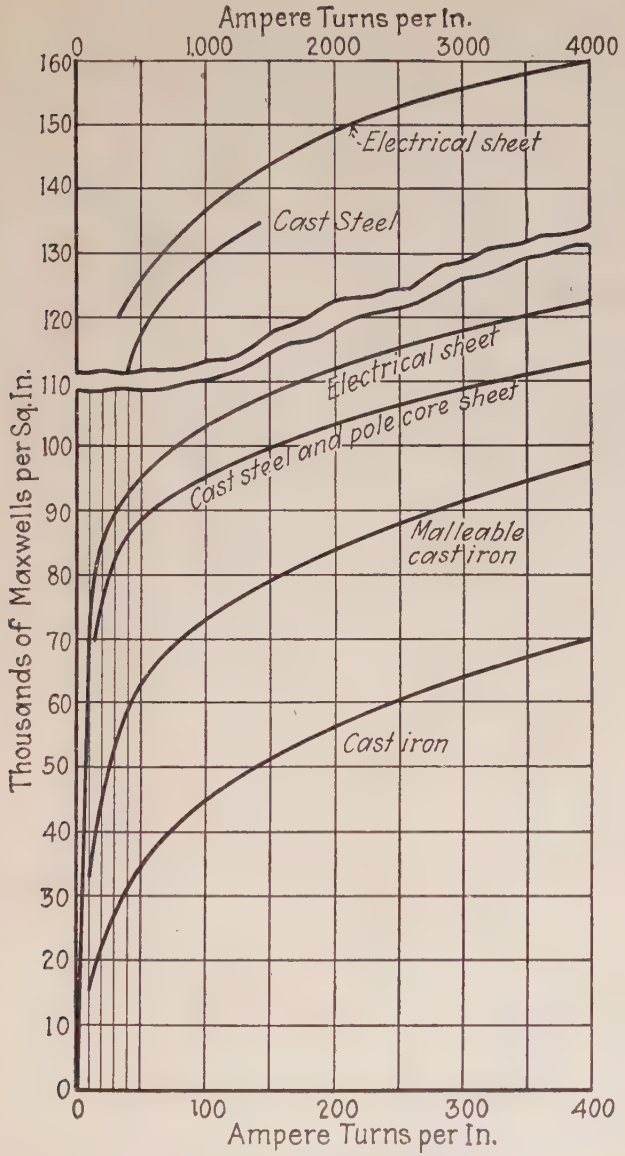


FIG. 22.—Magnetization curves.

CHAPTER XI

THE INDUCTIVE CIRCUIT; THE ANALOGOUS PROBLEM IN HEATING

The time required for the current to change in an inductive circuit often becomes a factor of importance to the designer. For instance, the swiftness with which an electromagnet will act depends upon how fast the flux builds up. A similar problem is found in the rate at which a piece of apparatus will rise or fall in temperature. The general treatment of these two problems should therefore be developed at the same time.

83. Voltage-current Relations in the Inductive Circuit.—Let a circuit having a resistance R and a constant inductance L henries have applied to it a constant voltage E at a given time, this time being taken as zero.

Students so often indicate confusion as to the signs in the general equation

$$E = iR + L\frac{di}{dt}$$

that a short discussion of the matter will not be out of place. Using the positive sense of E as the positive sense in the circuit, the current begins to rise in the positive sense; then di is positive. The "increase" in time is positive, if any sense whatever is ascribed to it. L is a number. Then the last term in the equation is positive on its own merits. The first term, iR, is of course positive if i is positive. All the equation really does is to state how the applied voltage is distributed over the circuit, and the two terms on the right-hand side are simply two parts of a whole. The second term exists by virtue of the fact that the changing current gives a changing flux, which induces a voltage. This voltage is either with or against the current. If with, it would tend to increase it, which is easily shown to be absurd. If against, the induced voltage is negative. A positive part of the applied voltage is required to equalize this negative induced voltage; the remaining positive part sends the current through the resistance.

Note that it was said above that the induced voltage was *equalized* by part of the applied; many people use the term *overcome*. This is an incorrect usage of the word and should be avoided. If something is overcome, the overcoming factor is, by implication, greater, and such is not here the case.

The general equation is then

$$E = iR + L\frac{di}{dt}. \tag{1}$$

84. The Time Constant.

If

$$R = 0,$$
$$E = L\frac{di}{dt}.$$
$$dt = \frac{L}{E}di. \tag{2}$$

Integrating from $t = 0$ when $i = 0$ to $t = T$ when $i = I_0$,

$$T = \frac{L}{E}I_0.$$

Bringing R back into the problem,

$$I_0 = \frac{E}{R}, \text{ where } I_0 \text{ is the ultimate current.}$$
$$T = \frac{L}{E}\frac{E}{R} = \frac{L}{R} \tag{3}$$

$T = L/R$ is called the **time constant** of the circuit; the current has actually a final value of $I_0 = E/R$, but L/R gives the time in seconds it would take the current to rise to this value if impeded by inductance only.

85. Rise of Current.—Rearranging Eq. (1),

$$\frac{E - iR}{di} = \frac{L}{dt},$$
$$\frac{Rdi}{EiR} = \frac{Rdt}{L}.$$
$$\int \frac{Rdi}{E - iR} = \int \frac{R}{L}dt.$$

$$-\frac{R}{R}\log_e(E - iR) = \frac{R}{L}t + C, \text{ a constant of integration.}$$

when $i = o$, $t = o$ (to evaluate C).

$$-\log_e E = C.$$

$$-\log_e (E - iR) + \log_e E = \frac{R}{L}t. \tag{4}$$

$$\log_e \frac{(E - iR)}{E} = -\frac{R}{L}t.$$

$$E - iR = Ee^{-\frac{R}{L}t}.$$

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right). \tag{5}$$

$$= I_0\left(1 - e^{-\frac{R}{L}t}\right). \tag{5a}$$

If t is taken equal to L/R, the time constant,

$$i = I_0(i - e^{-1}).$$

$$= 0.632\, I_0. \tag{6}$$

Thus it is often said that the time constant is the time in seconds taken for the current to reach 63.2 per cent of its final value.

If in Eq. (5a) i is taken as $\frac{1}{2}\, I_0$,

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}t}.$$

$$\log_e \frac{1}{2} = -\frac{R}{L}t.$$

$$t = 0.693\frac{L}{R}, \tag{a}$$

or, in $t = 0.693L/R$ seconds, the current becomes one-half of full value. In Eq. (5a) if i is taken as three-fourths of I_0, then seven-eighths, etc., thus cutting the remaining half of the final current value in two, the total times taken will be respectively twice, three, four, etc. times as much as the first interval amounted to; that is, if the final current will be 16, and if in 2 seconds the current rises to 8, then in 4 seconds it rises to 12; in 6 seconds, to 14; in 8 seconds, to 15; etc.; or more simply, with final value $I_0 = 16$, if 2 seconds are used up in going to 8 amperes, the next 2 seconds will add 4 amperes; the next 2 seconds, 2 amperes, etc.

All of these characteristics are shown in the curve of current rise (Fig. 23a).

86. Current Decay.—Suppose the current has finally risen to a steady value I_0, due to E volts acting on R ohms. The work

immediately following, however, holds true for any current i. Let the circuit be short-circuited. The fall of current would now induce a *positive* e.m.f., which would *tend* to maintain the current, as it is usually stated. This e.m.f. actually *does* maintain the current, for it is the only e.m.f. in the circuit, being now equal at all times to the iR drop; however, it does not maintain it at a constant value. Therefore,

$$O = iR + L\frac{di}{dt}. \tag{7}$$

$$-\frac{di}{Ri} = \frac{dt}{L}.$$

$$\log_e i = -\frac{R}{L}t + C, \text{ a constant of integration.} \tag{8}$$

Evaluating C, when $t = 0$, $i = I_0$.

$$\log_e I_0 = C.$$

$$\log \frac{i}{I_0} = -\frac{R}{L}t.$$

$$\frac{i}{I_0} = e^{-\frac{R}{L}t}.$$

$$i = I_0 e^{-\frac{R}{L}t}. \tag{9}$$

If

$$i = \frac{1}{2}I_0,$$

$$\frac{1}{2} = e^{-\frac{R}{L}t},$$

and

$$t = 0.693\frac{L}{R},$$

or again, the same time is required for the current to make one-half of its ultimate change. As before, half of the remaining change will be made in the next and equal interval of time.

The curve of current decay (Fig. 23*b*) illustrates the above features.

87. Statement of the Problem of Heating. I. *Similarity to Inductive Circuit Problem.*—Passing now to the analogous case of the heating of a piece of electrical apparatus, it is first necessary to idealize the actual conditions somewhat. It is assumed that the piece is a compact body which, upon being put into service, generates heat within it at a constant rate; for example, a flat

iron, a coil on constant voltage, or a motor armature on constant load. (The rate actually and usually varies somewhat as the temperature changes.) It is also assumed that the heat stored, due to the thermal capacity of the body, is a constant amount per degree Centigrade; that the body will dissipate heat at a constant rate for each degree difference in temperature between

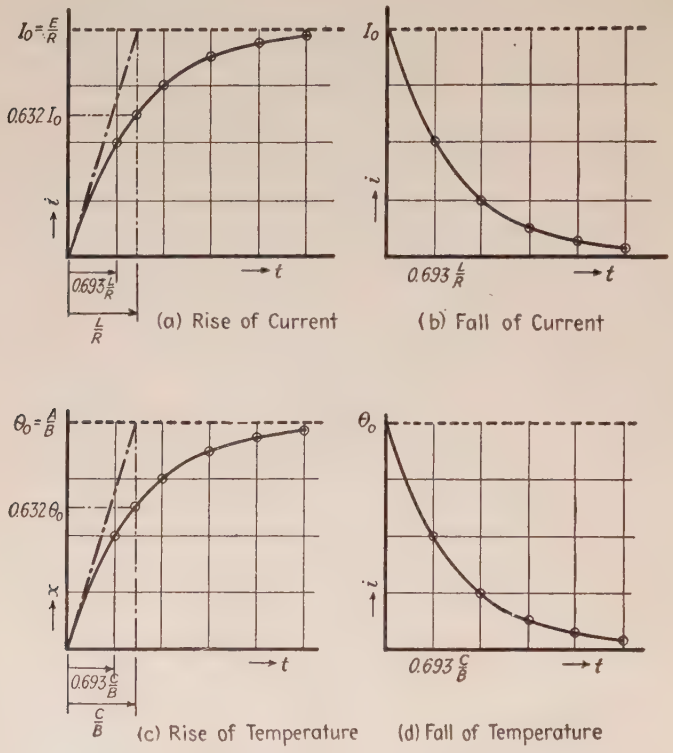


FIG. 23.—Current in an inductive circuit, and temperature change of a body.

it and the surrounding air; and that the temperature of the surrounding air is unchanged. The notation follows:

Constant rate of heat production, units per second..... $= A$
Rate of heat dissipation, units per second per degree centigrade difference in temperature..... $= B$
Heat storage capacity, units per degree Centigrade rise $= C$
Temperature rise (above surrounding air), degrees Centigrade..... $= \theta$
Time elapsed after beginning, seconds..... $= t$

At some time after the beginning or zero time, the temperature will have risen to θ degrees above that of the air, which was also the temperature of the body; a time of t seconds has elapsed; in a differential amount of time dt, the heat produced, must equal the heat dissipated plus the heat stored, or

$$A dt = B\theta dt + C d\theta. \tag{10}$$

$$A = B\theta + C\frac{d\theta}{dt}. \tag{10a}$$

This problem in its several aspects has already been solved; compare with Eq. (1). By replacing the constants and variables of this equation with A, B, C, and θ, t being common to both cases, the solutions for the inductive circuit can be adopted bodily. Figure 23*c* and Fig. 23*d* show the succeeding treatments.

II. *Time Constant.*—The time constant becomes

$$t = \frac{C}{B} \tag{11}$$

which is now the seconds required for the body to attain what will be its final temperature, if, instead of being allowed to dissipate heat, it is perfectly insulated so that all heat is stored; that is, the body will attain some final temperature Θ_0; but it will require an infinite time to do it in. If the body were insulated perfectly, it would attain that temperature by heat storage alone, in C/B seconds.

Note the character of C and B; C is the storage capacity in units per degree rise in temperature; B is the heat-dissipation rate in units per second per degree rise. Then C/B gives seconds.

Interpreting the time constant in terms of the actual case, it will take the body C/B seconds to attain a temperature rise of 63.2 per cent of the final temperature rise Θ_0.

III. *Rise of Temperature.*—The temperature equation is

$$\theta = \frac{A}{B}(1 - e^{-\frac{B}{C}t}), \tag{12}$$

or, using the relation found by Eq. (6*a*),

$$t = 0.693\frac{C}{B}, \tag{13}$$

whereby, in this many seconds, the temperature rise has gone to one-half its final value; in as much more time, it will have gone to three-fourths of its final rise; in as many more seconds, seven-eighths, etc.

IV. *Final Temperature.*—The final temperature rise Θ_0 will be such as will cause the heat to be dissipated as fast as it is produced, or

$$\Theta_0 = \frac{A}{B}. \tag{14}$$

V. *Cooling Off.*—After the body has attained a temperature rise Θ_0, if the source of heat production is stopped, the stored heat will gradually be dissipated until the body arrives at the temperature of the surrounding air. Corresponding to Eq. (7),

$$O = B\theta + C\frac{d\theta}{dt}. \tag{15}$$

$$x = \Theta_0 e^{-\frac{B}{C}t}. \tag{16}$$

$$t = 0.693\frac{C}{B},$$

which is the time required for the temperature rise to drop one-half of its value.

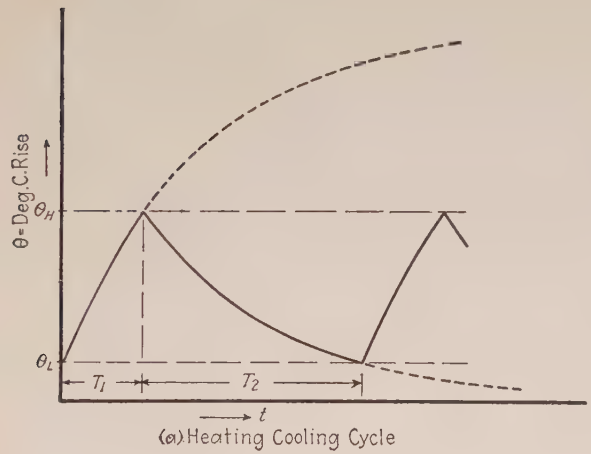
88. Heating-cooling Cycle.—If a body, such as an electromagnet, is alternately on and off service, its temperature is rising and falling. If the heating occurs at a constant rate and for the same period each time, and if the cooling period is each time the same, the temperature curve goes through a definite cycle, after equilibrium is established in the cycle. The first few cycles will be occupied in warming the coil to the state where thereafter the cycles are alike.

As before, let A, B, and C represent respectively the heat-production rate, the heat-dissipation rate per degree rise, and the heat-storage capacity per degree rise. Let θ be the temperature rise, T_1 the seconds of time elapsing during a heating period, T_2 the time for a cooling period, Θ_H the highest value of temperature rise, or the rise occurring at each cycle peak, and Θ_L the lowest temperature rise, or the rise occurring at the cycle minimum points. Figure 24*a* shows some of these values on the curve plotted there. In this drawing, the first cycle is not the actual beginning cycle, but is any cycle after equilibrium is established. The first temperature rise goes to Θ_H; if heating continued, the curve of rise would be as the dotted extrapolated curve portion shows. Then the heating is stopped, and the temperaure rise falls to Θ_{Li}; if cooling continued, this curve would follow the dotted curve to become asymptotic to the zero-rise axis.

1. *Heating Period.*—Take $t = 0$ when $\theta = \Theta_L$. Then, as

$$A = B\theta + C\frac{d\theta}{dt},$$

$$\log_e (A - B\theta) = -\frac{B}{C}t + a \text{ constant.}$$



(a) Heating Cooling Cycle

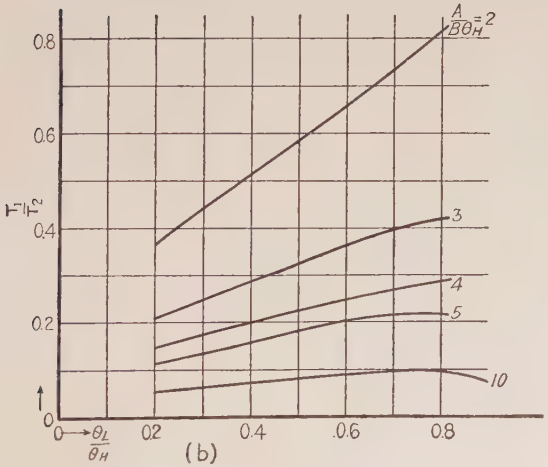


FIG. 24.—Curves for heating-cooling cycles.

Evaluating the constant for the time $t = 0$, $\theta = \Theta_L$,

$$\log_e (A - B\Theta_L) = 0 + \text{constant.}$$

Then

$$t = -\frac{C}{B}\log_e \frac{A - B\theta}{A - B\Theta_L}.$$

$$T_1 = -\frac{C}{B}\log_e \frac{A - B\Theta_H}{A - B\Theta_L}. \qquad (17)$$

II. *Cooling Period.*—Let $t = 0$ when $\theta = \Theta_H$.

$$0 = B\theta + C\frac{d\theta}{dt}.$$

$$\log_e \theta = -\frac{B}{C}t + a \text{ constant}.$$

$$\log_e \Theta_H = \text{constant}.$$

$$t = -\frac{C}{B}\log_e \frac{\theta}{\Theta_H}.$$

$$T_2 = -\frac{C}{B}\log_e \frac{\Theta_L}{\Theta_H}. \tag{18}$$

III. *An Example.*—A coil has a heat dissipation rate of $B = 0.5$ watt per degree rise, and a heat storage rate of 720 watt-seconds per degree rise. If the maximum temperature rise should be 40°, since $\Theta_0 = A/B$, the constant-duty heat-production rate would be $A = B\Theta_0 = 0.5 \times 40 = 20$ watts. At this rate, the heating period could last indefinitely; if the A-rate exceeds 20 watts, either the coil will overheat, or it must be given chances to cool.

Let the heat-production rate be 40 watts; if Θ_H be 40 and Θ_L be 20, how will the heating and cooling periods compare in length, and what is the length of each? Solving Eq. (17) for T_1, the heating period is found to be 584 seconds. Solving Eq. (18) for T_2, the cooling period is 998 seconds.

89. Curves for Solving Heat-cycle Problems.—If the ratio T_1/T_2 is found by combining Eqs. (17) and (18), the C/B factor drops out, leaving a fraction consisting of two logarithms. The only quantities appearing in the fraction are A, B, Θ_H, and Θ_L. Therefore, the division of time within the cycle depends only on these quantities.

The kind of solution desired in a given heating cycle case depends on what there is to start with. Practically every type of direct or inverted problem is found in practice. The time periods may be definitely fixed, requiring the heat-production rate to be found; or the heat-production rate may be fixed, leaving the cooling period to be found; in either case, the minimum rise Θ_L, will also be an unknown. There are other combinations possible. Such problems, from the engineering standpoint, are inherently cut-and-try problems; hence, every possible aid is justified in its use.

If various ratios of $A/B\Theta_H$ are taken, and for each ratio, if various ratios of Θ_L/Θ_H are taken, the ratios of T_1/T_2 can be

computed. The variations of two of these ratios may be used as independent variables, the values of the third ratio as parameters, and a system of curves can be plotted from which, by interpolation, any value of one ratio is found, given the other two. The author has calculated these values and the curves are given in Fig. 24*b*.

If any of these ratios approach certain critical values, the general equations (or the curves) often need not be resorted to, for easy approximate solutions may become possible.

Problems

1. A circuit has 10 ohms resistance and 0.002 henry inductance. (*a*) In how many seconds will the current rise to 90 per cent of its final value? (*b*) After current is established, if the supply switch is opened, and simultaneously the circuit is closed through a 10-ohm resistance, how long will it take the current to drop to half value?

2. A series of coils, each of the same external dimensions, is built. The different coil units are designed to give exactly the same magnetic results, but they are to operate on different voltages. Their heat losses must of course be the same. Assume for simplification that their space factors are the same. (*a*) How will coil resistance vary with voltage? (*b*) How will inductance vary? (*c*) How will the time constant vary?

3. A coil of 2,000 turns is placed on a laminated closed core made of electrical sheet, core section being 2 square inches. At a current of 4 amperes, the core flux density is 140,000 maxwells per square inch. The resistance of the coil is 50 ohms. Let the coil be energized at 200 volts, constant. The rise of current does not follow the ideal curve, because of saturation of the core. Work up a step-by-step graphical solution for obtaining an approximately correct curve of current rise.

4. A large generator has 12 poles; the flux per pole is 4×10^6 maxwells. The field coils are in series, there being 150 turns per coil. The resistance totals 3.6 ohms; field voltage is 120. In what time will the field current build up to within an ampere of its final value?

5. If, due to a high time constant, an electromagnet acts too slowly, will it help matters any if the coil is redesigned so that it has two equal sections operated in parallel?

6. A field discharge resistance is placed in parallel with the field circuit; its function is to permit field current to decay through it, rather than to be broken suddenly at the switch. Thus field insulation is protected. (*a*) If the discharge resistance is permanently in parallel with the field circuit and takes 10 per cent as much power as does the field, what momentary voltage will exist at the moment of switching off the field, in per cent of field voltage? (*b*) What if discharge watts equal field watts?

7. If the magnetic path of a coil is partly or wholly made of solid iron, eddy currents will flow in the core when the flux changes. What will this do to the time constant?

8. A coil has a heat-production rate of 20 watts, a heat-dissipating rate of 0.5 watt per degree Centigrade rise, and a heat-storage capacity of 720 watt-seconds per degree Centigrade rise. Find the time constant, the time for the temperature rise to attain half of full value, and the final temperature rise.

9. If the above coil is placed in service for 48 minutes and then is taken off the line and allowed to cool for 20 minutes, what is the final temperature rise?

10. For how long could this coil have a heat-production rate of 100 watts without exceeding a temperature rise of 50°C.?

11. If this coil has a heat-production rate of 60, followed by a cooling period, the cycle being repeated continuously; if Θ_H be 40° rise; and if the time of the cycle is to be 2,000 seconds, find the time for the heating and cooling periods, and the value of Θ_L. (Suggestion: Cut-and-try method; call the solution done when $T_1 + T_2 = 2{,}000 \pm 80$.)

12. Work out enough values for plotting an extra curve in Fig. 24*b*, whose parameter is $A/B\Theta_H = 2.5$.

13. For the above coil, if a heating cycle 30 seconds in duration is used, and if the heat production rate is 100 watts, find approximately the time for the heating period; Θ_H to be 40° rise, as before. (This problem is preferably solved mentally.)

14. A generator operated at full load has a time constant of 2 hours, and an ultimate temperature rise of 50°C., these values being for the armature. At half load the armature losses are one-third the full-load value. If a loading cycle of full load for, 1 hour, followed by half load for 20 minutes, is performed continuously, find the maximum and minimum temperature rises.

15. A steel cube 12 inches on a side is heated to 100°C. and then is suspended in air which is at 20°C. Assume that heat dissipation from all surfaces of the body averages up to the performance as shown for the 9- by 9-inch brass plate (Fig. 33). Working by 10° steps, obtain the approximate curve of temperature plotted against hours, down to a body temperature of 30°C.

CHAPTER XII

CONDUCTORS AND CONDUCTOR INSULATION

Except when resistance loss is desired, copper is almost universally used for the conducting material in all kinds of apparatus. The one exception of any consequence is aluminum, which finds its niche in certain specialized applications. Both metals are soft and ductile, are available commercially in uniformly high degrees of purity, and may be drawn in almost any section desired. The wire sections used are almost invariably round, square, or rectangular.[1]

90. Wire Gages.—The Browne and Sharp Company introduced the B. & S. gage; this gage system later had its name changed to the American wire gage, or A.w.g. Both names still are used, but A.w.g. is to be preferred. The gage of a wire refers directly to its dimension, not its sectional area; however, the A.w.g. system so chooses the successive diameters that a usable series of section areas is obtained.

91. Round Wire.—As the area, and not the diameter, is the part concerned with the conduction of current, it is evident that successive wire sizes should regularly increase the area of section. The A.w.g. system does just that. Aside from minor variations, the following statements apply to round A.w.g. wires;

1. Any wire is larger than the next smaller wire by 26 per cent, in sectional area.

2. The cube of 1.26 is 2; hence sectional area is doubled or halved by moving to the third wire size away from a given size.

3. Beginning with a given size, if three steps of three sizes are taken, the ninth size away is reached, which has eight times the section of the given wire or one-eighth, as the case may be. If the movement is toward larger wires, one more step adds 26 per cent to the section, and 8×1.26 is about 8×1.25, which is 10; that is, adding or subtracting 10 to or from the gage number changes the area by nearly a factor of 10.

[1] UNDERHILL, C. R., "Coils and Magnet Wire," McGraw-Hill Book Company, Inc.

4. A No. 10 wire has a diameter of about 0.1 inch, or 100 mils. Its section area is about 10,000 mils. For approximate calculations, or for mental checks on solutions, any wire size can be quickly gotten without reference to a table.

5. In many pieces of apparatus, the hot resistivity of copper is about 1 ohm for a circular mil-inch; thus the resistance of any size, for any number of inches, is easily figured.

92. Resistivity. I. *Copper.*—By the Annealed Copper Standard, adopted by the International Electrotechnical Commission in 1914, copper of 100 per cent conductivity has a resistivity of 10.371 ohms per circular mil-foot at 20°C.; the temperature coefficients also being stated, as follows: 0.00427 per degree for 0°C., and 0.00393 per degree for 20°C.; that is, on the 20° base, resistivity, or ohms for a uniform wire 1 foot long and 1 circular mil in section, is 10.371. The resistance increases 0.00393 parts in unity, or that many ohms per ohm, or by 0.393 per cent, per degree rise above 20°C.

Electrical copper seldom departs more than 2 per cent from the defined 100 per cent conductivity, unless it is hard drawn; hard-drawn copper is 4 per cent low in conductivity.

II. *Aluminum.*—Aluminum has a resistivity of about ~~1.70~~ 17.0 ohms per circular mil-foot, at 20°C., the corresponding temperature coefficient being 0.0039.

93. Weights.—Copper weights can be figured by using 0.32 pound per cubic inch; aluminum, 0.093 pound per cubic inch.

94. Square Wire.—Square A.w.g. wires are regularly drawn and used. Note that square and round wires have equal widths and diameters, respectively, for the same wire sizes; a No. 10 square then has nearly 27 per cent more area than a No. 10 round. The theoretical ratio is $4/\pi$, or 1.273; but as the corners of square wire are always rounded a bit, a factor of 1.26 can be used with good accuracy; 1.26 being easily remembered, due to its intimate connection with area changes in the gage system itself.

Wire smaller than Nos. 10 or 11 is not used in square sections, for its tendency in winding is to twist and wind cornerwise.

95. Insulation of Round and Square Wires.—The usual insulations and their thicknesses are indicated by the Wire Table (Table III). Besides enamel, cotton, silk, and their combinations, asbestos finds application in high-temperature coils.

96. Rectangular Sections.—Rectangular sections furnish one of the bad cases of non-standardization in the engineering field.

Different manufacturers, and in fact, different designers within the same plant, have in the past ordered a huge array of unrelated sizes. When an odd section is made, the wire company keeps the dies, hoping for further sales; and the apparatus manufacturer may put the size in stock, finding an occasional use for it. Thus unrelated sizes and sections are perpetuated. No general system being in use, such cannot be given here.

Small rectangular sections are insulated like round or square wire; thickness of insulation should be taken the same as that on the A.w.g. wire whose diameter is equal to the greater dimension of the rectangle section. Insulation on larger sections will be treated where the use of the section is taken up.

97. The Circular Mil.—The circular mil is often defined as the area of a circle 1 mil (0.001 inch) in diameter. This is a poor definition; it implies that, in order to be able to state the area of a surface in circular mils, the surface must be circular. The better definition is: A circular mil is an area *equal* to that of a circle 1 mil in diameter.

The circular mil was devised for the purpose of easily stating or finding the area of round wires; as circular areas vary with the square of the dimension, the circular mil area of a round wire is equal to the square of its diameter expressed in mils. The circular mil area of a square wire is approximately 1.26 times the square of a side dimension expressed in mils.

The A.w.g. system has as its last five sizes, 0, 1-naught, 2-naught, 3-naught, and 4-naught. Round wires of greater section are spoken of in terms of the circular mil area, as 100,000 circular mils, 1,000,000 circular mils, etc.

98. One Ohm per Circular Mil-inch.—Machines rated at a 40°C. rise, operated in an air temperature of 20° will have a *measured* temperature (surface temperature by thermometer) of 60°. In ordinary cases the "hot-spot" temperature within the winding will be roughly 15° higher, placing the hot spot at 75°. The mean copper temperature will be somewhere between 60 and 75°C. Using a value of 67.5°, the rise above 20° is 47.5. $10.37(1 + 47.5 \times 0.00393) = 12.3$ ohms per circular mil-foot. This amounts approximately to 1 *ohm per circular mil-inch*, a value much used in electrical design. *It should not be used when it is known that temperatures are going to be much different from those used in obtaining the value.*

TABLE III.—A.W.G. (B. & S.) WIRE TABLE

Diameters in mils

Gage number	Area, circular mils	Bare wire	Enameled	Enamel single silk	Enamel single cotton	Single silk	Double silk	Single cotton	Double cotton
8	16,380	128.0	131.0	....	139.0	....	....	136.5	142.5
9	13,090	114.4	116.9	....	123.9	....	....	121.4	126.4
10	10,360	101.8	104.4	....	110.4	....	....	107.9	112.9
11	8,226	90.7	93.2	....	98.2	....	....	95.7	100.2
12	6,529	80.8	83.3	....	88.3	....	....	85.8	90.3
13	5,170	71.9	74.5	....	79.5	....	....	77.0	81.5
14	4,109	64.1	66.6	....	71.6	....	....	69.1	73.6
15	3,260	57.1	59.6	....	64.6	....	....	62.1	66.6
16	2,581	50.8	52.8	54.8	57.8	52.8	54.8	55.8	60.3
17	2,052	45.3	47.3	49.3	52.3	47.3	49.3	50.3	54.8
18	1,624	40.3	42.3	44.3	47.3	42.3	44.3	45.3	49.8
19	1,289	35.9	37.9	39.9	42.9	37.9	39.9	40.9	45.4
20	1,024	32.0	33.8	35.8	38.8	34.0	36.0	37.0	41.5
21	812	28.5	30.3	32.3	35.3	30.5	32.5	33.5	38.0
22	640	25.3	27.1	29.1	31.6	27.3	29.3	29.8	33.8
23	511	22.6	24.1	26.1	28.6	24.6	26.6	27.1	31.1
24	404	20.1	21.6	23.6	26 1	22.1	24.1	24.6	28.6
25	320	17.9	19.4	21.4	23.9	19.9	21.9	22.4	26.4
26	253	15.9	17.1	19.1	21.6	17.9	19.9	20.4	24.4
27	202	14.2	15.4	17.4	19.9	16.2	18.2	18.7	22.7
28	159	12.6	13.8	15.8	18.3	14.6	16.6	17.1	21.1
29	128	11.3	12.5	14.5	17.0	13.3	15.3	15.8	19.8
30	100	10.0	11.2	13.2	15.7	12.0	14.0	14.5	18.5
31	79.2	8.9	9.9	11.9	14.4	10.9	12.9	13.4	17.4
32	62.4	7.9	9.0	11.0	13.5	10.0	12.0	12.5	16.5
33	50.4	7.1	7.9	9.9	12.4	9.1	11.1	11.6	15.6
34	39.7	6.3	7.1	9.1	11.6	8.3	10.3	10.8	14.8
35	31.4	5.6	6.3	8.3	10.8	7.6	9.6	10.1	14.1
36	25.0	5.0	5.7	7.7	9.7	7.0	9.0	9.0	13.0
37	19.4	4.4	5.1	7.1	9.1	6.5	8.5	8.5	12.5
38	16.0	4.0	4.6	6.6	8.6	6.0	8.0	8.0	12.0
39	12.2	3.5	4.0	6.0	8.0	5.5	7.5	7.5	11.5
40	9.6	3.1	3.6	5.6	7.6	5.1	7.1	7.1	11.0
41	7.84	2.8	3.2						
42	6.25	2.5	2.9						
43	4.84	2.2	2.6						
44	4.00	2.0	2.4						

CHAPTER XIII

COILS

An electrical machine or piece of apparatus that does not have one or more coils somewhere about it is the exception rather than the rule. The coil plays so important a part in so many different kinds of electrical devices that every electrical student is warranted in acquiring a considerable knowledge of coils and their characteristics. It follows that when one can bring to a successful conclusion a variety of types of coil designs, one has a working knowledge of electromagnetic theory as well.[1]

99. General.—Coils in general consist of a regular formation of turns of wire in a layer, with one or more layers. The coil wall nearly always has a rectangular cross-section. The shape of a single turn is nearly always round, or rectangular, or rectangular with corners rounded. The wire sections are round, square, and rectangular. But aside from these characteristics, coils display a great number of type variations. C. R. Underhill has a full-sized book on the subject, "Coils and Magnet Wire," and would be the last to claim that he has written exhaustively on his theme.

100. Fine-wire Coils. I. *Winding Machinery.*—Let all wires from about Nos. 28 to 44 be called fine wires, for present purposes. These wires are so small that to guide the finer sizes by hand in the winding process would be too tedious or impossible. This fact, coupled with the vast demand for ignition and radio coils (to cite only two applications), has caused much ingenious winding machinery to be developed.

II. *Paper between Layers.*—Ignition coils and the like need, as a rule, extra insulation between layers. This is usually provided for by using from one to four wraps of paper, of about a mil per sheet in thickness, between layers. The automatic machinery feeds the paper in and cuts it off. The paper insures even layers and a strong, self-supporting coil.

[1] Underhill, C. R., "Coils and Magnet Wire," McGraw-Hill Book Company, Inc.

Still, Alfred, "Elements of Electrical Design," McGraw-Hill Book Company, Inc.

III. *Self-supporting Coils.*—A variety of names covers a variety of such coils. A common machine-wound type has cotton strands interwoven with the wires. Everyone is familiar with the form of a ball of binder twine, or its smaller counterpart, the common ball of string; this is self-supporting, due to the back-and-forth lay of each thread or wrap. By interweaving cotton strands into the wire layers of a coil, a self-supporting coil is obtained which lends itself readily to being impregnated by virtue of the wick effect of the cotton strands. In inductive coils where it is desired to keep the turns apart to minimize capacity effect, the wire itself is crisscrossed in various ways, some of the schemes resulting in self-supporting coils.

IV. *Spool-wound Coils.*—Huge numbers of low-voltage coils are wound directly upon a permanent insulated spool; enameled wire usually is used, and the winding may or may not be haphazard. Telephone receiver coils give one example.

V. *Haphazard Winding.*—Formerly, many fine-wire coils for low voltages were wound without regard to regularity; this allowed a spool to be quickly wound full by hand feeding. Among the obvious disadvantages were the tendency to stress the insulation of the wire to breakdown point, lack of uniformity as between one coil and the next, and loss of space factor.

The demands of radio are responsible for great improvements. Ten years ago, No. 44 wire was a commercial curiosity; no one could produce it in quantities, much less wind it successfully. With the pressure of demand, methods and machinery were developed until now No. 44 wire is made and used in great quantities. All this development has reacted on the design of fine-wire coils. Most applications wherein haphazard windings were used now employ the machine-wound, regular-layer coil.

VI. *Insulation.*—The increased demand for fine-wire coils also helped to develop the art of making enamel insulation for wires. Cotton or silk cannot be uniformly applied to very fine wires without unduly taking up space. But enamel coatings can be thin and yet effective. Enameled, enamel single cotton, and enamel single silk are the usual insulations for fine wires.

101. Medium-wire Coils. I. *Not Wound on Spools.*—Sizes from Nos. 10 to 27 may be called medium-wire sizes, for present purposes. The larger the wire, the less the demand; therefore, the less available becomes automatic machinery for winding. Much of this wire is guided by hand, although large-production

jobs are turned out in great quantities by machine. These sizes are mostly large enough to be somewhat self-guiding; not much skill is needed to get the same number of turns into each layer, provided the first layer is sufficiently tight. Hence, paper between layers becomes mechanically unnecessary. When needed for insulating purposes, it is used.

II. *Spool Winding.*—Many relay, electromagnet, solenoid, field coils, and the like are wound directly onto permanently insulated spools. The coil may or may not be impregnated. If enameled wire is used, impregnation is difficult; so heat conductivity is low; but enamel stands high temperatures. Fibrous insulation takes the impregnating compound in good style. If the spool is a metal one, the coil has the advantage of being able to remain in stock indefinitely without change of shape.

III. *Mold Winding.*—Field coils and the like have a habit of occurring in too great a variety to permit of high standardization of spools; so many field coils are wound on adjustable molds. When the cheeks and core of the mold are removed, the coil is likely to spring out of shape somewhat. If the shape is important, the coil may be put into clamps until impregnated; the compound then stiffens the coil permanently. All compounds with an asphalt base are plastic, however, and coils so impregnated may, if stacked in a warm room, sag enough to make their future application in a given assembly difficult.

Many coils are impregnated and then taped to give them an outer mechanical protection; others are taped without being impregnated; still others are impregnated without being taped. The practice depends on the use to which the coil is put, how hot it may get, what handling it is likely to get, and how high its voltage may be.

102. Square-wire Coils.—Small square wires are impracticable, because square wire tends to bend on edge and become twisted. Thus No. 10 gage is about the smallest square wire used. The square wire gives a high space factor and high heat conductivity; but the greater likelihood of the insulation being rubbed through where square corners come in contact makes it poor material for any but low-voltage coils. For interpoles (commutating poles) requiring a coil wire ranging from No. 9 to as large as 0.25 inch on a side, square wire, cotton insulated and wound in from one to several layers, is much used. Many of these interpole coils must be tapered off, becoming smaller at the

armature end of the coil where space between main poles becomes constricted. This is arranged by failing to complete the outer layers, giving the coil a stepped-off effect.

103. Ribbon and Strap Coils. I. *Spirally Wound Ribbon Coils.*—These coils are made of copper drawn in rectangular form. If the ribbon is wound on flatwise, a coil of one turn per layer and many layers is formed. Lifting magnets of large size have several such coils, or "pies," installed. Power-transformer coils usually take this form. Heavy-current coils for circuit breakers are often made of a wide ribbon rolled up on itself like a spool of ribbon, with paper, asbestos, mica, or whatnot between turns.

A bare ribbon with edges exposed and with only thin asbestos between layers has a high space factor and maximum heat conductivity.

II. *Edge-wound Strap Coils.*—*Ribbon* and *strap* are synonomous terms, so far as the unwound copper is concerned, but when the rectangular conductor is wound *on edge*, a strap coil is produced. A typical example is found in the low-voltage field coils of water-wheel generators. The field current is so large that square or rectangular wire is required. The field system revolves, which puts high centrifugal forces on the coil; in many cases this eliminates the use of square wire wound in several layers. The ideal solution is to use copper strap wound on edge, with asbestos or shellacked paper between turns and nothing but varnish on the bare outer trap edges. Many heavy-current interpole coils are built likewise.

When there is plenty of room for such a coil, as for the series coils of compound generators, or for interpole coils in synchronous converters, large strap coils are often built with air spaces between turns. Coil ventilation is improved so that less copper can be used.

104. Embedding.—Embedding is a term applying to the tendency for wires of one layer to wind into the valleys formed by the turns of the layer inside of it, just as a regular stack of round steel bars naturally tend to embed into a triangle-hexagon formation as seen from the ends.

In ordinary coils the turns of a layer are helical; the next layer is a helix of reversed pitch, and one turn in this next layer normally will wind so that at two places in its revolution it falls into valleys, and at two other places it falls on top of another turn.

To secure complete embedding throughout the coil, each layer would have to be pitched like every other; each layer then would have to be separate from the next or be joined to it by the end of a layer being carried through the core hole to start over again. Naturally, this is not done.

Embedded coils, so called, are not wholly embedded coils. They are wound on special machines. The first turn wound on forms a true circle lying in a plane normal to the axis. Just as the turn threatens to climb on top of itself, it is sharply side stepped, to begin another similar turn. Thus for one layer. On the way back in the next layer, the turns embed into those of the first layer, except where the bend is made, which is placed so as to cross the bends in the turns of the first layer. The coil tends to thicken along the radius running through the cross-overs, but this tendency is somewhat reduced by the compressing of the wire insulation in the same region.

As stated above, the ordinary coil inherently is somewhat embedded, and Underhill states that the resulting theoretical gain in space factor is 7.2 per cent. In the coil problems given in this book, embedding is neglected unless otherwise indicated by the context.

105. Space Factor.—The coil wall, which is a cross-section of one side of the coil, invariably contains both metal and insulation, unless the special case of one bare turn of wire is considered. The space factor of the coil expresses the percentage of copper in terms of the total coil section.

1. Copper volume divided by coil volume equals space factor.
2. In the coil wall, copper section divided by wall section equals space factor.
3. Wire section divided by section area per wire equals space factor.

It is evident that the better (higher) the space factor, the smaller the coil can be made. Square wires have considerably greater space factors than round wires. The smaller the wire, the thicker its cotton or silk covering becomes in proportion to its diameter; hence, small wires have poor space factors. Paper between layers reduces space factor. Embedding improves it. A strap or ribbon coil, with thin strip insulation between bare-conductor layers or turns, has the maximum practicable space factor; the thicker the strap in proportion to the insulation, the better the space factor.

106. Manufacturing Variations in Coils.—This general heading is intended to cover the multitude of causes for coils being different from that which they were intended to be. To begin with, wire insulations, although remarkably uniform, are not absolutely so. The wire itself varies a trifle from the supposed gage diameter. The turns of a layer compress the wire insulation, tending to reduce the turns in the layer below the number counted upon. Coils usually must be compact, a fact which requires that the wire be under tension while being wound. Careless tension adjustment can easily stretch a wire until it really is the next size smaller wire. Coils wound on forms which later are removed are almost certain to spring somewhat. Hammering before removal and during winding removes some or all of this tendency; inexpert hammering also ruins many coils, due to insulation breakdown. These are only a few of the reasons why coils as designed and coils as built may be and often are so different as to require doing the job over again. The practicing designer has at his finger tips a collection of data representing experience factors by which, in a given design type, he can allow for known variations in designing his coil. The experience factors from one factory, using one set of processes and one group of skilled winders, will not duplicate the experience factors of another organization having its own—and different—personnel and methods. To attempt to gather together and print here a comprehensive survey of such factors for a large number of typical coil types would be to go entirely outside the proper scope of the book. The coil work herein done, or laid out to be done, must be carried out on the basis of simplifying assumptions; and all solutions of coil problems thus arrived at must be looked upon as tentative solutions, perfectly good ones, under the assumptions, but often in need of final corrections which could be applied only by working out the design within a given manufacturing organization.

107. Heat Flow Inside a Coil.—The direction of heat flow in a coil may be across the layers, or diagonally, or with the wires, or irregularly, depending on the case. If heat flow occurs regularly across layers, a two-dimensional field of flow lines and isothermal lines is established, as represented roughly in Fig. 25. It is assumed that thermal conductivity is uniform in all directions in the insulation.

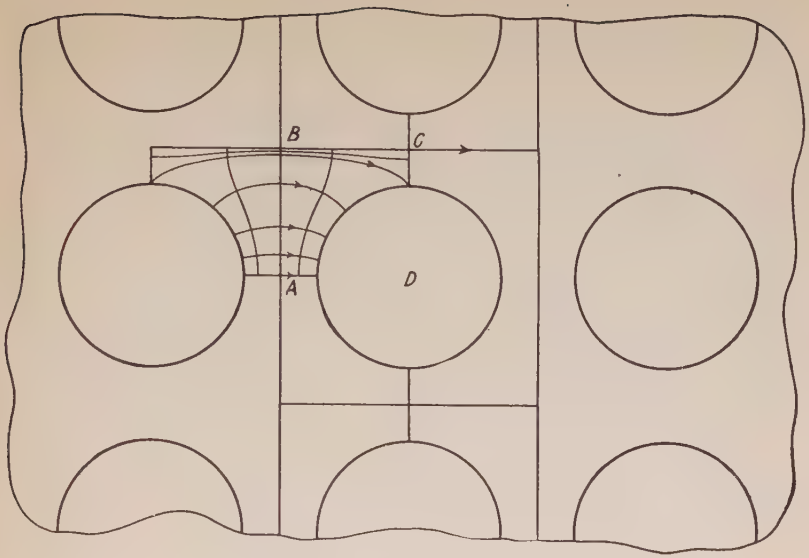


FIG. 25.—Heat flow in a coil.

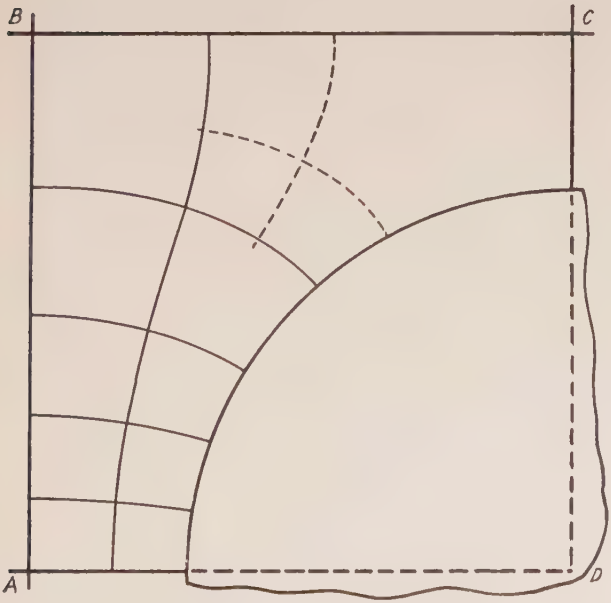


FIG. 26.—Heat flow field map in coil section.

The thermal conductivity of copper is 8.8 watts per inch cube per degree Centigrade, as compared with 0.005 for impregnated fibrous insulation. With such insulation the conductivity of the copper may be considered as infinite; thus the wire surfaces become isothermal (equipotential) surfaces. In Fig. 25, *AB* becomes by symmetry an isothermal line, and another goes through *C* which meets the surface of wire *D* and spreads around it. Some tentative flow lines are shown in the unit bundle *ABCD*.

For better work, the bundle *ABCD* is enlarged (Fig. 26), and the flow field is mapped in the usual way.

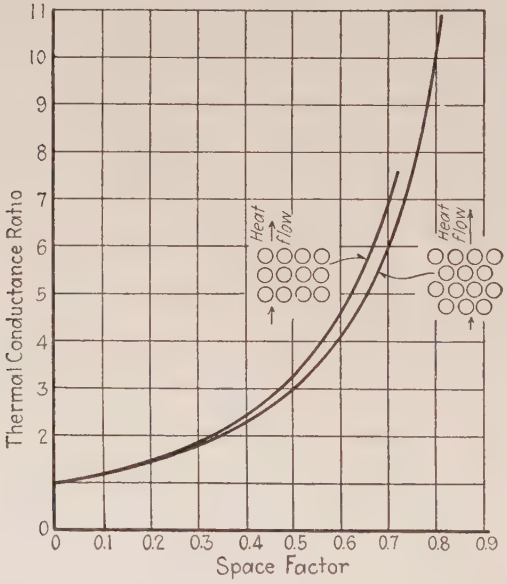


FIG. 27.—Thermal conductance ratio, heat flow in coils.

108. Thermal-conductance Ratio. I. *Definition.*—The thermal-conductance ratio may be defined as the heat conductivity of the coil divided by the conductivity of the insulation. In Fig. 26, if area *ABCD* were made entirely of insulation, it would have a certain conductivity; the actual flow path exhibits about 4.8 tubes of two squares each; hence the conductance of the actual path is 2.4 times what it would be for the bundle, if the entire space were filled with insulation.

II. *Non-embedded Coils.*—The conductance ratio would decrease as the space factor is diminished, approaching unity as its

lower limit. The writer mapped cases for various values of space factor, thus getting the upper curve of Fig. 27.

III. *Embedded Coils.*—With perfect embedding, the flow field is different; but as shown by the lower curve of Fig. 27, likewise derived by mapping flow fields for various space factors, *the conductance ratio is nearly the same as for non-embedded coils, for the same space factor.* Gain in heat-carrying capacity for given temperature differences thus is seen to be about proportional to gain in space factor; as perfect embedding gives a considerable gain in space factor over non-embedding, a definite gain in heat conductance is obtained.

109. Combined Thermal Conductivity.—The conductance ratio times the insulation conductivity gives the combined thermal conductivity of copper and insulation. Let J stand for the combined conductivity.

110. Parallel Heat Flow.—Let a long coil with flat sides be filled with a heat insulator, so that all heat produced by resistance must flow to the outer surface. For instance, a rectangular shunt field coil may be filled with cotton. Neglecting end surfaces and corner effects, the general flow of heat is parallel through a given coil wall. Looking at the intimate study of flow among the wires, the field of course is two-dimensional, as shown in Fig. 25; but having used such fields to find a conductance ratio and the combined conductivity, the assumption then is made that the coil is homogeneous and the flow rectilinear.

Let the coil wall be T inches thick, with a combined thermal conductivity, of J watts per inch cube per degree Centigrade, and a heat-production rate of W watts per cubic inch of coil volume. Consider a volume having 1 square inch on the inner surface as a base, and reaching through the coil wall to the outer surface. Let distances from the inner base be t inches. At t inches from this base, the heat produced which flows through a differential thickness dt is Wt. The differential temperature drop in dt is $d\theta$, and

$$d\theta = \frac{Wtdt}{J}.$$

$$\theta = \frac{W}{J}\frac{t^2}{2} \text{ in degrees Centigrade.}$$

$$\Theta = \frac{W}{2J}T^2 = \text{total degrees Centigrade drop through coil wall.}$$

111. Radial Flow in a Solenoid.—Let the same conditions hold, but change the coil to one of solenoid form. Let the inner coil radius be R inches. Consider a volume of 1 square inch base at the inner surface; the section area of the volume at t inches out is $(R + t)$. The watts produced inside the differential thickness dt at distance t are

$$\text{watts} = W\frac{R + (R + t)}{2}t.$$

$$d\theta = W\frac{2R + t}{2}t\frac{1}{J}dt.$$

$$= \frac{W}{2J}[2Rtdt + t^2dt].$$

$$\theta = \frac{W}{2J}\left[\frac{2Rt^2}{2} + \frac{t^3}{3}\right]$$

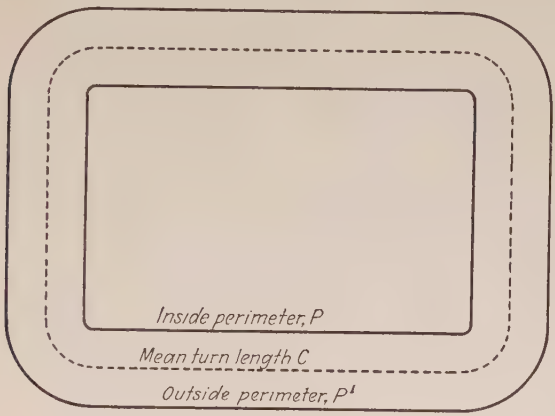
$$\Theta = \frac{W}{2J}\left[RT^2 + \frac{T^3}{3}\right].$$

112. Coil Relations. I. *Notation* (Refer to Fig. 28).—

C = mean turn length, inches.
N = number of turns.
I = amperes.
E = volts applied to coil.
R = coil resistance, ohms.
M = wire section, circular mils.
T = thickness of coil wall, inches.
H = height or length of coil, in direction of the layers.
P = inside coil perimeter.
P' = outside coil perimeter.
l = number of layers of wire.
t = turns per layer.
D = wire diameter, mils.
D' = insulated wire diameter, mils.
V = coil volume, cubic inches.
f = space factor.

II. *Relations.*—It is assumed that no embedding occurs, that pressure within the coil does not change wire insulation thicknesses, that no paper or separator is used between layers, and that the resistivity of copper is 1 ohm for a circular mil-inch. Under these simplifying conditions, it is easily seen that the following relations are true:

$M = D^2$ = wire section, circular mils, for round wire.
$M = 1.26D^2$ = section for square wire.
$M = 1.26D_1D_2$ = section for rectangular wire.
$H = tD'/1{,}000$ = coil height or length.
$T = lD'/1{,}000$ = coil thickness, if no extra insulation is used between layers.



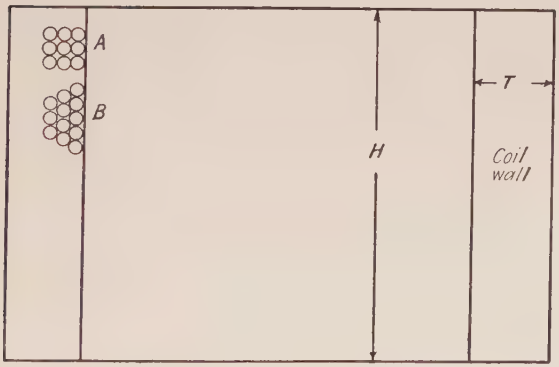


FIG. 28.—Coil description.

HT = coil-wall section, square inches.
$HTC = V$ = coil volume, cubic inches.
$lt = N$ = coil turns.
$HT1.273 \times 10^6$ = coil section, circular mils.
$fHT1.273 \times 10^6/M = N$ = coil turns.
$P + 2\pi T = P'$ = outside coil perimeter.

$P + \pi T = C$ = mean turn length.
NC = wire length, inches.
$NC/M = R$ = coil ohms.
$NIC/M = IR = E$.
$M = NIC/E$.
$NI = ME/C$.
$R = NC/M$.
$E = NIC/M$.
$f = D^2/1.273\ D'^2$ = space factor for round wire.
$0.32Vf$ = coil copper weight, pounds.
$0.32NMC/(1.273 \times 10^6)$ = coil copper weight, pounds.

113. Coils of Two Sizes of Wire in Series.—As the section area of a wire is about 26 per cent larger than that of the next smaller wire, in the A.w.g. table, it very frequently happens in coil design that the wire section desired is far enough from either of the two nearest sections to raise a difficult point. If the next smaller wire is used, the desired ampere-turns are reduced; if the next larger wire is used, an undesirable increase in ampere-turns may have to be accepted. The problem particularly arises in smaller factories, in which it is sometimes the custom to carry in stock only every other wire size.

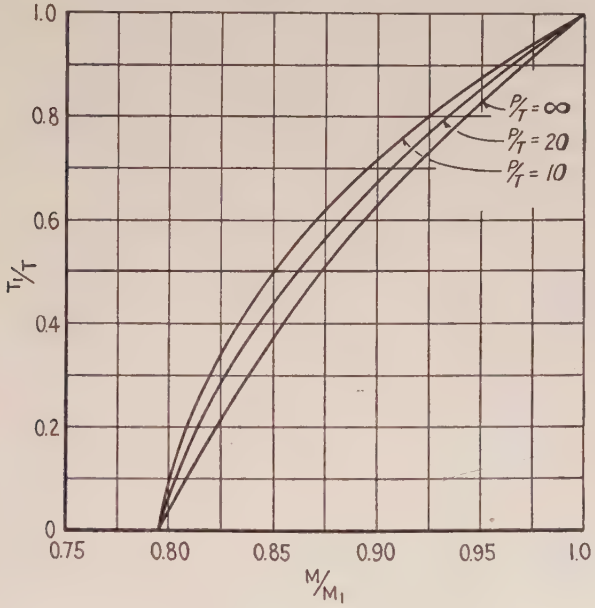
A method used sometimes to get around the difficulty is to combine the two nearest wire sizes by building a coil out of each, the two parts being placed in series with the larger wire inside. The solution of the problem involved presents algebraic difficulties out of all proportions to the size of the job. The writer has produced the curves shown in Fig. 29, which are useful in finding approximately how to divide the coil into its component parts.

In explaining the use of the curves, it is presumed that the coil design is already made in terms of the desired, but unobtainable, wire section. The coil thickness, height, perimeter, temperature rise, watts, turns, and ampere-turns are all known. In order to make this design, it is necessary to know the insulated diameter of the desired wire section, which is found by adding to the bare wire diameter as much as is allowed for the adjacent sizes in the wire table.

Now, instead of building this coil of a wire of desired section, which is fictitious, it is proposed to make it of some inner layers of the next larger actual wire, in series with some layers of the next smaller actual wire. As a general proposition, it is possible to do this and achieve a compound coil which nearly duplicates

the preliminary design as to dimensions, ampere-turns, watts, and temperature rise.

Let M be the circular mils of the desired section, T the coil thickness, and P the inside coil perimeter; let M_1 be the circular mils section of the next larger actual wire, and T_1 the thickness of the inner coil part made of layers of this wire. The ratios M/M_1 and P/T can be computed, and by interpolation on the curves, an approximate value for the ratio T_1/T is found. From



P = Coil inside perimeter
T = Total Coil thickness
T_1 = Thickness of larger wire (inner) winding
M = Circ. mils section of desired (unobtainable) wire
M_1 = " " " " next larger AWG size

FIG. 29.—Curves for dividing shunt coil into two parts.

here on, it is just a matter of figuring out all the remaining facts in the two-wire design, and checking to see if the compound design is close enough to that required to go as it is; if not, it will be nearly acceptable, and a small shift in the number of turns in a layer, or a change in a whole layer of one wire or the other, will bring the design to a proper finish.

114. A Coil Example.—A constant-duty, 115-volt coil must have a minimum strength of 600 ampere-turns. It is a round

coil, 3 inches in inside diameter, length of $H = 2$ inches. The coil is enclosed by the iron of the magnetic circuit, and the watts allowed per square inch of *total* coil surface are 0.3 to 0.4 Coil impregnated; wire, enamel single cotton covered.

Assume $T = 1$ inch.
$P = 3\pi = 9.43.$
$C = P + \pi T = 12.57.$
$M = NIC/E = 600 \times 12.57/115 = 65.5$; try No. 32.
$M = 62.4$ (giving less than 600 NI, but this is only preliminary).
$D' = 13.5$ mils.
$N = lt = 2{,}000 \times 1{,}000/(13.5 \times 13.5) = 10{,}960.$
$R = NC/M = 10{,}960 \times 12.57/62.4 = 2{,}210.$
E^2/R = watts = 6.0.
Area = $C(H + T)2 = 62.2$ square inches.
Watts per square inch = 0.0965, entirely too low.

Assume $T = 0.4$.
$C = 10.69.$
$M = 55.7$, using No. 32.
$M = 62.4$, with $D' = 13.5$ mils.
$N = 4{,}400.$
$R = 752.$
Watts = 17.6.
Area = 51.2.
Watts per square inch = 0.344.
$NI = 673.$

This design is satisfactory. Depending on how the coil is made, paper might be used between layers. This would modify above figures.

115. Coil Example Involving Heating.—A solenoid is to have a length of $H = 10$ inches, and an inside diameter of 2 inches. It is so placed that the heat-dissipating factor is 0.01 watt per square inch per degree Centigrade, the dissipating surface being the outer cylindrical surface only. The voltage is 115, the NI minimum is 5,000, and the hot-spot temperature rise is to be 55°C. Wire insulation, enamel single cotton covered; added coil insulation, impregnation only; embedding, none counted on.

This is a cut-and-try problem. The outer diameter is assumed at different values, until the desired temperature rise is obtained.

Outer diameter = 4 inches, assumed.
$C = 3\pi = 9.43$.
$M = NIC/E = 5{,}000 \times 9{,}43/115 = 410$, using No. 23 wire.
$M = 511$, $D = 22.6$ mils, $D' = 28.6$ mils.
$f = (22.8)^2/1.273(28.6)^2 = 0.499$.
$lt = 10{,}000 \times 1{,}000/(28.6)^2 = N = 12{,}230$.
$R = NC/M = 12{,}230 \times 9.43/511 = 226$.
$E^2/R = 58.5$ watts.
$V = HTC = 10 \times 1 \times 9.43 = 94.3$ cubic inches of coil volume.
Watts/cubic inches $= W = 58.5/94.3 = 0.62$.

Dissipating surface $= 10 \times 4\pi = 125.7$ square inches.
Watts/square inches $= 58.5/125.7 = 0.465$.
Surface temperature drop $= 0.465/0.01 = 46.5°C$.

Insulation conductivity = 0.005 watt/inch cube/degree Centigrade (Table V).
Thermal-conductance ratio = 3.2 for $f = 0.499$ (Fig. 27).
Combined thermal conductivity $= 3.2 \times 0.005 = 0.016 = J$.
Internal drop is

$$\theta = \frac{W}{2J}\left(RT^2 + \frac{T^3}{3}\right)$$
$$= \frac{0.62}{2 \times 0.016}\left(1 \times 1^2 \times \frac{1^3}{3}\right)$$
$$= 25.8.$$

Total drop (or rise) $= 25.8 + 46.5 = 72.3°C$.

For the next value of outside diameter, if T is increased to 1.25, C will not change enough to change the wire size from No. 23; hence the resistance will be raised roughly 13 per cent, and the wattage reduced accordingly. There will be more surface and less watts per square inch and per cubic inch. The total rise may come about right. The problem is dropped here, as the further steps are clear.

116. Coil Example Involving Heat Storage.—A coil is to be in service but once a day, and for a period of 1 minute. It operates at 115 volts; the minimum $NI = 2{,}000$; the maximum temperature rise is to be 60°C. or less. The coil space requires a solenoidal coil, such that inside diameter = 1 inch, the length $H = 2$ inches, and $T = 1$ inch. Is the design feasible? Coil impregnated; wire insulation, single cotton covered; embedding neglected.

$C = 2\pi = 6.28.$

$M = NIC/E = 2{,}000 \times 6.28/115 = 109.1$, using No. 29.

$M = 128$, $D = 11.3$, $D' = 15.8$.

$N = lt = 2{,}000 \times 1{,}000/(15.8)^2 = 8{,}000$ turns.

$R = NC/M = 8{,}000 \times 6.28/128 = 393$ ohms.

$E^2/R = 33.7$ watts $= A$, the heat production rate.

From Table V,

Copper thermal capacity, watts per pound per degree Centigrade = 180.

Insulation thermal capacity, watts per pound per degree Centigrade = 800.

Copper at 0.32 pounds per cubic inch, insulation at 0.035.

$f = D^2/1.273D'^2 = (11.3)^2/1.273(15.8)^2 = 0.402$ = space factor.

$V = HCT = 2 \times 6.28 \times 1 = 12.56$ cubic inches = coil volume.

$fV = 0.402 \times 12.56 = 5.06$ = copper volume.

$V - fV = 7.50$ = insulation volume.

Copper heat capacity $= 5.06 \times 0.32 \times 180 = 292$ watt-seconds/degree Centigrade.

Insulation heat capacity $= 7.50 \times 0.035 \times 800 = 210$ watt-seconds/degree Centigrade.

Total capacity = 502 watt-seconds/degree Centigrade.

For a 60° rise, assuming all heat stored,

Time $= 60 \times 502/33.7 = 89.5$ seconds.

The design is entirely safe, as the coil is on duty only for 60 seconds. Some heat also will be dissipated, in spite of the short time.

CHAPTER XIV

HEAT RADIATION

Some elementary properties of radiant heat[1] are discussed in this chapter, and a few simple cases of heat transfer by radiation are developed. Heat-radiation law seems not to be well understood by the engineering fraternity. As is illustrated within the chapter, many engineering books and some physics works contain confusing, misleading, and often erroneous statements.

The principle of multiple reflections and a method for handling cases in which this factor enters are developed herein. This kind of work has, of course, been handled in the field of illumination engineering, but it seems not to be generally understood as applied to heat-radiation problems even of the simpler kinds.

The writer wishes to express his thanks to Dr. E. F. Barker, Physics Department, University of Michigan, for his kindness in checking over the manuscript of this chapter, and in suggesting ways for improving it.

117. Nature of Heat Radiation.—Radiant heat waves are electromagnetic waves found in the infra-red part of the spectrum. They are invisible but nevertheless have the same general physical properties as light waves. A given surface may absorb or reflect light waves; so also with heat waves. Both travel in straight lines. Heat waves are reflected by a mirror surface exactly like light waves. Heat waves, like light waves, are transmitted by certain solids, liquids, and gases, and are absorbed by others.

118. Diathermancy and Athermancy.—These terms, though highly expressive of their meanings, seem to be almost forgotten. Diathermancy expresses the ability of a material to transmit heat waves; athermacy, its inability to transmit. A centimeter thickness of water is almost opaque to heat waves; it absorbs nearly all striking it. Pure air transmits almost or quite per-

[1] Preston, "The Theory of Heat," The Macmillan Company; Pender, Harold, "Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc.; Luke, G. E., "Cooling of Electrical Machines," *A.I.E.E.*, December, 1923 (see bibliography at end of Luke's paper).

fectly. Glass is highly athermanous. Impurities in air, such as water vapor, may make it highly athermanous.

A great deal of work has been done in recent years by physicists to determine the absorption properties of gases and liquids, but there appears to be no available tabulation such as would be of immediate use to engineers. It will be assumed in the work of this book that air (or a vacuum) transmits perfectly. The reason for making this statement, which might seem unnecessary to some, is found in the fact that sources of information ordinarily available to engineers, which might be expected to enlighten one as to the transmitting properties of a given material, not only yield no information, but as rule do not even mention the subject.

119. Emission.—The total heat energy radiated by a surface depends on its temperature and on the character of the surface. For a given temperature, a true black-body surface will emit more heat than any other surface.

120. Stefan's Law.—Stefan formulated the law of emission:

$$R = keT^4$$

where R is the rate of heat emission, k is a constant, e is the *emission coefficient*, and T is the absolute temperature. For the black-body, e is equal to unity; lampblack, the nearest practical approach to the black-body, has a coefficient of about 0.98, while that of a silvered surface is about 0.03.

In terms of usual electrical units,

$$\text{Watts per square inch} = 36.9e\left(\frac{\Theta + 273}{1{,}000}\right)^4$$

where Θ is degrees Centigrade.

For all surfaces except the black-body, the emission coefficient changes with the temperature; in general, it increases with the temperature, eventually approaching unity for incandescent bodies.

121. Absorption.—Molecules able to emit energy on a given wave length are also able to absorb energy of that wave length. Also, absorption ability and emission ability for given wave lengths are equal, if unity is taken as the base for each. Suppose a surface emits given quantities of energy only in given wave lengths; then it can absorb only those same wave lengths from radiation which may fall upon it, and it will transmit, or reject by reflection, energy of other wave lengths.

The emission coefficient merely compares the *total* emitting ability with that of the black-body. The black-body emits definite and maximum energy quantities in each wave length, whereas a random surface may emit selectively. If now the energy striking the surface is distributed over the spectrum exactly like that emitted by the surface, then emission and absorption coefficients are equal; but if otherwise, they are unequal.

122. Gray-bodies.—The gray-body may be described as one for which the energies emitted for all wave lengths are toned down by a proportionate amount as compared with the black-body; that is, if a gray-body has $e = 0.5$, it then emits, at the same temperature, half as much energy in *every* wave length as does the black-body. It follows that if gray-body radiation falls upon a gray-body, emission and absorption coefficients are equal.

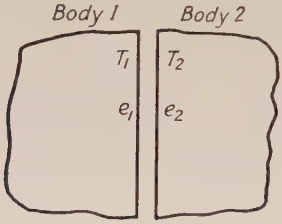
In order to make possible any simple attack on the radiation problem as it arises in engineering, it may be assumed in the following derivations that all surfaces dealt with are gray- or black-body surfaces; then e will be both emission and absorption coefficient.

123. Heat Transfer by Radiation between Parallel Surfaces.—Two bodies, 1 and 2, are placed so that their plane faces are near together and parallel; face area is to be large compared with separation distance. Let their emission coefficients be e_1 and e_2, and their absolute temperatures, T_1 and T_2. Let their areas be equal.

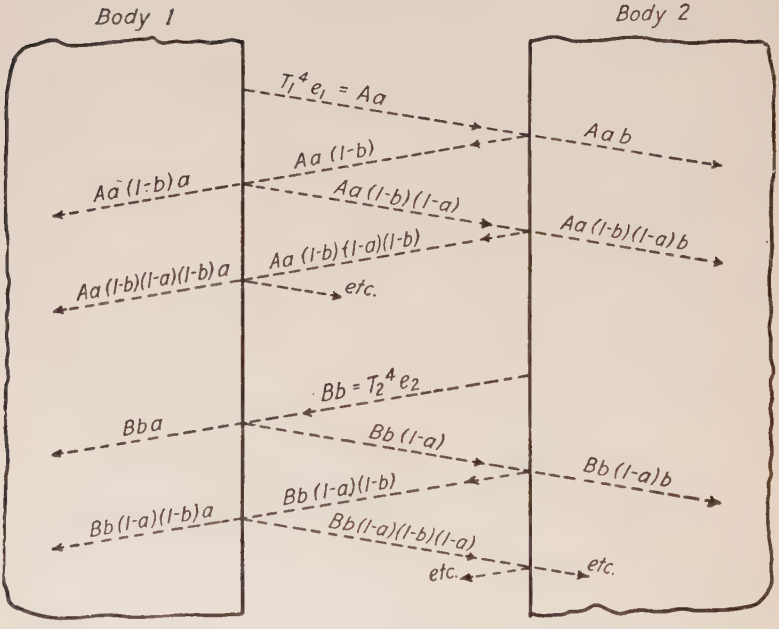
The total heat emitted from the face of body 1, in a given time, is proportional to $e_1T_1^4$ and that from face of body 2, to $e_2T_2^4$. The succeeding work is more easily written if the subscripts and exponents are eliminated by substituting aA for $e_1\ T_1^4$, and bB for the other expression. The actual conditions are shown in Fig. 30*a*, while Fig. 30*b* is made up in terms of the substituted values.

The net rate of heat transfer is also an interesting factor to determine. This can be stated by finding the net heat lost by body 1, or the net heat crossing the separating space, or the net heat gained by body 2. The first of the three methods will be followed. Body 1 loses initially its emitted energy Aa, all of which strikes body 2. Body 2 absorbs b times that, or Aab, and reflects back to body 1 an amount equal to $Aa\ (1 - b)$. Of this amount, body 1 absorbs $Aa(1 - b)a$, which is the first quantity gained by that body; the reflections and absorptions continue as shown by the scheme of Fig. 30*b*.

Also, body 2 initially emits Bb, all of which strikes body 1; and the subsequent history of this energy is shown, for the early terms, on the drawing. Thus it is seen that while body 1 loses



(a) Conditions



(b) Multiple Reflection and Absorption Scheme

FIG. 30.—Heat transfer by radiation between parallel surfaces.

only Aa, it receives by absorption certain decreasing quantities of both emissions.

If, temporarily, the factor $(1 - a)(1 - b)$ is replaced by X, the net heat lost by body 1, due to its own emitted energy, is given by the first of the two infinite series given below; and the heat

lost by it (actually a gain) due to the energy emitted by body 2 is given by the second infinite series. These are:

$$Aa - Aa(1-b)a - Aa(1-b)aX - Aa(1-b)aX^2 - \cdots$$
$$- Bba \qquad - BbaX \qquad - BbaX^2 - \cdots$$

Adding,

$$Aa - [Aa(1-b)a + Bba]\,[1 + X + X^2 + X^3 + \cdots].$$

But the series $1 + X + X^2 + \cdots = \dfrac{1}{1-X}$, if X is less than 1. This puts the above expression into the form (resubstituting for X),

$$Aa - \frac{Aa(1-b)a + Bba}{1-(1-a)(1-b)},$$

which reduces to

$$(A - B)\frac{ab}{a+b-ab}$$

or resubstituting for these symbols their actual meanings,

$$(T_1^4 - T_2^4)\frac{e_1 e_2}{e_1 + e_2 \oplus e_1 e_2}. \qquad (1)$$

minus

The rate of heat transfer in watts per square inch would be found by multiplying the above expression by $36.9 \times 1{,}000^{-4}$.

124. Heat Transfer by Radiation between an Enclosed Body and the Enclosing Walls: the Concentric Spherical Case.—A spherical body called body 1 (Fig. 31*a*) has an absolute temperature, an emission coefficient, and a surface area of T_1, e_1, and S_1 respectively, and the enclosing spherical wall has similar characteristics as shown. The total initial emission rate of body 1 is represented by $T_1^4 e_1 S_1$; replace this by the more easily written expression $Aa\mathfrak{A}$. Likewise, initial emission of the walls of body 2 may be written $Bb\mathfrak{B}$. As before, the net heat lost by body 1 will be sought.

$Aa\mathfrak{A}$ is initially emitted by body 1; all of it strikes the walls.
$b(Aa\mathfrak{A})$ is absorbed by body 2.
$(1-b)(Aa\mathfrak{A})$ is reflected by body 2.
$Bb\mathfrak{B}$ is initially emitted by body 2.
Hence, the total first quantity leaving body 2 is $[(1-b)Aa\mathfrak{A} + Bb\mathfrak{B}]$. Temporarily, let H stand for this quantity.

The next question to settle is, If the walls radiate a quantity H, what happens to it? Of this quantity, a part is going to be absorbed by body 1, and the remainder must again strike the

walls of body 2. Note first this fact: that the heat leaving any infinitesimal element of wall area will behave just like that from any other similar small area; hence the whole amount H may as well be treated at one time.

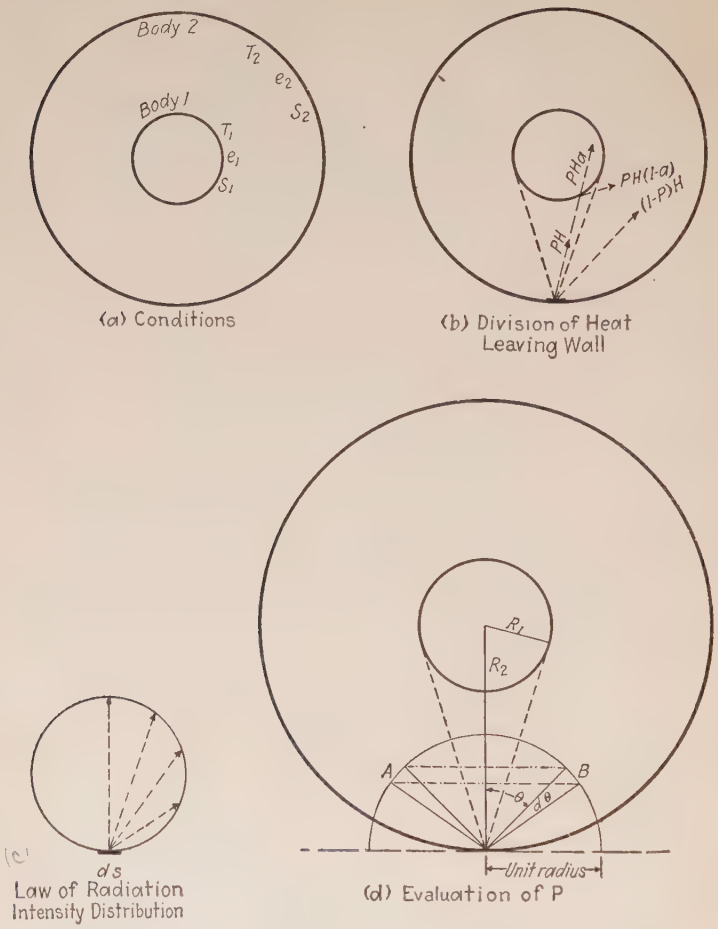


FIG. 31.—Heat transfer by radiation between concentric spheres.

Referring to Fig. 31*b*, a differential area dS_2 is shown, and as just stated, all the heat H may be considered as leaving from this small area. Now of all the heat H leaving the area, only that portion radiated out the solid angle subtended by body 1 will strike body 1. Let P be the fraction striking body 1, so that PH strikes body 1, and $(1 - P)H$ goes on past.

Of the quantity PH, aPH is absorbed by body 1 and $(1 - a)PH$ is reflected back. Thus the quantity H is divided so that

H is leaving the wall.

aPH is absorbed by body 1.

$(1 - P)H + (1 - a)PH$ goes on to strike the wall again.

The last quantity may be written $[(1 - P) + (1 - a)P]H$.

Of this amount striking the wall, that reflected is

$$(1 - b)[(1 - P) + (1 - a)P]H,$$

and aP times that is next absorbed by body 1, etc. The next reflection from the walls will be seen to be $\{(1 - b)[(1 - P) + (1 - a)P]\}^2H$, etc. Thus the beginnings of an infinite series expressing the increasingly small quantities absorbed by body 1 are obtained. The series is of the form $1 + X + X^2 + X^3 + \cdots$, where X is less that 1. Remembering that the initial heat lost by body 1 is $Aa\mathfrak{A}$, then the summing up of terms gives its net heat loss as

$$Aa\mathfrak{A} - aPH \frac{1}{1 - \{(1 - b)[(1 - P) + (1 - a)P]\}}.$$

It is now time to replace H by that for which it was substituted, giving

$$Aa\mathfrak{A} - \frac{aP[(1 - b)Aa\mathfrak{A} + Bb\mathfrak{B}]}{1 - \{(1 - b)[(1 - P) + (1 - a)P]\}},$$

which, when multiplication and cancellation are completed, gives

$$(A\mathfrak{A} - B\mathfrak{B}P)\frac{ab}{aP + b - abP}.$$

Introducing now the original symbols, the net heat lost by body 1 is

$$(T_1^4S_1 - T_2^4S_2P)\frac{e_1e_2}{e_1P + e_2 - e_1e_2P}, \qquad (2)$$

As the constant has been left out, this expression needs the multiplier 36.9 to make it give watts.

125. Distribution of Radiation over the Solid Angle.—If the intensity of radiation from a small flat surface element dS_2 were equal in all directions, the fraction P would be equal to the solid angle subtended by body 1, divided by 2π. But the distribution of radiant intensity, at any constant radius, is represented by the circular locus shown in Fig. 31*c*. The surface radiates most intensely along the normal. Referring to Fig. 31*d*, if I is the intensity of radiation at unit radius along the normal to the

elemental area dS, the intensity at unit radius along a line at angle Θ to the normal is $I \cos \Theta$.

The annular ring AB defined by the differential angle $d\Theta$ has an area equal to $2\pi \sin \Theta d\Theta$, and the radiation through it is expressed by

$$I \cos \Theta 2\pi \sin \Theta d\Theta.$$

The total radiation through the solid angle indicated by the plane angle Θ is

$$2\pi I \int_0^{\Theta} \cos \Theta \sin \Theta d\Theta$$
$$= \frac{\pi I}{2} (1 - \cos 2\Theta).$$

Now the solid angle subtended by body 1 is such that the sine of its plane angle Θ is R_1/R_2. The radiation out this cone is

$$\frac{\pi I}{2} (1 - \cos 2\Theta)$$
$$= \frac{\pi I}{2} (1 - \cos^2 \Theta + \sin^2 \Theta)$$
$$= \frac{\pi I}{2} \left(1 - \frac{R_2^2 - R_1^2}{R_2^2} + \frac{R_1^2}{R_2^2}\right)$$
$$= \frac{\pi I}{2} \left(\frac{R_2^2 - R_2^2 + R_1^2 + R_1^2}{R_2^2}\right)$$
$$= \pi I \frac{R_1^2}{R_2^2}$$

when $\theta = \pi/2$, the total heat radiated becomes

$$= \pi I.$$

Therefore,

$$P = \frac{\pi I \dfrac{R_1^2}{R_2^2}}{\pi I} = \frac{R_1^2}{R_2^2} = \frac{S_1}{S_2}.$$

This makes it possible to write the expression for heat-transfer rate in watts for the concentric spheres as

$$\frac{36.9}{1{,}000^4} S_1 (T_1^4 - T_2^4) \frac{e_1 e_2}{e_1 \dfrac{S_1}{S_2} + e_2 - e_1 e_2 \dfrac{S_1}{S_2}}. \qquad (2a)$$

126. The Stefan-Boltzmann Law.—If in the above expression the enclosed sphere is made so small that the ratio of S_1/S_2

becomes negligibly small, or if e_2 is made unity by virtue of using black-body walls, then the expression degenerates into

$$S_1 e_1 (T_1^4 - T_2^4), \qquad (3)$$

which is the Stefan-Boltzmann law. This law was originally formulated from experiments made with small enclosed bodies, the enclosing walls being blackened; yet many modern writers have made the mistake of supposing that this law would hold for *all* cases of enclosed bodies. One has no difficulty in finding plenty of evidence of a confused state of mind on this point.

127. Large Sphere in Close-fitting Enclosure.—If the concentric spherical case is specialized by making the two radii very large compared with their difference, it is easy to show that the concentric-sphere expression approaches as a limit Eq. (1) which is for flat parallel surfaces.

128. Concentric Cylinders.—The treatment is exactly like that for the spheres, up to the point where P is to be evaluated. It is then found that P again comes out equal to the ratio of the two surfaces, or in this case, to R_1/R_2. Proof that $P = R_1/R_2$ is not given here.

129. Irregular Cases.—In practice, radiation cases appear in infinite variety. Unless some kind of perfect symmetry is present, a mathematical solution is out of the question. If the case is near some limiting condition, such as a small body in a large room, the Stefan-Boltzmann law (Eq. (3)) would apply very well, as it would also if the walls have a very high coefficient. If the enclosed body is fairly round or fairly cubical and is enclosed by a fairly small room, the body being near the center, the shortest approximate solution would be obtained by guessing at what would be the equivalent concentric spherical case and solving that case instead of the real one. Cases which are very lengthy along one axis, as is a cable in a duct, might be attacked by dividing the surfaces of both wall and body into a fairly small number of parts, and solving by tabulating the multiple reflection and absorption terms in some manner. The job would be lengthy but by no means impossible. In most cases, the series is rapidly converging.

130. Popular Errors Made in Handling Radiation.—The writer has made a search through many representative engineering books, handbooks, and physics texts, and some interesting facts have come to light. Using, as above, T_1 to represent the

absolute temperature of the body, T_2 is variously stated by different writers to be the temperature of the surrounding air, the temperature of the surrounding objects, the temperature of the surrounding space, and the temperature of the surrounding walls. Nor is this all.

Every work on physics referred to either stated that the Stefan-Boltzmann law (Eq. (3)) holds for any case of a body completely enclosed or else, by failing to give exceptions, allowed one to make such a deduction. Most engineering works follow that lead and state the Stefan-Boltzmann law as a general law. One handbook may be excepted: for enclosed bodies, it gives the law derived above (Eq. (1)) for parallel surfaces and makes no exceptions.

The derivations above, for parallel surfaces and for spheres, are original with the writer; but the spherical case at least must be available somewhere in the literature on illuminating engineering, and very likely elsewhere, if it could only be found. The point is, that sufficient information on the laws of heat radiation is not to be found in the usual references available to the engineer, and of that which is available, much is misleading and much erroneous.

131. Emission Coefficients.—Coefficients for a few surfaces follow:

TABLE IV.—EMISSION COEFFICIENTS FOR HEAT RADIATION

Lampblack	0.98
Oils, varnishes, insulating materials	0.80 to 0.90
Matt surface paper, white	0.90 or better
Copper, oxidized	0.74
Cast iron, freshly machined	0.25
Cast iron, oxidized	0.65
Silver, polished	0.03
Tinned surface	0.50
Steel surface, oxidized	0.75

132. Emission by Grooved, Scored, or Corrugated Bodies.—Most of the books dealing with heat radiation say that a body emits radiant heat in proportion to its area. This is true only when no heat radiated from a portion of the surface is able to strike some other part of the same surface, as when the body is spherical, cylindrical, cubical, etc.

Imagine a hollow sphere, with no holes through the surface into the interior. Outer surface area would determine the radiation. Now let a small round hole be cut through to the interior. Would radiation be determined by the sums of the remaining

parts of both outer and inner surfaces? Not at all. Granting that the material has a fair emission coefficient, then the window constitutes a practical equivalent to a black-body surface. This is true, because if any radiation from another source enters the window or hole, multiple reflections and absorptions inside the sphere will insure that only an extremely small portion of the radiation entering will escape; and if the window acts to take in and keep all radiation passing through it, then it acts as a black-body surface. The radiation of this sphere would then be determined by the remaining outer surface and its coefficient, with an added quantity from the window which has a coefficient of unity; cutting the window has raised the radiation of the sphere.

Consider a corrugated surface with respect to radiation falling upon it. That which enters a valley will be partly reflected, but by no means will all of the reflected portion escape; if the valley is narrow and deep, a good share of the reflection will again strike the walls of the valley, again to be partly absorbed and partly reflected. Thus it is perfectly evident that the corrugated surface will absorb a good deal more radiation than would an enveloping smooth surface having the same emission coefficient.

This case is mentioned, not only because it brings out the facts, but also because there is a rather popular belief that a transformer tank, corrugated, will radiate no better than if it were left smooth. This is true *only* in case the emission coefficient is unity.

In a given case, if the surface is studied with respect to radiation falling upon it, the radiation it absorbs can be found which can then be compared with that which it would absorb if a smooth enveloping surface were present instead. The actual emission coefficient can then be multiplied by the ratio of these two quantities, and this higher coefficient, worked in conjunction with the enveloping surface, can be used. This gives an effective surface and an effective coefficient, good either for absorption or emission calculations.

133. Emission Coefficient of a Slot.—Suppose a rectangular slot is cut into the plane surface of an emitting body. If the slot is long axially, the radiation intensities distribute according to two-dimensional laws. A study of this case shows that the portion of all the heat originally emitted from slot walls and bottom which at once escapes through the slot opening is just equal to that which would be emitted from that part of the plane surface which was removed by creating the slot. But, if the slot is, say,

as deep as it is wide, then two-thirds of the initial emission strikes the slot walls and bottom, there to be partially absorbed and partially reflected; the reflected portion partly escapes out of the opening, thus adding to the total loss of heat.

The writer made a study of the square-slot case, finding that if $e = 0.5$, the slot emits 50 per cent more heat than the surface it replaced. This can be stated in terms of coefficients, by saying that if actual surface $e = 0.5$, "effective" $e = 0.75$. For the square slot, as wide as it is deep, the following approximate values obtain:

Actual Emission Coefficient	Effective Coefficient
0.25	0.48
0.50	0.75
0.75	0.89

The effective coefficient must, of course, be used in connection with the surface replaced by slot opening.

The findings mean that if a surface has $e = 0.5$, its emission can be raised by 50 per cent by attaching fins to it at right angles so as to create adjacent slots of square section.

134. Selective Emission as Affecting Choice of Paints.—Consider the case of an enclosed body painted so as to emit selectively, the enclosing walls being painted so as to emit selectively also; if selectivity for the two paints agree, as when the same paint is used, good radiation transfer for the enclosed body is possible; but if the selectivities greatly disagree, transfer by radiation may be seriously reduced, and this may happen without the knowledge of persons concerned. Until more data are experimentally obtained, cases hinging upon radiation effects will bear close investigation.

Seemingly, color has little to do with emission coefficient. For establishing the best cooling conditions, the apparatus and the surrounding walls should be painted with a black-body paint.

Problems

16. An enclosed motor is installed in a small underground chamber for pumping. In equivalent terms, the case corresponds to a sphere that is a black-body, with a 2-foot diameter, enclosed by concentric spherical walls with a 4-foot diameter. The walls have a coefficient of 0.6, wall temperature being 20°C. With motor at 40° rise, find watts radiated.

17. The surface of an armature runs at 70°C., while the pole faces opposite run at 60. A pole face has 50 square inches, there being four poles. If

both surfaces are black-bodies, find the watts transferred to the pole faces by radiation.

18. There are two parallel surfaces, A and B. A is maintained at 300°C. and has a coefficient of $e = 0.8$; B is at 20°C., with $e = 0.3$. Can heat loss by A be reduced by inserting a thin plate between the two surfaces? Try it out by inserting, halfway between, a plate having negligible thickness and $e = 0.5$ on both sides. What temperature will the insert have?

19. A radiation problem involving concentric cylinders is roughly approximated by a tall round transformer tank located in a small room. The radii are 3 and 5 feet. For tank, $e = 0.9$; for wall, $e = 0.8$. Tank at 65, wall at 25°C. Find watts radiated per vertical foot.

20. Idealize the vacuum-bottle case by assuming the inner bottle to be a closed cylinder, of a height equal to four times the diameter, this being freely suspended inside an enclosing cylinder. The container when full holds 1 pound of water. The opposing surfaces of the two cylinders are silvered, the space between being evacuated. Let the container be filled with water at 0°C.; let the outer jacket be brought to 30°C. and held there. How long before the water reaches 10°C.? Solve step by step (difficult mathematically).

21. Suppose a 50-watt, 110-volt, metal-filament bulb has a filament 18 inches long which operates at 1300°C. The glass bulb, which is evacuated, runs at 120°C. Find the diameter of the filament and its resistivity in ohms per circular mil-inch.

CHAPTER XV

HEAT DISSIPATION

Heretofore, the student of electrical engineering has usually been given little training in the laws of heat flow and heat dissipation. As nearly all electrical devices either have an overload temperature limit or else are limited in capacity for steady load by temperature rise, it follows that the engineers designing apparatus, those buying apparatus, and those responsible for operation after installation should all have as a part of their basic training a considerable amount of work in thermal problems of various types.

This chapter deals with heat dissipation from heated surfaces to air. Enough natural convection data are included to handle specific types of natural cooling problems and to indicate the procedure for unusual cases. Experimental data on the cooling of odd shapes are meager. In the work on forced convection cooling,[1] the writer believes that the most trustworthy results, as published at the present writing, are herein set forth; no attempt was made to assemble material not of interest to the electrical engineer.

No material on the Langmuir film theory, is included, nor is the later work of Rice and others dealing with the method of dimensions and the properties of gases. For information on these subjects, the reader must be referred to the original papers.

135. Terminology.—Many writers in the past have referred to the area which is dissipating heat to its surroundings, as the *radiating surface.* As will be seen, radiation plays an important part in heat dissipation, only in case air velocities are very low. The fact is that a surface loses heat by the three processes—radiation, conduction, and convection. The surface should in general then be referred to as the **dissipating surface,** not the radiating sur-

[1] The data included in this chapter are taken mainly from the reports of G. E. Luke. This able investigator kindly gave the author permission to draw freely from his publications only a few months before his untimely death.

face. When the general loss of heat is mentioned, it should be called dissipation (or by a kindred term), not radiation.

136. Dissipation Coefficient.—Calculations concerning heat enter so universally into engineering and physical work of all kinds that every sort of unit and every sort of expression for stating the heat properties of a material or state have been used. Let one attempt to gather together a large fund of information on the thermal properties, and he will find himself using all kinds of conversion factors. Much of this sort of trouble can be removed if the use of one type of expression will be insisted upon. In this book, all thermal coefficients will begin with "watts per —." All dissipation coefficients will be stated in terms of *watts per square inch per degree Centigrade rise or difference.*

137. Radiation Laws.—These have been developed in the preceding chapter.

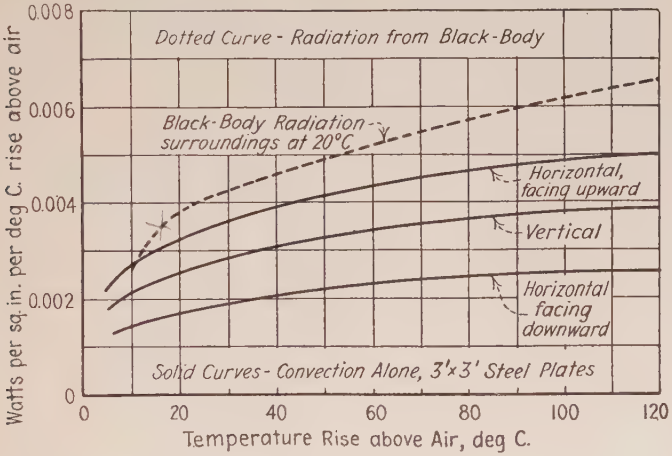


FIG. 32.—Dissipation by natural convection alone, and by radiation alone.

On the basis of e for the radiating surface being unity and with room walls at 20°C., the dissipation coefficient for pure radiation is given by the dotted curve in Fig. 32.

It is easily seen that with usual temperatures of surroundings, if the radiating surface has a low temperature *rise*, such as 20°, its radiation is radically affected by its surroundings. But if the rise becomes more than, say, 60°, the exponent of 4 in the radiation law minimizes the effect of the surroundings. For these higher temperature rises, radiation often is calculated as if the second term of the above equation amounted to zero.

The relative importance of radiation for different conditions of cooling will be brought out in the succeeding work.

138. Conduction through Air.—Dead or still air is one of the worst heat conductors known. In all cases, air must be present under special conditions in order to be or remain still. But those special conditions, unfortunately, occur only too often. No matter how fast air moves or is forced to move past a solid surface, a film of air is held next to the surface; and heat must be conducted through this blanket of air. All usual fabricated insulating materials are more or less full of voids, and unless these voids are later filled with some compound, the entrapped air greatly contributes to the temperature rise. The low thermal conductivity of still air is seen by referring to Table V and comparing it with other materials.

139. Natural Convection.—Unless the air near to a heated surface is baffled, convection currents are bound to be present. The heated air near the surface expands and rises, and cooler, denser air replaces it. The process adds greatly to the cooling otherwise already present due to radiation.

It was only somewhat recently that any extensive tests for free natural convection dissipation were made. Total dissipation by pure natural convection varies with the character of the body, surrounding conditions, and the 1.25 power of the temperature rise. As the dissipation coefficient is a matter of "per degree rise," the coefficient varies with the 0.25 power. From tests by Griffiths and Davis, the dissipation coefficients for polished metal plates held in three positions are given by Fig. 32. The effect of position upon the freedom of flow of convection currents and therefore upon the dissipation is clearly indicated by the curves.

140. Natural Cooling: Radiation and Convection.—Luke[1] supplies curves for three plates, by which the writer has computed the dissipation coefficient and obtained the curves (Fig. 33). The large black plate dissipates more watts per square inch due to its ability to create greater air speeds than does the small black plate. The brass plate has a low coefficient due to its very low radiation ability.

It is evident from the data given so far that, for black surfaces held stationary in unobstructed and otherwise still air, convection and radiation each account for something like half the total

[1] Data for this chapter taken from Luke's paper as published in the *Journal, A. I. E. E.*

dissipation. Thus radiation, hence the surface character, is a very important factor for such devices as enclosed motors and control coils, or other unventilated and stationary devices.

141. Natural Cooling: Irregular Bodies.—It is easily shown that a corrugated or wrinkled surface, if it is a black-body surface, *radiates* no more heat than would the minimum enveloping surface. But an increase of surface, obtained by corrugations running the same way as the convection air currents, will increase convection dissipation. On the other hand, corrugations running horizontally might easily reduce flow and convection dissipation.

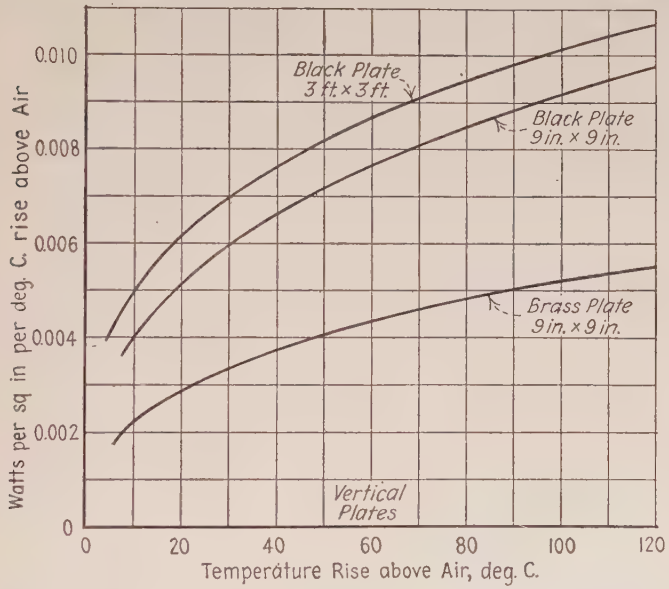


Fig. 33.—Dissipation by natural convection and radiation, still air.

In highly irregular surfaces, the total dissipation usually is increased somewhat, but not in proportion to the surface.

For railway motors with car at standstill and using the total surface, equivalent of the cylinder, Luke uses a coefficient of 0.013 watt per square inch per degree Centigrade rise; at car speed of 10 miles per hour, 0.026.

142. Natural Cooling of Horizontal Cylinders.—The dissipating ability of a wire depends upon its diameter. Small diameters either help to reduce the air film next the surface, or to speed up the convection currents, or both. Luke gives results

obtained with wires and cables tested in a still room for diameters up to 1.25 inches; Heilman ran tests on horizontal black iron pipes up to diameters of 18 inches. The writer has figured the Heilman dissipation rates in terms of dissipation coefficient and put the two sets of results on the same axes (Fig. 34). (Note the change of scale at 1-inch diameter.) The two investigations match up very well indeed; the pipes show higher coefficients, as would be expected, as they have better radiation-emission coefficients than wires and cables.

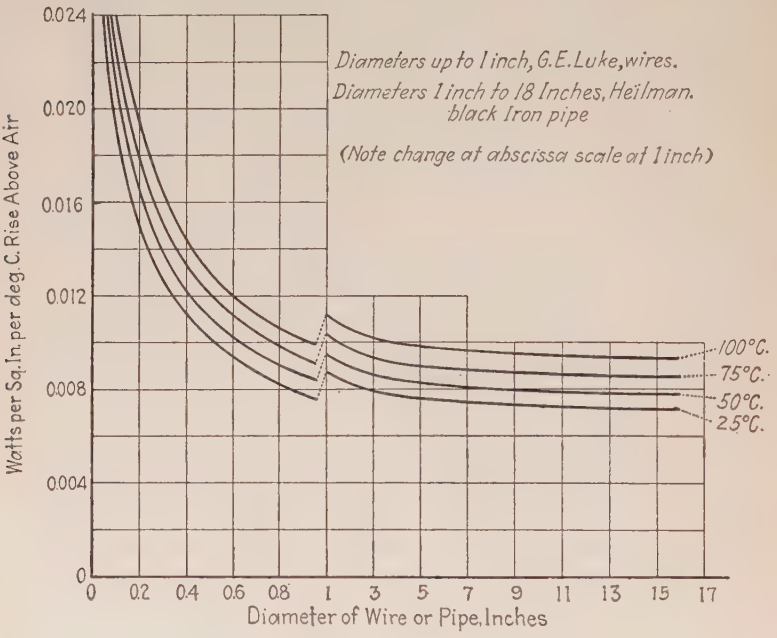


Fig. 34.—Dissipation from horizontal wires and black iron pipe.

143. Forced Convection.—In Nature, all air currents are natural convection currents. As Nature has a rather large laboratory, it is not uncommon for wind velocities to mount to 100 miles an hour; and when things are really going on a grand scale, twice this figure is possible. But all electrical apparatus is limited in size, and so cooling convection currents move very slowly.

By stimulating the otherwise sluggish natural air currents to high velocities, great increases in heat dissipation are obtained. It is possible greatly to expand the dissipating surface of a transformer and thus to arrive at sufficient natural cooling for this

stationary device. But a rotating machine presents a strict limit to having its total surface expanded. An armature or stator can only be divided safely into so many bundles, or perforated by so many axial ducts; and the surface thus obtained is totally inadequate, from the natural ventilation standpoint, to rid the machine of a normal amount of loss. High air velocities must be attained.

Until very recently, almost no trustworthy data on dissipation by forced convection currents were at hand. But among other things, the removal of commutation as a capacity limit on direct-current machines and the demand for larger and larger alternating-current machines have placed emphasis on the fact that it is now temperature that limits capacity.

Through such recent investigations as those of G. E. Luke, the dissipation of a given surface, for given forced-air velocities, has been determined for many cases.

144. Surface Friction: Turbulence.—It is found that, with forced convection, a rough surface dissipates more heat than a smooth one. The dissipation does not rise as fast as the air friction losses; nevertheless, the gain in dissipation economically offsets the increase in windage loss up to air velocities of several thousand feet per minute.

Turbulence is the term used to define that condition of air flow obtaining when the smooth streamline flow of low velocities is broken up into eddies and cross-currents. For a given duct there is an air velocity that is critical: below it, flow is uniform; above it, turbulent. For round ducts, the equation is

$$V_c = \frac{(225 \text{ to } 330)}{d} \left(\frac{T}{273}\right)^{1.75}$$

where

V_c is the critical velocity in feet per minute.
T = absolute temperature of the air, Centigrade scale.
d = duct diameter in inches.

For example, if the temperature of the air is 30°C., or 303 absolute, and d is 1 inch, V_c is about 340 feet per minute.

There is a sharp increase in dissipation when turbulence occurs. The reason, of course, is that turbulent flow scours the dead air film away from the heated surface far faster than does a smooth air flow. For forced convection, velocities below the critical velocity are not worth using. In self-cooled, direct-current

armatures with radial ventilating ducts, if the duct air velocity is below the critical value, the duct had better not be there.

It appears that there are degrees of turbulence, as caused by differing shapes of orifice or passage; and that dissipation is radically increased by increase in turbulence, for a given air velocity. Little is known of this highly important subject, but enough is known, as indicated by the following accounts of investigational results, to show that it is important.

145. Test Methods.—As most of the results later to be given are due to Luke, his method of making measurements should be noted.

The most difficult feature of all is to measure air velocities with accuracy. The anemometer, the pitot tube, and the orifice meter methods were in general found inaccurate as compared with the specific-heat method. In this method, advantage is taken of the fact that with air weighing 0.074 pound to the cubic foot at 25°C. and 29.92 inches of mercury, its specific heat of 0.2418 enables it to absorb 1 watt-minute of energy per cubic foot for a rise of 1.765°C. Thus

$$\text{Cubic feet per minute} = \frac{1.765W}{\theta}$$

where

W = watts absorbed by the air.

θ = resulting air temperature *rise*, beginning at 25°C.

Then,

$$V = \frac{\text{cubic feet per minute}}{\text{square feet of duct area}}$$

where

V = average duct air velocity, feet per minute.

It follows that if the watts given up to the air are known, and if inlet and outlet air temperatures are measured, average velocity becomes known. Thermocouples were used to measure temperature difference between cool and heated air. Several hot junctions were placed in the outlet, and their cold junctions in the inlet; all were placed in series. Suitable baffles and screens were used to mix the air at each end thoroughly before going past the thermocouples, which were distributed over the openings. Reference should be made to the original papers for details. Suffice it to say that high accuracy is obtained by this method.

In a given case, the dissipation coefficient was calculated by

$$k_d = \frac{\text{watts loss}}{S\left(\Theta_s - \frac{\Theta_a}{2}\right)}$$

where

k_d = dissipation coefficient, watts per square inch per degree Centigrade.
S = ventilating surface of duct in square inches.
Θ_s = average duct surface temperature rise above inlet air.
Θ_a = temperature rise of outlet air.

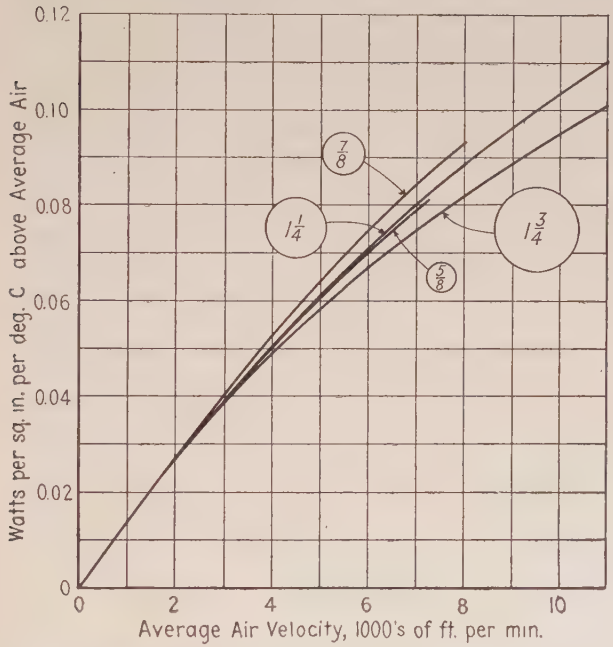


FIG. 35.—Dissipation from interior surface of smooth tubular ducts.

In finding the watts given up to the air, proper means were taken for finding the stray watts loss of the apparatus. Total watts minus stray watts gave watts absorbed by the air.

Be it noted that this method for finding the coefficient finds the *average coefficient* for the whole surface; other means were resorted to for finding the coefficient for various parts of the surface, as explained later.

146. Average Dissipation Coefficient for Tubes.—The average dissipation from the inside surface of smooth tubes 36 inches long gave results as shown in Fig. 35. The effect of using the same duct perimeter with various types of section shape shows up clearly in Fig. 36. The increase for odd sections cannot be due to changes in velocity or surface friction but only to increased eddying, or turbulence.

To increase the turbulence and yet have a round duct, a spacer was inserted in one of the tubes and given two twists in

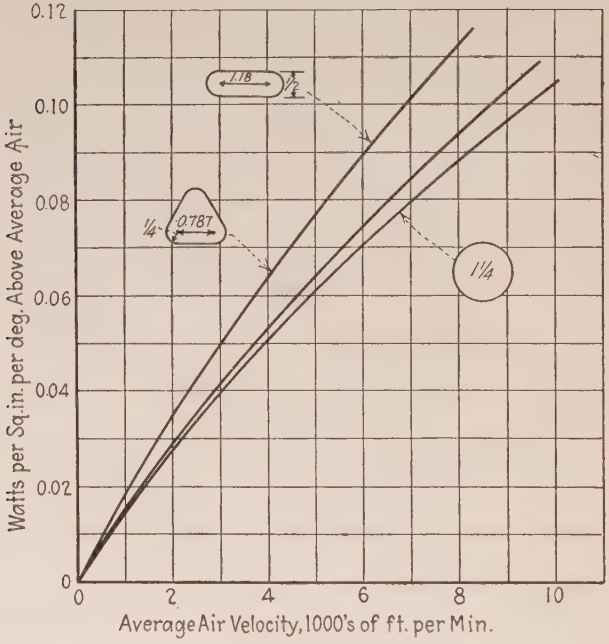


FIG. 36.—Dissipation from interior surface of smooth ducts of equal perimeters.

its 36-inch length. The two air paths were forced to corkscrew about the axis twice. The surprising increase of 50 per cent in dissipation coefficient resulted, as shown by comparing the solid curves (Fig. 37).

147. Average Coefficient for Axial Ducts.—In machines, an axial duct pierces the laminations; hence any duct, in machine, transformer, or whatnot, running through the laminations, is called an axial duct. The irregularity of laminations in manufacture and assembly gives to these ducts a rough surface which

should raise both friction and dissipation. Static air-pressure tests showed the friction increase, and the upper dotted curve in Fig. 37 shows a 25 per cent (or greater) increase in dissipation coefficient over the smooth round duct of the same diameter.

A disadvantage of the rough axial duct is that it tends to collect dust, which is a heat insulator. The lower dotted curve in Fig. 37 was obtained after putting on a layer of coal dust 0.030 inch thick. As dust both lowers dissipation and tends to start or

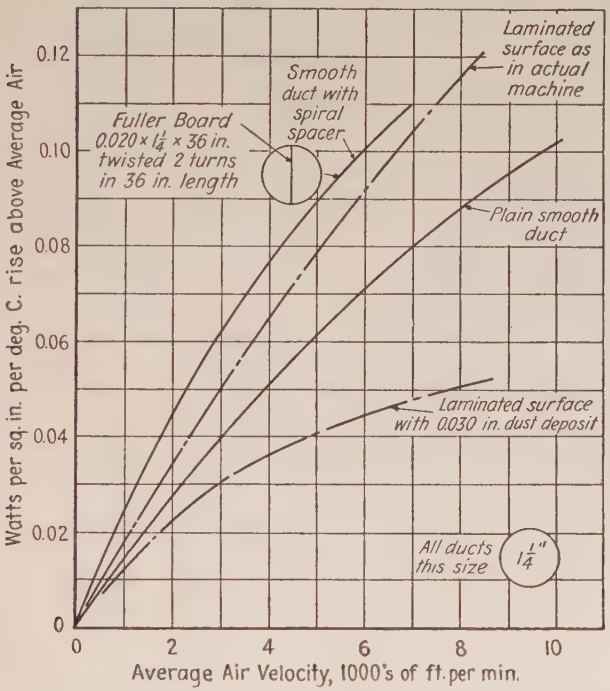


FIG. 37.—Dissipation from interior surface of tubular ducts.

favor the starting of fires in large machines, recirculation of clean air offers great advantages, and this idea has already met with success in a number of installations.

All ducts mentioned so far were 36 inches in length.

148. Varying Dissipation along the Duct Length.—Results of various investigators working along the general lines outlined above showed unexplained discrepancies. It was thought that variation in turbulence along the duct had much to do with these discrepancies. Luke, therefore, arranged a test by which the

dissipation rate for any position along the duct could be determined.

An axial duct 39.5 inches long and 1.25 inches in diameter was built up by assembling "washers" cut from laminations. Thermocouples placed along the duct measured the iron temperatures; inlet and outlet air temperatures were accurately measured. Thermocouples placed in the heat insulation surrounding the iron tube enabled stray losses to be computed. The thermal conductivity across the laminations was found by separate test to be 0.040.

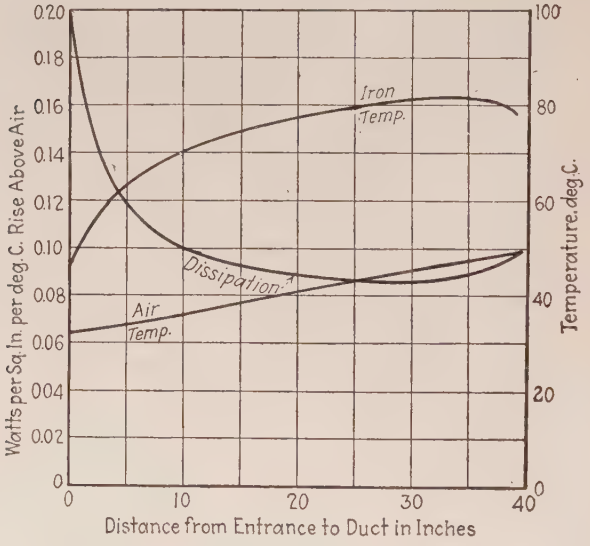


FIG. 38.—Dissipation and temperature, for round 1¼ in. laminated duct, 39¼ inches long. Average air velocity 5,760 ft. per min.

Any duct section, such as a section 1 inch long axially, had developed a known wattage, by means of the heating element wrapped around it. This wattage was dissipated as stray loss by absorption by the duct air and, depending on which section it is, by conduction axially to the next section. Calculations similar to the type mentioned later, along Lamme's "unit-package" idea, were made, and the results plotted in Fig. 38 were obtained.

Discussing first the end conditions, it is evident that the duct iron temperature must drop somewhat toward the ends, as the curve shows, due to end losses. Hence, the temperature rise of air above surface falls at each end; yet the dissipation coefficient,

toward both ends, *rises*. The coefficient at or near the entry is double that near the middle of the duct length. Air follows to a large extent the hydrodynamic laws of water flow, and it is common knowledge that a stream entering a sharp-edged orifice suffers a contraction—the "vena contracta" effect. The entry to this tube was square faced, and the vena contracta results in great turbulence at and near to the entry. Diverging air exhibits increased turbulence, and this is shown by an increasing coefficient at the exit end.

The cause for discrepancies between *average* coefficients for ducts of different length as used by different investigators now becomes apparent.

149. Coefficients for Ducts of Different Lengths.—The turbulence in the duct mouth would be present in either long or short ducts. In a very short duct the entire flow would be highly turbulent, giving a correspondingly high dissipation rate, average value. From the results just spoken of, Luke computed the average coefficients for various ducts, all being 1.25 inches in diameter. These coefficients are given by Fig. 39. The duct 2 inches long is little more than a short orifice, giving a coefficient nearly twice that of a duct infinitely long.

150. Reduction of Turbulence by Converging the Opening.—Wood-facing blocks placed against the entry of the above duct were used; to find the effect of rounding the corners to smooth out the air flow at the entry, rounded-entry blocks were tried out. In every case so tried, the dissipation coefficient for the part of the duct surface at and near to the entry dropped about 15 per cent. Further along in the duct, at a distance equal to 12 diameters, the coefficient was approximately unchanged. At 6 diameters, the drop in coefficient was about 5 per cent.

151. Concentric Ducts.—A concentric duct is formed by placing a cylindrical jacket over a cylinder and forcing the air along the gap between the two.

A short duct was tested by Luke, having square entry, a 7-inch length, an inner diameter of 3 inches, and outer diameter of 5.375 inches. Although the surfaces were smooth, the turbulence was such that the curve coincides closely with that for a 7-inch-long, round, 1.25-inch axial duct through laminations, such as could be interpolated for in Fig. 39. By rounding off the faces of both jacket and cylinder with wood blocks, a converging or smooth air entry was obtained, with the characteristic reduction of 15

per cent in total dissipation for a length so short that it is all near the entry and subject to its turbulence.

A long concentric duct with diameters great compared with radial dimension gave an average coefficient approximately equal to that of a parallel duct between plates with parallel flow, given later herein.

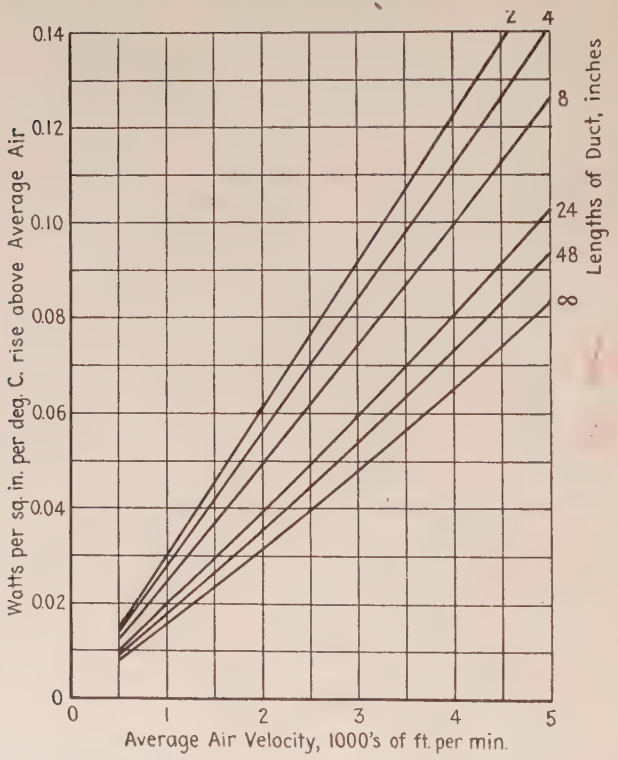


FIG. 39.—Average dissipation, round 1¼ in. axial ducts. Entry, square.

152. Turbulence and Friction.—Surface friction and turbulence each raise the coefficient. But friction operates in itself by causing turbulence. Therefore, if other factors, such as square-entry conditions, cause sufficient turbulence, friction may not add to that turbulence. Exactly this result was found by Luke. One experiment will be mentioned.

To test for the effect of roughness of surface on the short 7-inch concentric duct above mentioned, the cylinder was furnished with a heating element made of close-wrapped 0.0155-inch enamel

copper wire. No definitely certain increase in dissipation was to be found. In a longer duct, the regions far from the entry would exhibit increased dissipation due to increased friction turbulence.

153. Parallel Ducts.—Two hot steel plates were separated by different distances, the plates being 8 inches wide and 22 inches long. For a separation of 0.5 inch, the duct thus formed would be 0.5 by 8 inches in section. All separations from 0.25 up to

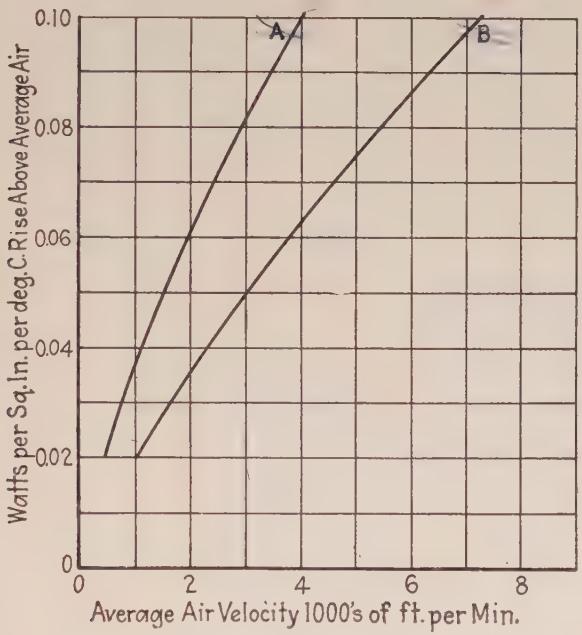


FIG. 40.—*A*, Average coefficient, parallel duct (see text). *B*, Average coefficient, radial duct (see text).

1 inch gave approximately the same average coefficients. Curve *A* of Fig. 40 is for a 0.5-inch spacing; it gives, within about 3 per cent, the coefficients for 0.25- and 0.75-inch spacings; 1-inch spacing coefficients are about 7 per cent below these.

The same general conditions of turbulent flow at the entry are to be expected as was found for the round duct. In the absence of tests on the parallel duct, it is likely that one may assume, with fair safety, that identical variations in dissipation coefficient occur.

154. Radial Ducts.—Machines are more frequently cooled by radial ducts left between bundles of laminations than by any other method. Luke's apparatus for investigating this type of cooling consisted of two annular steel discs placed opposite each other at various spacings, their inner diameters being 10 inches and outer diameters 24 inches. Tests were made with outward air flow.

The average dissipation coefficient was figured by using average air velocity at the mean disc radius (8.5 inches) and total inner surface of the discs. *For unobstructed flow,* the average coefficient was about 60 per cent higher than for the parallel ducts. This big increase is due to two factors: The radial flow in this case is for a shorter length which gives a high turbulence effect, and where turbulence is greatest at the inner air entry velocities also are greatest.

To simulate actual conditions, steel spacers, corresponding to the ventilating fingers of ordinary spacers as used in machines, were introduced. There were 35 of these, $\frac{1}{16}$ inch thick and 7 inches long. They were placed radially, thus reaching the whole radial length of the duct and giving a spacing equal to their width. In addition, 35 blocks of wood were arranged around the edge. The blocks were 0.75 by 1.75 inches, with the longer dimension placed radially; their thickness was equal to duct spacing used. These blocks simulated the effect of coils crossing a radial duct.

In computing the average coefficient as given in curve *B* of Fig. 40, the velocity at the mean radius was again taken, and the *area now used included the exposed areas of the steel spacers.* Note that the coefficient is again (curve *B*) about 60 per cent above that for a parallel duct measuring 8 by 22 inches.

It is easy to show that the steel spacers will have a temperature within a few degrees of that of the surfaces of the duct; hence their effectiveness is nearly in proportion to their area. They furthermore probably add to the turbulence. And although the wood blocks reduced disc area in the outer region, they created a *high-velocity, short-duct, high-turbulency flow* in that region. It is quite evident that the heat dissipation from the sides of teeth and from spacer areas and coil areas in the restricted tooth neighborhood is a very important factor. It is also quite evident that short radial ducts in general are more effective per square inch than long ones, because of turbulence effect.

155. Dissipation from Outer Tube Surface : Forced Convection. Hughes tested for this type of dissipation by means of copper tubes placed across a moving current of air, the average air velocity being measured. Figure 41 gives the coefficients.

156. Rotating Cylinder.—Luke used a micarta cylinder, 25 inches in diameter and 36 inches long, with a heater element of monel metal ribbon wound around the outer surface. When

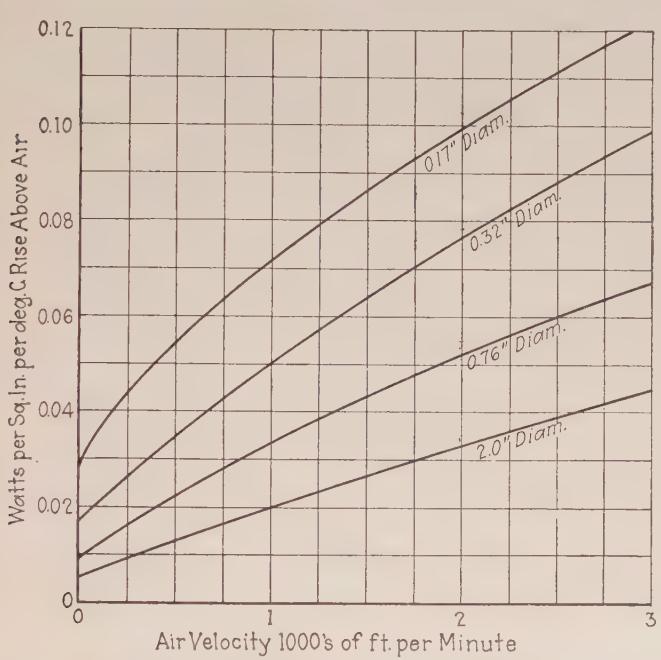


FIG. 41.—Dissipation from outer surface of copper tubes placed across moving air stream.

operated open to the air, the coefficients as given by the lower curve of Fig. 42 were obtained.

157. Axial Air-gap Ventilation.—A tin housing was arranged to cover the above rotor so as to leave a $\frac{1}{2}$-inch air gap; air chambers at either end allowed air temperatures to be measured, while air was forced from one end to the other. With the rotor at rest, the dissipation coefficient varies as shown by the upper curve of Fig. 42. This case is really that of a concentric duct; the curve practically duplicates the parallel-duct curve of Fig. 40.

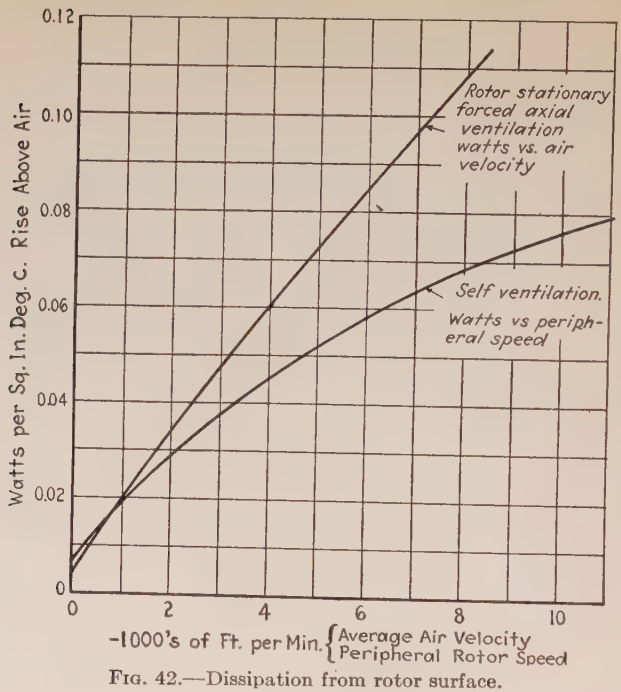


FIG. 42.—Dissipation from rotor surface.

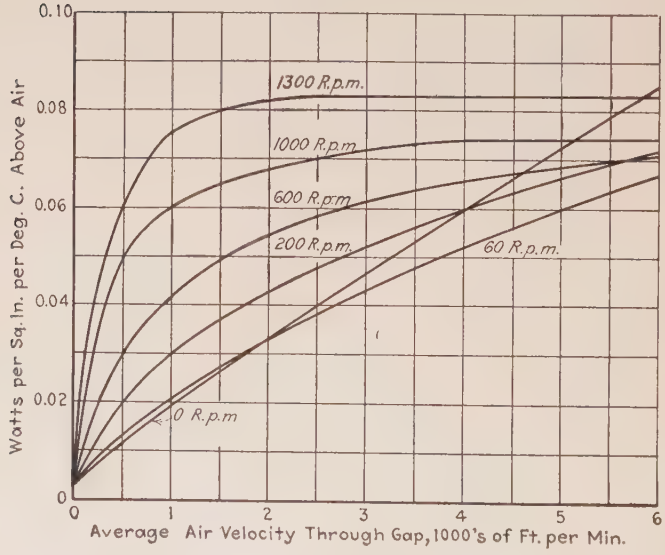


FIG. 43.—Dissipation from surface of a 25-in. diameter rotor—forced axial ventilation through ½ in. air gap.

Figure 43 shows what happens when both forced axial ventilation and rotation take place. At low rotation speeds, increase of duct air velocity gives good increases in dissipation up to high velocities; but for high rotor speeds, there is a limit to benefits derived from increasing duct air velocity. At high rotor speed, the air next to the rotor is carried around with the rotor, and apparently no increase in turbulence occurs to strip off the hot air film.

158. End-connection Dissipation.—The coil parts extending beyond the slots of the core are called end connections, or end coils. It is usually necessary that the end connections should at least cool themselves; and in short-core machines, quite a

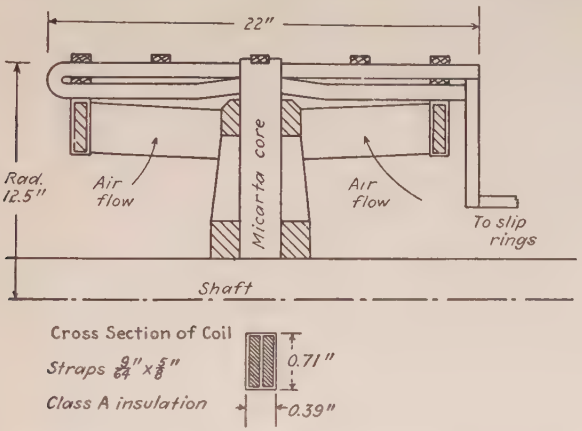


FIG. 44.—Experimental model for determining dissipation from rotating end windings.

portion of core and slot losses can often be conducted to the end connections to be dissipated. Luke's apparatus for investigating end-connection dissipation is shown in Fig. 44.

The surface used in finding the coefficient was the entire coil surface, equal to section perimeter times mean turn length. The winding was a 600-volt, class A insulated winding. The temperature rise measured was that of the copper above the air, and not the rise of the outside insulation surface. The curves of Fig. 45 give results both with free air access and with inner air access blanketed off. The dotted curve is calculated to show what the coefficient would have been if the given temperature rises had been measured in the insulation surface.

Adjacent coil sides were spaced $\frac{1}{8}$ inch.

159. Forced Ventilation of Coils.—To find the effect of coil spacing and coil arrangement on rate of dissipation, an air tunnel (Fig. 46) was constructed and two frames of coils were placed in it near to each other; the coils could be placed either parallel or crossing. There were twelve coils per frame; a section of the coil was 1 by 2.25 inches; the tunnel was 12 by 12 inches. In

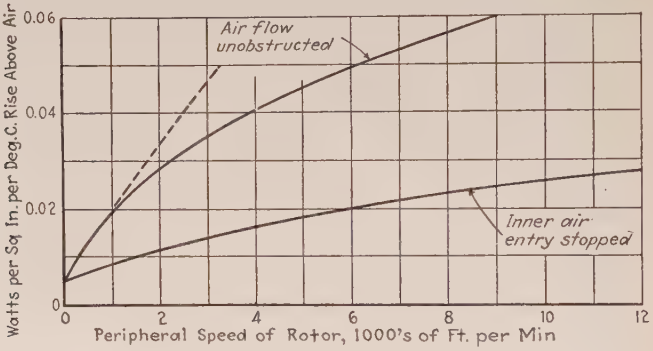


Fig. 45.—Dissipation from surfaces of rotating end windings. (Temperature rise, copper above air.)

this case, temperatures of the outer insulation surface were measured, instead of copper temperatures, as before. The surface used was again the entire coil surface. The coefficients are given in Fig. 47.

160. Limitations of Present Knowledge.—It will be evident to the discerning reader of the literature on dissipation of heat that

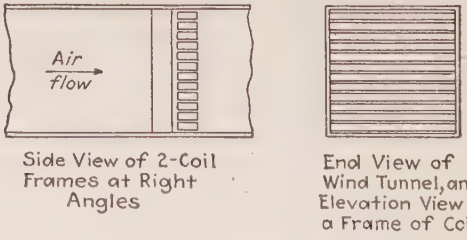


Fig. 46.—Experimental model for testing dissipation from end windings.

much remains to be done. The more rapidly the fund of knowledge is extended by experimental and mathematical investigations, the sooner will it be possible to improve designs. There are always two possible benefits to be derived from heat studies. First, exact knowledge, when available, enables fairly exact

predictions of dissipation and temperature rise to be made for a given design. Second, if exact knowledge, quantitatively speaking, is not available, then a knowledge of tendencies can at least be used to improve on present methods, even if to an unknown degree.

It will be noted that there are no data given in this book on pressure drops necessary to maintain given conditions of air flow. Published data of this kind are notably meager and incomplete.

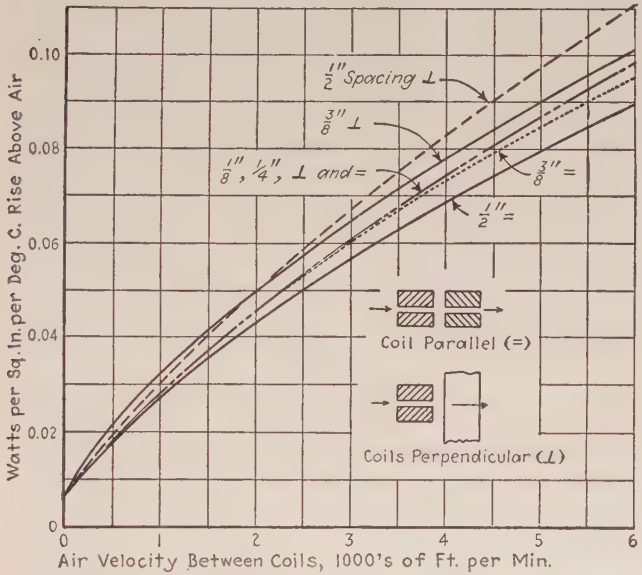


FIG. 47.—Dissipation from surfaces of coils in air tunnel. (Temperature rise, outer insulation surface above air.)

It is to be hoped that research work will be done in the near future to correct this deficiency.

The interested reader, to get a full conception of what has been done and what remains to do, is referred to the published papers of the men who have recently contributed by their investigations; a bibliography is included at the end of this chapter.

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CHAPTER XVI

THERMAL PROPERTIES: THERMAL-CALCULATION METHODS

The general process of getting rid of heat from a surface to the surrounding air or other medium, if called by any one name, should be called *dissipation*. The general process by which heat is transferred within bodies is defined by the term *conduction*. Dissipation already has been treated in the chapter devoted to that subject. Thermal conduction will be handled in this chapter.

The three phenomena of dissipation, conduction, and heat storage having been considered, they can be brought together in various ways by the study of problems of different types. In steady-state problems, where ultimate temperature rise is the prime consideration, heat storage or capacity drops out. If it is a matter of confining temperature rise to some limiting value during a very short period, dissipation may drop out as a negligible factor and leave thermal capacity, and perhaps conduction, in the problem. Simple parts of larger problems may involve conduction alone. The most difficult type of problem is like that of the railway motor; its different main parts are subject to different heating and cooling rates and have differing storage capacities; with a given fluctuating load or a given load cycle, the job of predicting the temperature rise of a given part becomes very interesting. Some of the simpler types of problems and a method of solution for each type will be given here.

161. Thermal Capacity and Thermal Conductivity. I. *Table of Thermal Properties.*—The thermal properties of the more common electrical apparatus materials are given in Table V.

The thermal capacity values are given in the most convenient units—watt-seconds per pound per degree Centigrade rise. A watt-second is 1 joule.

The thermal conductivity values are also in convenient units—watts conducted per inch cube per degree Centigrade difference. Aluminum, for instance, will conduct 5 watts across an inch cube for a difference of 1°C. between opposite faces.

TABLE V.—WEIGHTS AND THERMAL PROPERTIES OF MATERIALS

Material	Pounds per cubic inch P_c	Thermal capacity, watt-seconds per pound per degree Centigrade K_s	Conductivity, watts per inch cube per degree Centigrade (if insulation, taken *across* grain or fiber) K_f
Aluminum, commercial	0.093	433	5.0
Copper, commercial	0.32	180	8.8
Steel, solid, electrical sheet	0.283	225	1.15
Steel, 0.014 laminations	0.28	225	1.10
across laminations	....	....	0.045
Air at rest, 1 atmosphere (constant pressure)	0.428	453	0.0007
Asbestos paper	0.08	373	0.006
Cotton, compressed	....	690	0.0008
Fullerboard, varnished	0.035	700	0.0035
Impregnating compound	0.035	1,000	0.006
Impregnated cotton, paper	0.035	800	0.005
Paraffin	0.029	1,125	0.007
Mica, molded	0.09	390	0.003
Paper	0.035	700	0.0033
Varnished cambric, cotton	0.035	700	0.0065

II. *Conductors.*—Aluminum and copper have fair thermal capacities, 433 and 180, respectively, and very high conductivities, 5.0 and 8.8. It is typical of good conductors that they are good heat conductors and poor heat storers. Also, conductors have high densities.

III. *Electrical Steel.*—Electrical steel stores somewhat more heat per degree rise than does copper but is a far worse conductor of heat. Note particularly that heat conductivity *with* laminations is far better than that *across* laminations.

IV. *Air.*—Air at rest is about the worst heat conductor known. The figure for compressed cotton, given as slightly higher, is a close second; of course, all cotton is mostly a matter of some cotton and a lot of entrapped air. The low conductivity of air explains the great care with which manufacturers try to exclude air pockets and films from insulation wherever it is practicable to do so.

V. *Insulation.*—The various insulations, as compared with the electrical conductors, are strikingly poor heat conductors. Fabric or spun or wrapped insulations entrap a great deal of air, and it is likely that the usual insulation with very low heat conductivity is such because of dead air entrapped. When the minute air pockets in insulation are filled with any of the impregnating compounds, the general conductivity rises greatly.

Mica in the natural state, by comparison with other insulators, is a fine conductor of heat; but it is almost always split up and rebuilt, with shellac between layers. This brings it down to the value given in the table, or lower; 0.003 should be used as the optimistic value.

Although all insulations are low in density, they tend to make up for it by having a high thermal capacity in units per pound. The result is that if a coil has a high space factor, the heat storage capacity of its insulation may well be neglected; but if space factor is low, serious error (by playing too safe) is made if insulation heat storage is forgotten.

For the purposes of this book, enough representative thermal data are given to enable a good number of types of problems to be solved; but anyone having real need of a wider fund of information must refer to proper sources. The manufacture and application of insulating materials is a large and complex field, and only the surface of it can be scratched here.

162. Symbols Used in This Chapter.—In the problem work to follow, the following symbols will be used:

W = total watts lost, produced, conducted, stored, or dissipated.

w = watts loss per pound of material, or heat production rate per pound.

p_c = pounds per cubic inch, for the material.

Θ = temperature, degrees Centigrade, or temperature rise.

k_s = thermal capacity of the material, watts per pound per degree Centigrade rise (subscript s to denote heat *storage*).

k_d = dissipation constant, watts per square inch per degree Centigrade rise (subscript d to denote *dissipation*).

k_f = thermal conductivity of material, watts per inch cube per degree Centigrade (subscript f to denote *flow*).

163. Parallel Flow: No Heat Production. If the conduction is through a homogeneous medium and along parallel lines of flow,

simple arithmetic solves all problems. The temperature drop Θ, over a length of L inches with the flow through a section area of A square inches is

$$\Theta = \frac{WL}{Ak_f}.$$

164. Non-parallel Flow: No Heat Production.—If the flow is along radial lines in a cylindrical mass, with no heat produced within the mass, the solution is simple. Given extreme radii of R_1 and R_2 and a cylinder length L, the differential drop over a differential cylindrical element is

$$d\Theta = \frac{dr}{2\pi rL}\frac{W}{k_f}.$$

$$\Theta = \frac{W}{2\pi Lk_f}\log_e\frac{R_2}{R_1}.$$

If the flow lines are curvilinear and yet give a two-dimensional field of flow and equitemperature lines, the field can be mapped into curvilinear squares; the treatment by this method is by now a familiar story.

165. Parallel Flow with Heat Production. I. *Derivation.*—Let the flow be along parallel lines, and let all the watts loss in the case be produced within the body. Let the temperature difference between the ends of the body be denoted by Θ. The body is L inches long, with a section area of A square inches. Measuring distance from the high-temperature end, the loss between that end and a differential element at distance x is

$$Ap_cwx,$$

and the drop through the differential distance dx is

$$d\theta = \frac{Ap_cwx}{Ak_f}dx.$$

$$\Theta = \frac{p_cw}{k_f}\frac{L^2}{2}.$$

II. *Application: Flow across a Core.*—A bundle of laminations 3 inches thick has a hot-spot plane in its center. For parallel flow acoss laminations to a duct, L is 1.5 inches, k_f is 0.045, and p_c is 0.28. At $w = 3$ watts per pound,

$$\Theta = \frac{0.28 \times 3 \times (1.5)^2}{2 \times 0.045}$$

$$= 21.0°\text{C. total drop.}$$

III. *Application: Flow along Core.*—Using all preceding values and a total flow distance of 1.5 inches, k_f is 1.1; Θ becomes

$$\Theta = \frac{21.0(0.045)}{1.1}$$
$$= 0.86°\text{C}.$$

IV. *Application: Flow along a Conductor.*—A copper conductor is being worked at a value of 400 circular mils per ampere. Assume a section of 1 square inch. Over a length L of 5 inches, all heat produced must be conducted to one end. At a resistivity of 1 ohm per circular mil-inch, the resistance totals $R = (5 \times 1)/(1.273 \times 10^6)$ ohms. Current is $I = (1.273 \times 10^6)/400$. $W = I^2R = 39.8$ watts. Watts per cubic inch $= p_c w = 39.8/5 = 7.98$. $k_f = 8.8$.

$$\Theta = \frac{7.98 \times (5)^2}{8.8 \times 2}$$
$$= 11.33°\text{C}.$$

If L is doubled, Θ is quadrupled. Thus, end connection will be able to help dissipate heat conducted to them, but much conduction over many inches is out of the question.

166. The "Unit-package" Method.—For want of any generally recognized title, the author suggests this name for the method about to be outlined. This method was suggested by B. G. Lamme. Luke's investigation of variation of dissipation coefficient along an axial duct involved the application of the method, and it will now be seen how the coefficient curve was obtained in that case. Reference is made to Fig. 38.

The duct is divided into unit packages, say, 1 inch long. The temperature curve for the iron gives the iron temperature for each package. Subtracting from the watts produced by each package the stray watts lost to the outer surroundings, the net watts to be accounted for are obtained.

Knowing the thermal conductivity axially, by using it and the temperature drops from one package to the next, axial watts conducted to or from a package are computed.

The package at the entry will now dissipate a known number of watts to duct air, equal to its own watts plus those conducted to it. (The iron temperature curve shows that conduction is *toward* the end package.) The dissipation coefficient for its duct surface is calculated; also, the increased temperature of the duct air, due to taking up the dissipated watts, is figured. The

next package rids itself of its own watts, minus the watts conducted away from it, plus the watts conducted to it. The coefficient and the air temperature are again computed. Thus for the whole length. If the final air temperature does not check with the measured value, the "heat balance" is off, indicating an error in the calculations.

167. Unit-package Method for Machines.—Kelly[1] first reported the application of this method to the predetermination of temperature rise of alternators. As used here, it is a cut-and-try method, for instead of temperatures of iron and copper being known, they are sought. The elements of the method can be successfully demonstrated in terms of a much simpler application: that of predetermining the temperature rises of the iron for the same duct of the preceding article.[2]

It is assumed that the watts loss per package, the air inlet temperature, air velocity, and the dissipation coefficients for all package surfaces are known. To simplify present language, let stray losses be neglected.

The temperature of the first package of iron is *assumed.* Its dissipated watts are at once calculated, as is also the increased air temperature. The excess (or deficit) watts of the first package are now conducted to the next package, and the axial temperature drop (or rise) in so doing is found. This gives the temperature of the next package. Its dissipated watts are found, and the new air temperature. Continuing to the last package, if an iron temperature, dissipation coefficient, and air temperature are attained of just the right values to dissipate all remaining watts, than the original guess at iron temperature was correct (barring errors made on the way). If not, the excess or deficit watts will tell how to correct the guess.

This is a laborious process when applied to the complex unit package of an alternator or induction motor. But it has been applied and with notably good results in predicting temperatures. When the manufacturing concern signs a contract to put half a million dollar's worth of material and labor into one unit, its engineers have good reason to go to somewhat laborious lengths to see that temperature guarantees will mean something.

[1] KELLY, RALPH, "Internal Temperatures of A. C. Generators," *A. I. E. E.*, February, 1917.

[2] LUKE, G. E., "Surface Heat Transfer in Electric Machines with Forced Air Flow," *A. I. E. E.*, November, 1926.

168. Other Thermal Problem Types.—It is out of the question to attempt to list all the practical types of problems that confront the engineer. Some have been given above. Others will appear, as occasion arises, in the book.

Problems

22. A cylindrical transformer tank is 8 feet in diameter and 12 feet high. At 40°C. rise of tank wall, neglecting top and bottom, how many watts will be dissipated?

23. The elongated, somewhat dome-shaped, nickeled top of a 500-watt electric iron has about 30 square inches of surface. When the iron is not in motion, if the top temperature rise becomes 140°C., approximately what wattage will be lost from the top? If the top is painted black, what will be the loss?

24. A 100-foot length of No. 8 bare copper wire is strung horizontally in still air, the air being at 20°C. A constant current finally heats the wire until its resistance measures 0.0903 ohm. Find the temperature of the wire, and the current.

25. A copper conductor of 250,000 circular mils carries 400 amperes. If air temperature is 20°C., what is the conductor temperature?

26. An iron-clad solenoid is mounted horizontally. The iron cylinder is 8 inches in diameter and 13 inches long. At a surface rise of 40°C., what constant wattage loss will it dissipate?

27. A copper cube is painted black. The cube weighs 50 pounds. It is heated up to a temperature rise of 100°C., and then is suspended in still air. First, decide which coefficient curve (for plates), as given in this book, will be applicable to this case. Then find, by taking several steps, approximately how much time elapses before the temperature rise falls to half value.

28. A copper tube, having an inside diameter of 1 inch, is bent into the form of a *U*. The *U* is submerged in boiling water until the length submerged is 8 feet. Air at 20°C. is blown into the tube at a velocity of 4,000 feet per minute. How hot is the air coming out at the other end of the tube?

29. If the above perfect tube is replaced by a like tube which has been found on the junk pile and which has been much stepped on and kinked, will the emerging air be hotter or cooler?

30. A part of a certain laminated core has an iron loss of 2 watts per pound. Let a series of round ducts now be made through the core, arranged in regular rows so that four adjacent holes or ducts lie on the corners of a square. Let the duct center spacings be 6 inches, and let the duct diameter be $1\frac{1}{4}$ inches. If by this operation the loss per pound in the remaining iron goes up to 2.1 watts per pound, find the duct air velocity needed to cool the core so that the core temperature rise is 50°C.

31. For the cylindrical surface of an open-type, self-ventilated motor, with a usual arrangement of poles present with usual small air gaps, Kapp's formula for temperature rise is $550/(A/W)(1 + 0.1v)$, where the temperature rise is in degrees Centigrade above the air, A is the surface in square centimeters, W is the watts dissipated, and v is the peripheral velocity in

meters per second. Use this formula to find watts per square inch per degree rise, for peripheral velocities of 2,000, 4,000, and 6,000 feet per minute, and compare with the dissipation from Luke's self-ventilated rotor.

32. An industrial motor is to be totally enclosed. The exterior is roughly cylindrical; diameter, 24 inches; length, 24 inches. At an overall motor efficiency of 87 per cent, what will be its continuous rating?

33. A knife switch blade (copper) has a section of 0.1 by 0.8 inch. If it takes a short-circuit current of 5,000 amperes, how long (in seconds) will it take to reach a 200° rise?

34. How many pounds of air per 8-hour day must be blown through a 10-horsepower motor operating on full load at 86 per cent efficiency, if air temperatures are: incoming, 25°C.; outgoing, 40°C.?

35. A radial duct in an armature core has a $\frac{3}{8}$-inch spacing. The spacer fingers are steel strips, $\frac{3}{8}$ by $\frac{1}{16}$ inches in section. At a certain radius they are placed 1 inch apart. Air velocity is 1,000 feet per minute. Air is at 30°C., and duct wall temperature is 70°C. Find the temperature drop from edge of spacer to center of spacer, and find what part of total watts dissipated can be credited to spacer surface.

36. An electromagnetic brake for loading motors on test is made by mounting a stationary two-pole electromagnet inside a revolving steel shell. The shell is 12 inches in diameter and 8 inches long. The shell thickness is 1 inch. The motor end of it, which fits the motor-shaft extension, is closed, this part also being 1 inch thick. Torque on the electromagnet is measured.

Assuming no dissipation, how quickly will a 5-horsepower load cause the shell to rise 150°C.?

Using all outward surfaces of the shell, at 150°C. rise, what constant load will it carry? (State your assumptions.) The speed is 1,750 revolutions per minute.

37. A copper block has a section 4 by 4 inches square, with a length of 8 inches. If a center hole, 3 inches in diameter, is bored across the block, find the heat conductance in the lengthwise direction of the block, in watts per degree (a field mapping job).

38. A low-resistance standard shunt is made of strap, 0.05 by 0.75 inch in section, and 8 feet long. On checking it carefully, the resistance is found to be low. A saw slit, 0.020 inch wide and 0.50 inch deep, is cut into the strap from one side, at a point several inches from one end. Find the per cent increase in resistance.

39. A smooth laminated core (without slots) 16 inches long, is assembled on a small shaft. Core diameter is 12 inches, and shaft diameter is 3 inches. The ends are heat insulated. By means of poles placed at the ends, enough flux of varying density is sent axially through the core to give a watts loss per pound of 2, with a rotation speed of 1,000 revolutions per minute. Find the shaft temperature rise above the surrounding air. Assume no axial heat flow.

40. A long solenoid, considering the active coil, has an inside diameter of 4 inches and an outer diameter of 8 inches. The wire in No. 10, double cotton covered, with no paper between layers. It is vacuum impregnated. The outside surface is painted black. The solenoid is stationary and is

placed horizontally. Neglect end-surface dissipation. Find maximum temperature rise above air, for a constant current of 5 amperes.

41. A slot has six conductors, each 0.10 by 0.40 inch in section. They are arranged three wide and two deep. Between copper and tooth, insulation is 0.040 inch; between straps, 0.010 inch; between copper and slot bottom, 0.080 inch; between upper and lower copper, 0.080 inch; between copper and air, 0.080 inch thick. The copper is working at 300 circular mils per ampere.

(*a*) If all heat is conducted outward to the air gap, find the slot bottom temperature, if gap air is at 25°C. Do not neglect the drop in the copper.

(*b*) If all heat is conducted to the tooth iron, find maximum rise of copper above tooth.

42. Dissipation-coefficient Curve for Short Duct.—Round punchings are made with 1.25 and 3.25 inches for diameters of central hole and outside, respectively. Enough of these are stacked to make a duct 10 inches long. A uniform heater of resistance ribbon is wrapped around the outside, with insulation outside of that. For simplicity, neglect heat leakage from outside and ends. Neglect radial temperature drops in the iron, but take axial drops into account.

Air is forced through at a velocity of 2,000 feet per minute, entering at 20, and leaving at 35°C. Iron temperatures, measured every inch axially, are given as follows, beginning with that of the air entry end: 40, 59, 70, 76, 81, 84, 87, 89, 90, 89, 85. Divide into 10 unit packages, and get curves of air temperature and dissipation coefficient.

43. Temperature-rise Prediction for Short Duct.—A duct is built identical with the above, except that the outer diameter of the punchings is 5 inches. The heater develops 300 watts. Again, neglect heat leakages. Duct air velocity is to 4,000 feet per minute, air entering at 20°C. Plot the dissipation-coefficient curve from the following values which are taken every inch axially, beginning with the entry end: 0.10, 0.076, 0.065, 0.057, 0.051, 0.050, 0.049, 0.050, 0.055, 0.061, 0.070. (Do not mistake these coefficients for actual physical values; their curve is considerably different from that of an actual case.) Plot the curves of air temperature and iron temperature. Use cut-and-try method; assume temperature at center of first unit package.

CHAPTER XVII

ELECTROMAGNETIC DEVICES

It is desirable to introduce the study of some electromagnetic devices at this point, to give an applicational emphasis to a considerable part of the preliminary work covered so far. The student is hardly ready to begin the *design* of a complete device at this point; hence the chapter is introduced for its values as a study section. The type of problems treated will readily suggest similar problems which can be made up and assigned to the student.

169. Simple Magnetic-circuit Problem.—Given, the data on a magnetic circuit: to work out the data for and plot the saturation curve. Let the eccentric core of Fig. 48 be built up of electric sheet laminations to an axial depth of 2 inches. The air gap is 1.4 inches wide and 0.08 inch long. It is assumed that the flux field in the gap is rectilinear, and leakage is neglected. The flux density in the core varies from point to point. It might be thought that a very accurate solution for core ampere-turns, for a given flux, could be gotten by dividing the core up into a great number of sections, so that the density in a section would be practically constant. But this would be wasted effort, for it has already been decided to neglect leakage; and the error so introduced renders absurd laborious efforts at high accuracy in other parts of the problem.

Nevertheless, there is a technique involved in deciding on what sections to take in the core. To use one section, with an assumed or guessed at "average section area," would be very bad engineering. If the student should learn any one thing first, in dealing with such a circuit as this, it would be to become very wary of any averages or average sections. *After* one has had his judgment tempered by some experience gained in working out a circuit in *too much detail*, he may then with greater safety guess at how few sections may safely be used.

Each half of this core has been divided up as shown, into four sections. Note distinctly that the sections do *not* have equal

lengths. Again, to divide such a core into equal-length parts is to indulge in either too much work, or too much error. In the thin part of the core, where the densities are high, and therefore where the m.m.f. per inch may become very high, *short* sections are used. In the wide part, *longer* sections are used, thus preserving a like degree of accuracy throughout.

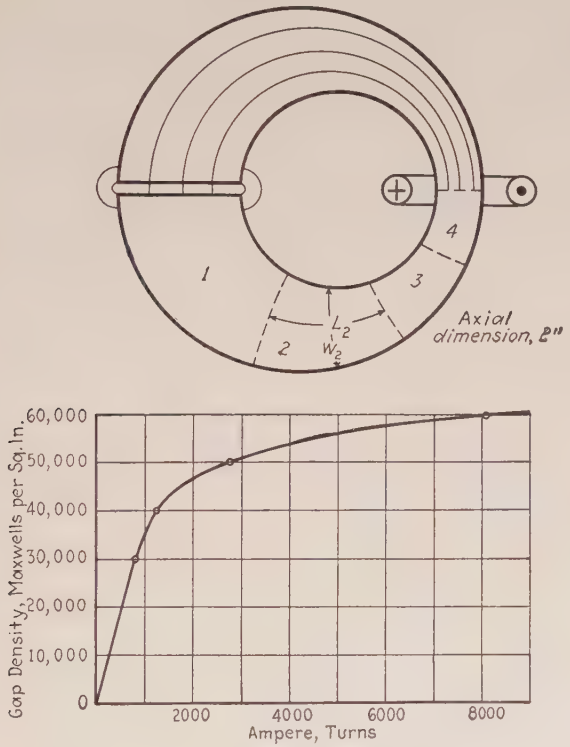


FIG. 48.—Core of varying section, with air gap.

Densities ranging from 30,000 to 60,000 maxwells per square inch in the gap are taken. Although the outside core dimensions are not given here, the effective lengths and widths of the core sections are given in the table of solution data; and it is seen that a gap density of 60,000 will give nearly 160,000 for density in the narrow section of the core.

If a given section had constant permeability, its effective length and width could be found very closely by subdividing it once each way and measuring the two distances along the flux and

potential subdivider lines. This method still holds with good accuracy, provided sections in regions of high density are taken reasonably short. The effective lengths and widths in the solution were so obtained.

For gap ampere-turns, $0.313BL$ applies, L being 0.08. For core ampere-turns per inch, the electrical sheet curve in Fig. 22 is used.

SOLUTION

Section	Air gap		1		2		3		4		Total	Flux
	L	*W*	2*L*	*W*	2*L*	*W*	2*L*	*W*	2*L*	*W*		
	0.08	1.4	3.0	1.3	2.6	0.9	2.0	0.65	1.8	0.54	*NI*	
B	30,000		32,300		46,600		64,500		77,700			
NI/inch	.....		5		6		10		12			
NI	750		15		15		20		22		840	84,000
B	40,000		42,800		62,200		86,000		103,700			
NI/inch	.....		6		10		20		100			
NI	1,002		18		26		40		180		1,266	112,000
B	50,000		53,700		77,800		107,300		129,500			
NI/inch	.....		7		20		140		650			
NI	1,253		21		52		280		1,170		2,776	140,000
B	60,000		64,200		93,300		129,000		155,000			
NI/inch	.....		10		45		650		2,850			
NI	1,505		30		117		1,300		5,130		8,082	168,000

The saturation curve, in terms of gap B *versus* total NI required of the coil, is plotted in Fig. 48.

Right here is the place to review the solution and digest the experience gained. It is seen that for gap densities below 30,000, the entire core NI could have been neglected, with an error in total ampere-turns of less than 10 per cent. And for the highest densities, core section 1 still has, for most practical purposes, a negligible NI drop. At high density, nearly all core drop occurs in the sections 3 and 4; if better accuracy is desired, these sections could further be split up.

If it is now desired to take leakage into account, the fringe of flux along each of the four sides or edges of the air gap could be assumed to follow two-dimensional laws. Studying the field out to a radius of 0.5 inches or so would include the major portion of the leakage field. The tabulated values of core densities could still be retained, and the *apparent* gap density could be raised by adding to tabulated gap densities by a due proportion of density.

170. Round-type Short-stroke Tractive Magnet.—Figure 49 illustrates this type of electromagnet. Mechanical details are omitted. Let the following specifications prevail:

Voltage = 230.

Tractive effort = 5,000 pounds.

Outside temperature rise = 40°C.

Armature stroke = 0.20 inch.

Duty: Applied at fairly steady, frequent intervals, equivalent to being on the line one-third of the time.

There are any number of magnets that can be designed to fit these specifications. But a given manufacturer, with fixed standards, agreed-upon allowances and clearances, and machine tools, jigs, fixtures, and processes already on hand and in use, will be able to produce the magnet in only one or a very few forms and do it cheaply. This fact causes further specifications to be written at once, and it is assumed that they are:

Dimension B = 0.25 inch (clearance around coil).

Dimension C = 1.0 inch (for insulation and coil retainer).

Coil section to be square.

Magnet body and armature, cast steel.

Magnetic path section area—constant (giving constant flux denxity).

For cooling, an approximate value for the total external area will be used, including that of the armature; and the dissipation coefficient will be taken from the curve for the 9- by 9-inch vertical black plate (Fig. 33). This curve gives 0.0067 watt per square inch per degree rise, and $40 \times 0.0067 = 0.268$ watt per square inch, average rate. But the magnet is used for only one-third of the time, frequent-interval duty. Then $0.268 \times 3 = 0.805$ watt per square inch, using actual watts *when on duty*.

If a saturating iron density is used, excessive ampere-turns will require an excessively large coil. If a very low density is used, say, 40,000, an excessive amount of iron is put into the job. There is, between low and high limits, no one best density to use. Such problems must either be solved in a practical way because the manufacturer insists that the designer shall use standard parts, etc., thus limiting his choice in various ways; or else an economic problem is made out of it, and every factor of any cost consequence is taken into account.

Proceeding with the problem, assume that the use of 100 pounds to the square inch, giving a density of 84,800 in the gap (and everywhere else) will be satisfactory.

If it were not for the fact that the magnetization curve for steel must be used and that this curve has no equation, it would be possible to gather all relations into one set of equations, and by direct solution, design the magnet. This is even possible under the conditions, for an empirical equation for the magnetization

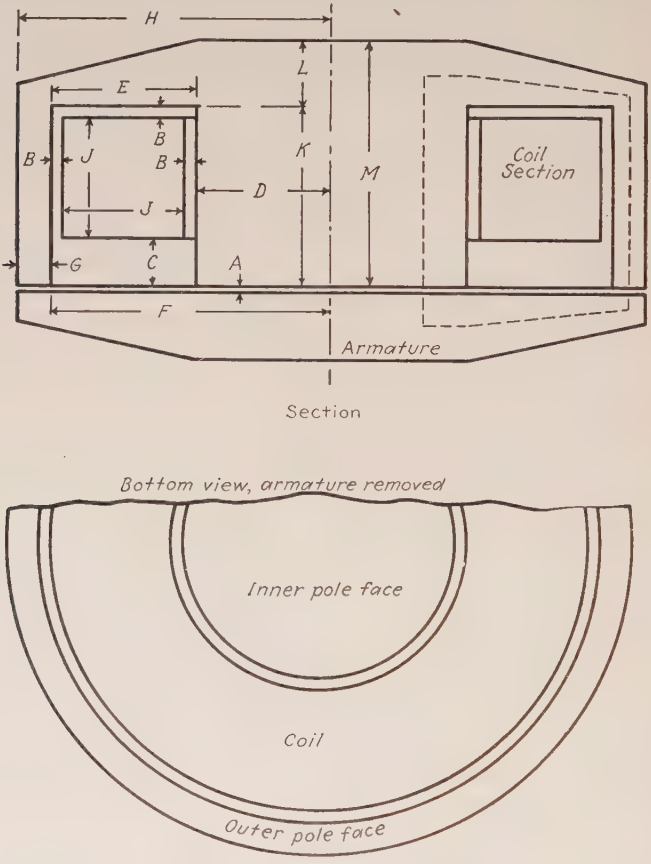


FIG. 49.—Round type magnet.

curve can be found. Occasionally, this type of solution is worth using. In fact, if a designer knew that he was going to have a great deal to do with this type of magnet, he might find the direct solution (mathematical solution) valuable. For the purpose here, a cut-and-try solution is the only practicable method of procedure.

Due to the fact that all densities are to be the same, each pole face must have the same attraction and the same area. The radius D of the inner face is then found, for an area of 5,000/(100 × 2), to be 2.82 inches.

ROUND-TYPE, SHORT-STROKE, TRACTIVE MAGNET

Voltage	E	230
Tractive effort, pounds		5,000
Pounds per square inch		100
Gap density	B	84,800
Total of pole-face areas		50
Dimensions:		
A		0.20
B		0.25
C		1.0
D		2.82
E		3.00
F		5.82
G		0.646
H		6.47
J		2.5
K		3.75
L		1.41
M		5.16
NI per inch, steel		35
Effective path length, steel		17.75
Steel core NI		622
Gap NI		10,600
Total minimum NI	NI	11,222
Mean turn radius		4.32
Mean turn length	C	27.1
Desired circular mils	M	1,320
Used circular mils	M	1,624
Used wire size	A.w.g.	18
Wire insulation		e.s.c.
Insulated wire diameter	D'	47.3
Turns to fill space $J \times J$	N	2,790
Ohms, at 1/circular mil-inch	R	46.5
Current	I	4.95
Actual NI	NI	13,800
Dissipation:		
End areas $= 2\pi H^2$		263
Side area $= 2\pi HM$		210
Total dissipated area		473
Watts	W	1,140
Watts per square inch		2.41
Temperature rise	Θ	120

Dimension E, the annular coil space width, is now *assumed* to be 3 inches. The various steps are then tabulated in a straightforward manner in the solution table given. The only point about which there may be lack of clarity concerns the finding of the *effective* or *mean* magnetic path length in the steel. (*Mean* path length means nothing specific, unless defined; *effective* path length is either a length which has been proved to be a correct one to use for a specific case, or else it is the length one *intends* or *proposes* to use, with the belief that its use will be sufficiently accurate.)

For most practical cases, a length as shown by the dotted line is good enough. As this is known probably to be wrong and wrong to an unknown degree, there is nothing to be gained by fussing about and trying to find the length of the dotted line with split-hair accuracy. A practical way is to sketch the section and *measure* it with a rule. It is advisable to note this fact, however, that a flux line found in the middle of the outer pole would, in going down the inner pole, be located at about 70 per cent of the radius of the inner pole from the magnet center. (Prove this.)

Jumping over the interior of the solution to the end of it, 2.41 watts per square inch are found, and 0.805 is allowed to attain a 40° rise. $40 \times (2.41/0.805) = 120$, which is *not* the rise at which this magnet will operate; for, since this design, if built, would operate above a 40° rise, then it would have a higher dissipation coefficient than the one used. This magnet will not be built, however, and so there is no further interest in its exact performance.

The next step is clear: Dimension E must be increased, and the solution is repeated thus until about a 40° rise is attained. The remaining steps are to be completed by the student.

As a final check on interior temperatures, a rough solution for conduction drop from coil-section center to magnet surface should be made. The coil and the space about it would be filled with compound. A way for finding approximate internal temperature rise is left to the ingenuity of the student.

171. The Series Lockout Magnetic Switch.—Remote control of motors is now a common thing. When a direct-current motor is started by pushing a button, the control panel devices throw the motor on the line with full armature resistance in. The armature current rises suddenly and then falls as the motor accelerates. When the falling current drops to a predetermined

value, a switch automatically closes, acting to short-circuit part of the starting resistance. The current rises again, and upon falling again to the previous value, another automatic switch acts; it continues thus until all resistance is cut out. In this way the acceleration is accomplished with any desired degree of smoothness, depending on the number of switches used.

These switches are closed by a magnet of the series lockout type. The armature current goes through the coil of a switch; the design is such that the switch refuses to close on the rise of current, but closes on the fall of current. When it does close, its action, by interconnection, puts the next switch into service so that it in turn acts as the one before it did.

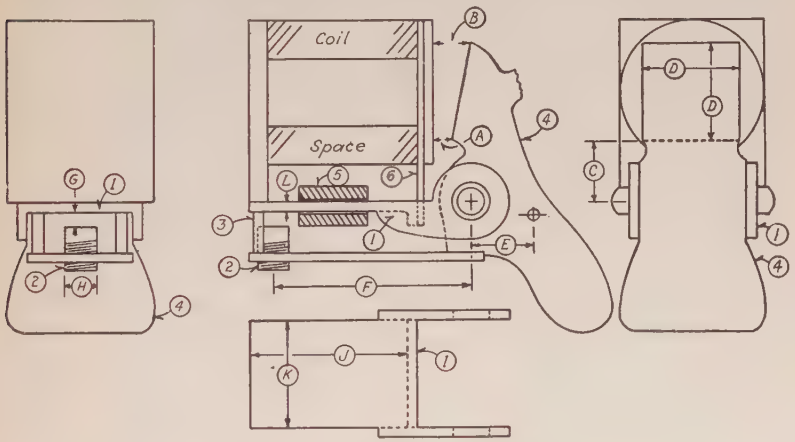
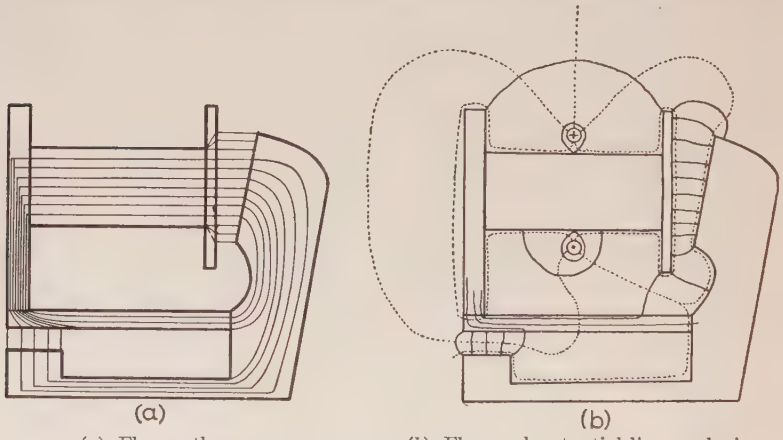


Fig. 50.—Series lockout switch magnet.

The essential features of the magnetic design are shown in Fig. 50. The clapper or armature, part 4, is hinged to the magnetic frame. The main air-gap flux passes through the coil and down the back piece. At the bottom of the back piece the flux path splits into two parts, one part being through a saturated bridge, part 1, upon which a short-circuit turn, part 5, is mounted; the other part being across the lockout air gap G to the round lockout plug, part 2, thence along the lower piece which is part of the clapper assembly, and to the clapper. The paths unite in the region of the hinge pin. Part 6 is non-magnetic, and so are the stop pins, part 3. Figure 51*a* gives a simple representation of the flux path; Fig. 51*b* shows a rough analysis of flux and equipotential lines as due to one turn.

The peculiar action of the device is best understood by beginning with a high current which is falling. The high current causes a heavy flux, most of which takes the path through the saturated bridge. The other lower path has a long air gap—the lockout gap—to offer reluctance, whereas the bridge path has only the reluctance of its own steel, and that of very short air gaps around the bearing-pin region. If now the plug in the lockout gap is adjusted properly, the torque due to attraction at the lockout gap, plus the torque due to the clapper weight assembly, will be greater than the torque due to main gap attraction.

As current falls, fluxes in all parts reduce. But the two gap fluxes fall at a greater rate than does the flux in the bridge, which



(*a*) Flux path. (*b*) Flux and potential line analysis.

FIG. 51.—Series lockout magnet switch.

was saturated. Therefore, main-gap flux falls less fast than does lockout-gap flux, for main-gap flux includes the slowly falling bridge flux. Then, if all parts are properly designed, there will be a current value for which closing torque will be greater than total lockout torque.

To study events during current rise, it is evident that if low currents will tend to close the clapper gap at one time, they will at another. But by damping the bridge path with a short-circuit turn, the very sudden rise of current causes enough damping current to flow around the bridge to keep bridge flux from building up until after a high value of current is reached. It is easily seen that this action will prevent closing for suddenly rising currents.

To acquire an appreciation for the relative proportioning of parts in this type of switch magnet, a problem should be worked out. The problem may be stripped of its worse aspects by making simplifying assumptions.

1. The damping action of the damping coil for *falling* currents will be neglected.
2. M.m.f. drop for all parts except the two gaps and the bridge will be neglected.
3. The flux in the lockout gap will be assumed to be rectilinear, with a section equal to the area of the plug itself.
4. The clapper main gap face will be assumed to be square, and the gap flux here is assumed to be cylindrical, following arcs of circles whose center axis is at the intersection between face of clapper and face of coil pole.
5. These flux field assumptions eliminate leakage from consideration.

The problem will be, given the specifications for such a magnet, to find all values of all quantities over the working range and to find for what coil NI the switch will close. The key to the problem consists in first finding, for each air gap, the gap flux and the gap torque in terms of the NI for the gap. For instance, torque of the main gap is stated as a function of main-gap ampere-turns. This requires that a differential equation be set up and later integrated. For a practical solution it is undoubtedly foolish to use the calculus on either gap. But anyone can make a practical or rough solution. It is urgently necessary that one should be able to work the beveled-gap problem, and here is a good opportunity to try it out. About one student in three is able to set up and integrate the differential expression for torque and get it right the first time; and no student can know whether he has it right or not, until he applies a rough check.

SERIES LOCKOUT SWITCH MAGNET

Clapper assembly weight, pounds ————

Dimensions

A ————	*G* ————
B ————	*H* ————
C ————	*J* ————
D ————	*K* ————
E ————	*L* ————
F ————	

Functions

Let main gap $NI = M_1$. Let lockout gap $NI = M_2$.

1. Main-gap flux = function of (M_1) = _____
2. Main-gap torque, pound-inch = function of (M_1) = _____
3. Lockout-gap flux = function of (M_2) = _____
4. Lockout-gap torque = function of (M_2) = _____
5. Computations:

Assumed bridge B							
bridge NI/inch							
bridge NI							
bridge flux							
Lockout-gap NI							
Lockout-gap flux							
Lockout-gap torque							
Main-gap flux							
Main-gap NI							
Main-gap torque							
Clapper weight torque							
Total lockout torque							
Coil NI							

6. Graphical. Plot main-gap torque and total lockout torque *versus* coil NI.
7. Find coil NI when switch closes.

The solution form given will be found to have enough places for given data and enough steps for making the computations to enable a given problem to be worked out. The following set of specifications is suggested:

Clapper weight, 2.0 pounds.
Dimension $A = 0.10$ inch

$B = 0.25$	$G = 0.10$
$C = 1.00$	$H = 1.00$
$D = 1.50$	$J = 3.00$
$E = 1.00$	$K = 2.00$
$F = 4.00$	$L = 0.10$

Carry out the computations for bridge densities of from 80,000 to 105,000, varying by increments of 5,000. (This magnet will close at about 1,050 coil NI.)

172. The Lifting Magnet.—Figure 52 shows the general proportions of a large lifting magnet of the round type. A quite small magnet, say, 15 inches in diameter, would be wire wound;

but the large ones require ribbon coils. The extremely varying duties imposed on a lifting magnet, coupled with the fact that it is subject to the worst of impacts, require that it be made as one solid mass.

The shell is of high-quality magnetic steel. It is ribbed to give better heat dissipation and better stiffness. The pole shoes, being subject to great wear, are removable to allow of easy replacement. The coil assembly is protected and retained by a ribbed manganese steel casting. This piece should be wholly nonmagnetic; if magnetic, it would act as a magnetic short-circuit on the useful flux. Manganese steel has a very low permeability, and it is so hard that it must be cast and ground; machine tools will not cut it. These properties make it the only possible choice for retainer material.

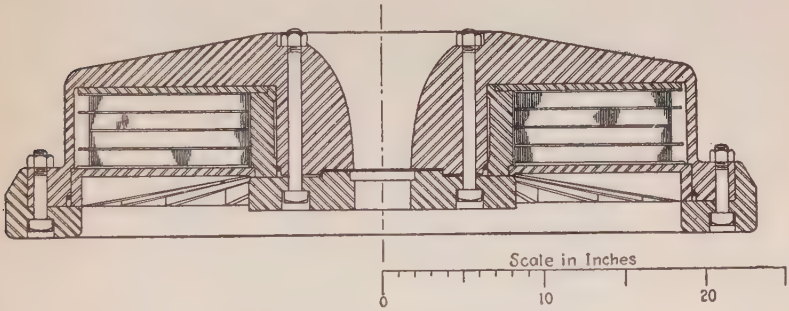


FIG. 52.—Round lifting magnet.

The magnet shown is of Cutler-Hammer design. Figure 53 shows the coil unit being lowered into the body casting. The coil spool is of steel, which forms part of the useful magnetic core circuit. The coil is split up into four pies in series, each pie being a ribbon coil, with 5-mil asbestos between turns. A mica-asbestos plate separates the pies. After assembly, the magnet is piped to an impregnating system; for 3 hours the magnet interior is subjected to a 20-inch vacuum, the coil temperature being maintained at 200°C. by current passing through it. After the moisture and air are thus removed, the pump is reversed, forcing the compound into all voids at a pressure of 100 pounds for 3 hours more.

To prevent insulation breakdown due to inductive kick when the current is broken, the control is arranged to connect a discharge resistance across the magnet when the control handle is

thrown to the "off" position. Retained magnetism often causes loads to be dropped sluggishly; when quick release is desired, the control handle is thrown from "lift" to "drop," which reverses the current through a series resistance.

The limiting factor in these magnets, which puts them almost in a class to themselves, is not directly temperature rise, but is due to it. If for instance, copper temperature is raised from 20 to 274.5°C., the resistivity is doubled; the "hot" ampere-turns would be one-half the "cold" ampere-turns, and attraction would drop more than in proportion, it varying with the square of

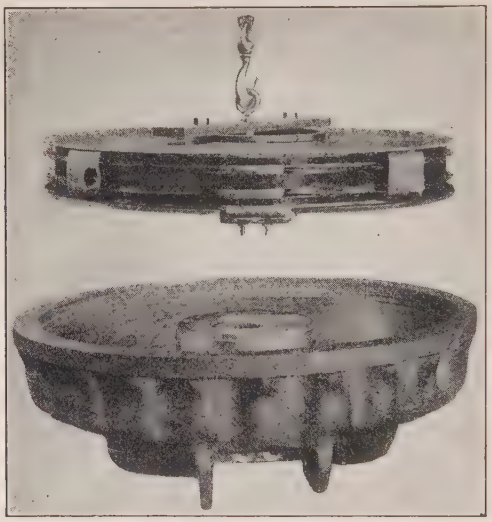


FIG. 53.—Lifting magnet being assembled. (*Courtesy The Cutler-Hammer Mfg. Co.*)

the flux. Such a high temperature, even if insulation could be found that would stand it, would be uneconomical.

To gain some experience with this type of design, a given case will be followed through. The section shown in Fig. 52 is drawn to scale, with scale included. It is assumed that it is a section of a Cutler-Hammer, 45-inch magnet, which is rated at 220 volts and 33 amperes at working temperature. The duty is assumed to be for an all-day period on half-time excitation cycle. It is assumed that the mica separators, called mica plates, are 0.1 inch thick and that asbestos between turns is 0.005 inch thick. The problem is to find probable temperatures and to solve for the

coil data. Dimensions will be taken from the section drawing as needed.

173. Study of Given Design: Lifting Magnet.—Referring to Art. 141, Luke recommends 0.026 for k_d, the watts per square inch per degree rise, for the enveloping surface of a railway motor when the car speed is 10 miles per hour. This will be as good a figure for dissipation coefficient as any, unless the duty cycle could be defined exactly, and lifting and lowering speeds were known; in which case, a performance test of the magnet itself would be referred to.

Sketching around one side of the section a line to represent the enveloping surface, the perimeter so obtained measures about 55 inches. The radius to its center of gravity is 11.5 inches. $2\pi 11.5 \times 55 = 2{,}975$ square inches.

Watts $= 33 \times 220 = 7{,}260$, but for half-time use, average watts $= 3{,}630$. Watts per square inch $= 3{,}630/2{,}975 = 1.22$. Surface rise $= 1.22/0.026 = 46.9$°C. rise above air. Assuming an ambient temperature of 50°C. (hot day), the magnet surface temperature is 96.9°C.

Drops in the steel shell can be neglected, because of larger errors entering unavoidably into other temperature-rise computations. Within the coil, consider possibility of radial (horizontal) heat flow. To go outward, heat would have to cross about an inch of compound from copper to steel shell. This is a big barrier to flow (note that this inch is required in order to extend the mica plates to prevent creepage breakdown between coils), and it leads to the temporary assumption that no heat can cross it. Inward flow, except from inner turns to the inner pole core, is also difficult, for inward flow means concentration, with fast-increasing temperature gradients. Vertically, flow is much easier.

Assume the center mica plate to be at a constant and maximum temperature and that all flow is vertically away from it. Neglect drop in the copper. Going upward, all the watts of the second pie from the top must cross one plate above it. This plate has a conduction area equal to its mean perimeter times the coil-wall thickness. Its mean radius measures 13.0, and coil wall measures 9.5 inches. Plate-conduction area $= 2\pi 13 \times 9.5 = 776$ square inches. Average watts per pie $= 3{,}630/4 = 907.5$. Using 0.004 for k_f the conductivity coefficient, watts per inch cube per degree, the drop through the plate is $(907.5/776)/0.004 = 300°$

for a 1-inch plate, or 30°C. for a 0.1-inch plate. The top mica plate must conduct the wattage of two pies, and, hence, has a 60° drop.

As the magnet surface temperature is 96.9°C., and as there is no telling within quite a few degrees what it actually will be, it may as well be called 100. Adding the drop through the top mica plate, the top coil pie has a temperature of 160°C. Adding the drop through the next plate below, the middle pies show 190°.

But this is based on the assumption that all flow is vertical. The average pie temperature, $(160 + 190)/2 = 175$, is high. Allowing for radial flow, the average copper temperature may be taken as 160°C., which is a reduction of 15°.

Copper resistivity, at 140° rise above 20° standard, is 10.371 $(1 + 140 \times 0.00393) = 10.371 \times 1.55 = 1.61$ ohms per circular mil-inch approximately.

The mean turn radius of the coil measures 13 inches; C, the mean turn length, is $2\pi\ 13 = 81.6$ inches.

Total coil resistance $= E/I = {}^{22}\%_{33} = 6.57$; letting R stand for the ohms per pie, $R = 6.57/4 = 1.67$ ohms.

The vertical dimension of the coil, including four pies and four mica plates, is 4.5 inches; $4.5/4 = 1.125$; $1.125 - 0.10 = 1.025$, the copper ribbon width.

Let N stand for the unknown turns per pie, and X stand for the unknown thickness of the ribbon in inches. The resistance of a pie and the coil-wall thickness of 9.5 inches can each be stated in terms of N and X. The two equations can be solved. For a resistivity of 1 ohm per circular mil-inch, with R, N, C, and M representing pie resistance, turns, mean turn, and circular mils, respectively, $R = NC/M$. The ribbon section in circular mils is $1.025 \times 1.273 \times 10^6$. Hence, for the pie, with resistivity of 1.61, resistance is

$$1.67 = \frac{N81.6}{1.025 \times 1.273 \times 10^6}. \qquad (1)$$

The coil-wall thickness is

$$9.5 = N(X + 0.005). \qquad (2)$$

These equations give a quadratic; the results of its solution are:

$$X = 0.0164.$$
$$N = 444.$$

Among a total of over 600 students in the writer's design course during the past ten years, the writer has not found one capable of

solving, *in a reasonable time*, two such equations as these, with the certainty of being right the first time through. This is no criticism of student ability; for it is doubtful if any normal man lives who can solve such equations infallibly. The point is that one's first solution is entirely worthless until checked. The writer will not pay any attention to a solution unless evidence of proper checking is presented.

The checking is easily done in this case.

$$\frac{444 \times 81.6}{1.025 \times 0.0164 \times 1.273 \times 10^6} = 1.695 \text{ ohms},$$

which checks 1.67 original ohms closely enough.

The total coil m.m.f. is $4 \times 444 \times 33 = 58{,}600$ ampere-turns.

174. Discussion of Lifting-magnet Design.—The proper design of a lifting magnet rests more upon a base of experience than on theoretical factors. The most that theory can do is to direct the trend of the design. Theory cannot say what maximum operating temperatures can be withstood; but it does say that if the mica plates, for instance, are made much thicker than 0.10 inch, the internal temperature rise will reach an uneconomical value. Disregarding heating, it could probably be shown that the best proportions of coil and magnet dimensions, once attained in one size of magnet, should be nearly or quite duplicated in other sizes. But remembering internal heat flow paths, vertical flow being much more easy than radial, it is seen that the larger the diameter, the flatter the general shape; that is, with radial conduction of heat as the unimportant factor, the magnet is found to expand radially, but not nearly so fast vertically. In such rough but exceedingly valuable ways, theory vastly helps to direct the practical design. From here on, practical knowledge, based on long experience, must rule. Theory cannot, for instance, be used to make predictions as to how much snow-covered pig iron a given magnet can lift, or from what height a magnet could be dropped onto a slab without being cracked wide open.

CHAPTER XVIII

DIRECT-CURRENT MACHINE CONSTRUCTION

The following notes on the construction features of direct-current machines apply to sizes ranging from 1 to 500 horsepower, and for ordinary voltages of from 110 to 600; also, high-speed machines requiring special construction are not considered.

It is impossible here to give anything like complete information on the subject of construction or any of its phases. Only a minimum of facts pertaining to usual or representative construction practice is presented. The object of this chapter is to round out the background of the student to such an extent that he can follow the later discussions of the direct-current machine with intelligence and can work out design problems without being confused as to names of parts, the terms used, etc. Thus this chapter becomes useful for reference purpose in so far as the student is concerned; those actually concerned with construction in a professional way will have their own sources of information.[1]

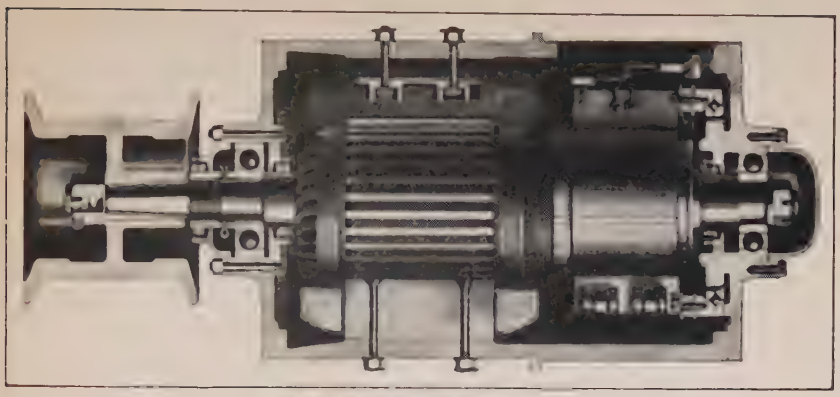
175. The Armature. I. *Laminations.*—The laminations, or punchings, are made of 14-mil (0.014-inch) electrical sheet, the quality varying somewhat with the type of machine to be built. After the punchings are punched from the sheet, they are annealed, both to relieve punching strains which raise the iron loss and to coat the lamination with oxide. This oxide film is a fairly good insulator. Later, an automatic system puts on and dries a coating of insulating varnish; thus eddy currents between punchings are effectively reduced. The punchings are assembled under a pressure of roughly 100 pounds to the square inch.

II. *Core Assembly.*—Refer to Figs. 54, 55, and 58. It should be noted that the whole of the laminations assembled into an armature is often referred to as the *core*, and one punching is often

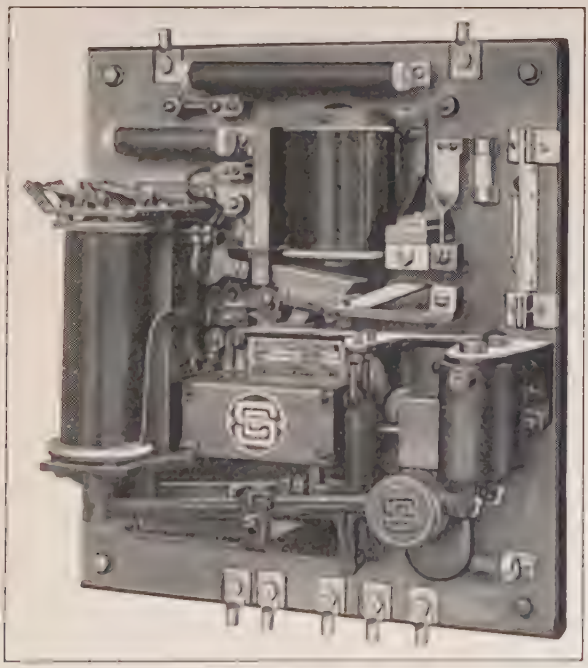
[1] Still, Alfred, "Elements of Electrical Design," McGraw-Hill Book Company, Inc.;

Gray, Alexander, "Electrical Machine Design," McGraw-Hill Book Company, Inc.;

Slichter, W. I., "Design of Electrical Machinery," John Wiley & Sons, Inc.



(*a*) Train-lighting generator.



(*b*) Control panel.

FIG. 54.—Train-lighting equipment. (*Courtesy Safety Car Heating & Lighting Company.*)

called a *core punching;* whereas, when the *magnetic circuit* is being discussed, only that part of the punchings below the teeth and slots is called the core.

The *end plates,* generally made of cast iron and placed at each end of the core, exert the axial force needed to hold the core punchings firmly together. In the smallest machines the end plates may be held together by rods which run through the core itself and which are upset in the end plate, thus forming a riveted construction, or the end plates may be attached to the shaft. In either case, the laminations are strung on the shaft itself. No internal core ventilation is possible in this assembly.

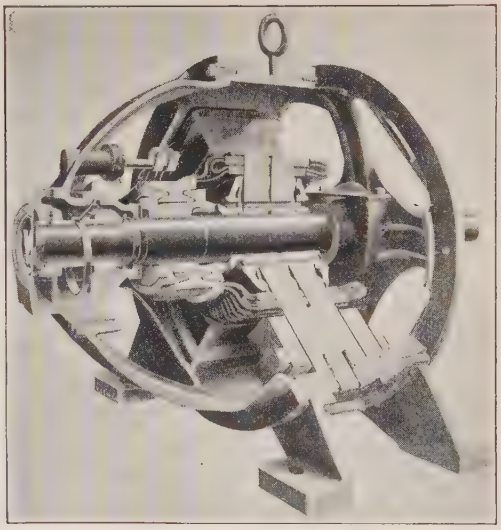


FIG. 55.—SK motor assembly. (*Courtesy Westinghouse Electric & Mfg. Co.*)

A core *hub* or *spider* is used in larger units, say, with armature diameters above 8 or 10 inches. This hub may be nothing but a cast-iron or steel tube, upon which end plates and core are mounted. If only a tube, it cannot provide internal ventilation; hence its use might be questioned. The factor justifying its use relates to manufacturing, assembly, and repair; for with this type of assembly, the armature unit can be completed, except for commutator, without the shaft; or in repair, a shaft can be pressed out and a new one installed without disturbing the core assembly, as would be the case if the end plates were attached to the shaft itself.

The *hub* is often known as the *spider*, when the size becomes large enough to give it radial arms upon which punchings and end plates are mounted. Thus only enough core depth below slots for maintaining proper core densities need be used, resulting in a saving of lamination material. The round piece punched out of each armature disc becomes available as a blank for use in a core punching of a smaller machine. The spaces between the spider arms provide air access under the core, which enables radial ventilating ducts to be used.

In these larger sizes (armature diameters of, say, 10 inches or more) an extension on the spider or on the end plate next to the

FIG. 56.—SK frame, wound. (*Courtesy Westinghouse Electric & Mfg. Co.*)

commutator is provided, over which is pressed the *commutator hub*. The commutator hub may or may not also have a press fit with the shaft. In either case, the shaft can be pressed in or removed *without disturbing* the entire armature assembly, an important factor in both assembly and repair.

III. *Ducts and Finger Plates.*—Small motors, such as the 1-horsepower motor, have no ducts. It is usually impracticable to cast teeth on the end plates which will hold in the tooth punchings; yet some axial compression must be put on the teeth to prevent them from bending out axially and from vibrating. This is often accomplished by using five or six heavy punchings at each end (about $\frac{1}{16}$ inch thick, each) whose tooth parts are stiff enough to do the job well enough.

From 5 horsepower and up, four poles or more are used, enabling form-wound armature coils to be applied. As is shown elsewhere, these coils need a straight extension beyond the slot for a short distance. The space for this extension is provided by the fingers on *finger plates*, which are assembled between the end plates and the laminations.

Formerly, *finger plates* were made of cast brass or cast iron. They are now made by spot-welding steel strips to a punching which is identical with the armature punchings but usually is somewhat thicker. The strips could easily be crushed over, as they are only $\frac{1}{16}$ inch thick, and usually $\frac{3}{8}$ inch wide; therefore, they are given a couple of bends. This modern finger-plate construction is better in every way; it is lighter, it permits of less eddy-current loss within the plate, and it gives maximum air flow access through itself.

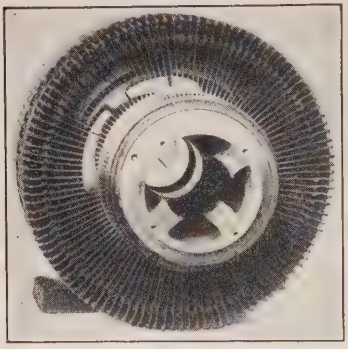


Fig. 57.—Commutator end of armature. (*Courtesy Westinghouse Electric & Mfg. Co.*)

The *radial ducts* are spaced by introducing finger plates just like those used at the ends of the core. "Fingers" and "spacers" are identical. When the hub or spider has good air access between its arms, air is drawn in under the core and thrown out through the finger plates in the ducts and also at the ends of the core. This provides very effective self-ventilation, provided the speed is high enough to give a turbulent air flow through the duct spaces.

176. The Winding. I. *Coils.*—Nearly all armature windings of any practical consequence are of the double-layer type; the exception is in the case of small and fractional-horsepower motors.

The *mush coil*, loosely wound, is variously applied to small armatures, or in the smallest armatures, the coils are wound on by machine. In either case, the machines are two-pole units as a rule, and it would be difficult to place formed coils into slots that are nearly opposite. Hence, the coils as they are applied are often made to fill their slots completely, the end parts of the coils being put on in some symmetrical pattern. The first coils have shorter end connections than the last ones, for the last ones

have to go on top of the first ones. The wires in the slots are haphazard.

Nearly all *formed* coils are wound *double layer*. All coils are alike; the coil sides filling the upper half of a slot return in the lower half of another slot nearly or exactly one pole pitch away. The wires in the slots are regular (see chapter on Windings).

II. *Bands and Wedges.*—Coil sides are retained in the slots either by *bands* or *wedges*. Either method is used on smaller machines, but wedges have the preference for the large units.

The bands are made of one layer of tightly wrapped piano wire, the band surface being flush with the armature surface. Bands are placed at from 3 to 6 inches apart. Each band is about ¾ inch wide. Under a band is found a bundle of punchings smaller than the normal punchings by enough to sink the band to the proper level. Tinned clips surround the band at about every pole pitch, the piano wires being all soldered together at these points. Band losses sometimes are excessive. This subject has been rather extensively studied (see bibliography). As the outer surface of the coils is exposed except where covered by bands, banding favors coil ventilation. To replace a coil, all bands must be removed.

Wedges fit into niches left in the sides of the teeth; the wedge is wider than the coil. This is a disadvantage in that where the wedge crosses a radial duct, it restricts the air flow. The wedge, made of maple or fiber, is a heat insulator, and forms a rather effective barrier to heat flow outward from the coil. The wedge is deeper than the band and so requires a deeper placing of coils, and deeper slots; narrower tooth roots and higher tooth-root densities result. On the other hand, not all wedges need be removed in order to replace a coil.

III. *End Connections.*—The end connections of very small armature coils form a compact mass. With form-wound coils of double layer, the end connections of larger units form a helical pattern. In small four-pole machines (say, of 10-inch armature diameter) the tooth-root width is so small that the inner edges of the inner end-connection layer are very close together. In order to get an appreciable ventilating space between coils, it would be necessary to carry the end connections out so straight that the axial length of the machine would be unduly great. Instead, the end connections should be cramped over as sharply as possible until inner edges are together so that the machine is

as short as possible; also, the more compact the end connections become, the better they will conduct heat to the outer surfaces, where it is dissipated.

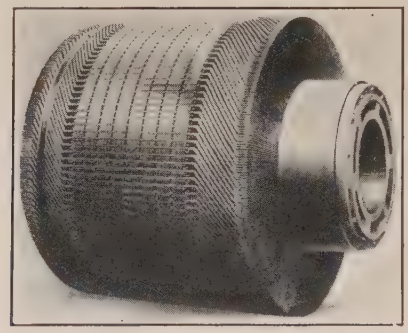


FIG. 58.—Type Q generator armature. (*Courtesy Westinghouse Electric & Mfg. Co.*)

In larger units (say, of 20-inch armature diameter or more) if the end connections are placed at from 30 to 45 degrees with the shaft, effective ventilating spaces can be left between them.

FIG. 59.—Type Q generator, rear end. (*Courtesy Westinghouse Electric & Mfg. Co.*)

Air flow through the end-connection gridwork of upper and lower crossed coil ends will be cut off, however, if, due to lack of stiffness in the coils, a treated tape or cloth separator must be used between upper and lower layers.

Formed coil end connections must be retained against centrifugal force, by the use of bands (see Fig. 58). In the smaller units, all that is needed at the commutator end is one band, and the stiffness furnished by the coil leads as soldered to the commutator. At the other end and, in large units, at both ends, the band must be given a *coil support ring* to react against. Without this support ring, the band would become loose and ineffective.

177. The Commutator.—The commutator bars are made of hard-drawn copper, pulled through a die that gives them the proper wedge shape. When assembled with the mica segments, a compact circle is formed. The built-up mica segments are

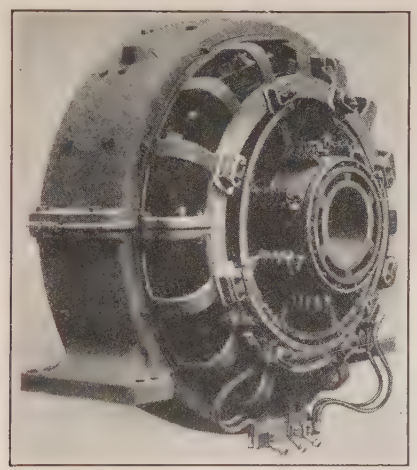


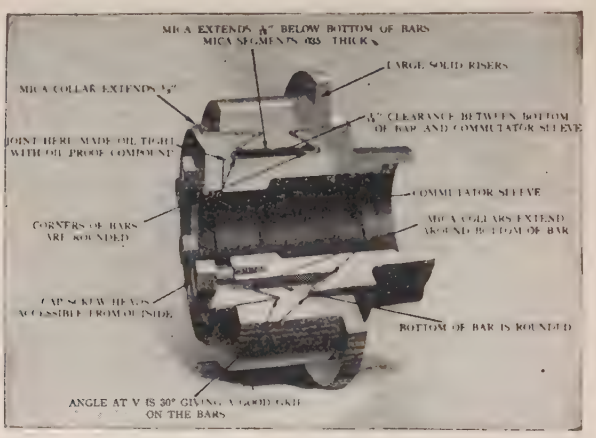
Fig. 60.—Type Q generator. (*Courtesy Westinghouse Electric & Mfg. Co.*)

$\frac{1}{32}$ inch thick; the segments are likely to wear more slowly than the bars and hence now are usually kept undercut by a $\frac{1}{16}$ inch or so. Steel or cast-iron V-rings (Fig. 61) mounted on the commutator hub are drawn towards each other by a ring nut or by bolts, thereby acting, through built-up mica V-rings, on the inner slopes of the V's which are machined on the ends of the assembled bars. The axial compression of the V-rings creates a radially acting inward component of force on each bar, thus drawing the bars together and causing a tangential reaction between them.

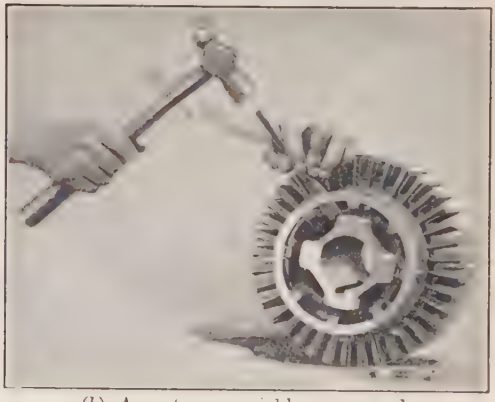
In small machines the coil leads are bent inward and are themselves soldered into slots cut in the stubby risers which are left on the bars when the commutator is machined. Large machines with strap or heavy-wire coils have risers soldered into the bars

which extend outward to the level of the winding; the leads are then soldered into the risers (see Fig. 57).

The commutator surface is made longer than the axial length of the brush assembly surface to permit of end play necessary


MICA EXTENDS 1/16" BELOW BOTTOM OF BARS
MICA SEGMENTS .033 THICK
LARGE SOLID RISERS
MICA COLLAR EXTENDS 1/2"
1/16" CLEARANCE BETWEEN BOTTOM OF BAR AND COMMUTATOR SLEEVE
JOINT HERE MADE OIL TIGHT WITH OIL PROOF COMPOUND
COMMUTATOR SLEEVE
CORNERS OF BARS ARE ROUNDED
MICA COLLARS EXTEND AROUND BOTTOM OF BAR
CAP SCREW HEADS ACCESSIBLE FROM OUTSIDE
BOTTOM OF BAR IS ROUNDED
ANGLE AT V IS 30° GIVING A GOOD GRIP ON THE BARS

(*a*) Commutator assembly.



(*b*) Armature assembly, unwound.

FIG. 61.—Commutator and armature. (*Courtesy the Reliance Electric & Engineering Company.*)

for preventing the wearing of grooves in the commutator and in sleeve-type bearings.

As noted above, the commutator assembly may be pressed directly on the shaft in small machines or, in larger sizes where it becomes possible, is pressed onto an extension from armature

hub or end plate; with the latter type, removal of the shaft (or assembly) will not disturb the armature assembly.

178. Brush Assembly.—Whereas brushes used to be placed at an angle with the commutator surface, they are now, in industrial machines at least, usually placed normal to the surface. They fit closely but not bindingly into steel (or, for bad atmospheric conditions, brass) boxes. Only in the smallest machines is the contact between brush and box relied on to carry the current. Otherwise, a pigtail is attached to the brush to shunt the current around this uncertain contact area. In various ingenious ways the brush box is insulated from but carried by the end bell or bracket. All of the brushes at one place make up a brush *set*, and if their boxes are carried by an insulated rod, this rod is called the *brush stud*. "Three brushes per stud" is an expression which thus becomes understandable.

Brushes rarely are used having an axial length of more than 1.5 inches. It is better, for several evident reasons, to use several brushes than one brush, say, 6 inches long. Two adjacent brushes on a stud are separated by twice the thickness of the brush-box wall, which is, if pressed steel, not more than $\frac{3}{32}$ inch. This is the first factor tending to lengthen the commutator beyond the net axial length of the brushes per stud. The second factor is this: The brushes of one stud should not trail those of the next exactly but should be staggered by at least the above separation; this to help in keeping a smooth-worn commutator surface. The third factor arises from needed end play. End play of $\frac{1}{4}$ inch is plenty for the smallest units, and it is doubtful if more than $\frac{1}{2}$ inch is ever needed.

Brush dimensions vary axially by $\frac{1}{8}$-inch increments, and on the chord, by $\frac{1}{6}$-inch increments, by *Electric Power Club Rules*.

179. Poles.—Only one type of pole is discussed here, and that is the type in which the whole pole is built up of $\frac{1}{16}$-inch punchings, held together by long axial rivets (see Figs. 62 and 63). The reason for laminating a pole is to reduce iron loss in the pole face, as caused by flux variations occurring here when the teeth of the armature sweep by. Thus only about $\frac{1}{4}$-inch depth of the pole face needs to be laminated, but most companies find, taking all things into account, that it is preferable to build up the whole pole from similar punchings than to use a cast pole core with a laminated pole shoe attached to it.

In the built-up pole (completely laminated) the axial length of the pole core, which is inside the coils, is the same as the length of the pole shoe; but the chord dimension of the core is always less than the chord of the face span, by a considerable amount. There are several reasons for this. Economy demands that the pole face shall cover about 70 per cent of the pole pitch. If now the entire pole is made as wide as the chord of the pole face, the field windings would either interfere or else leave so little space between them that they could not be kept cool. By indenting the pole sides, ventilating space is provided, field copper is saved due to reduction in mean turn, and a flat shoe extension is provided upon which to rest the field coil. The axial length of the pole is often made less or more than the armature core length by

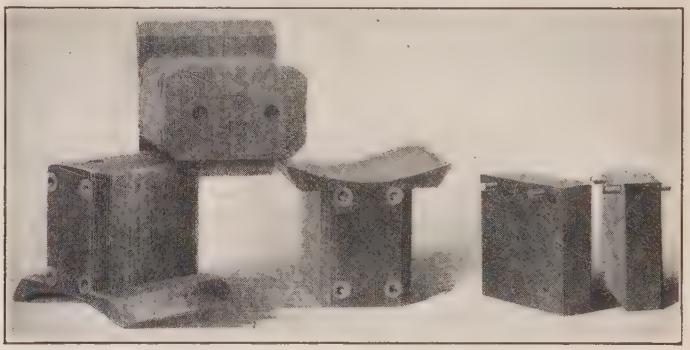


FIG. 62.—Various pole pieces. (*Courtesy the Reliance Electric & Engineering Company.*)

about twice the average air-gap length, in order that magnetic pull on the armature will remain fairly constant during the oscillation allowed by end play; if pole and armature are of the same length, free oscillation is in a large measure prevented.

The pole seats, if present (or else the entire inside surface of the yoke) are machined in a boring mill; hence, the pole punchings are made with a circular arc to fit to the yoke. Tap bolts reaching through the yoke are screwed into the pole body; the riveted assembly of pole punchings forms the pole iron into such a compact mass that it can be bored and tapped entirely satisfactorily.

It is good practice to round off the corners of the pole core, for field coils inevitably have rounded inner corners. This rounding may be done with the air chisel after assembly, or by varying the widths of the end punchings in the punching process.

180. Interpoles.—The interpoles (commutating poles) are generally rectangular pieces of rolled or forged steel, except in the largest sizes; in these, a laminated structure is used. Saturation of the interpole flux circuit is highly undesirable. Where the steel interpole joins a cast-iron yoke, reasonable interpole core densities may be saturating cast-iron densities; to avoid this condition of local yoke saturation, the laminated interpoles often are made with a wide base. Interpoles are also held to the yoke by tap bolts (see Fig. 63).

181. Interpole Coils.—Round wire is used in the smallest machines, square wire (Fig. 66) for the intermediate, and a strap coil, with asbestos or air in the space between adjacent turns (Figs. 63 and 64), for the large sizes. When space between coils

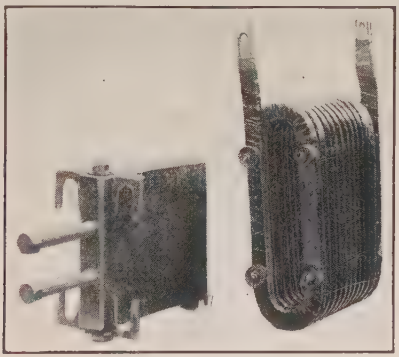


FIG. 63.—Interpole core and coil. (*Courtesy Westinghouse Electric & Mfg. Co.*)

is small, as in the smaller four-pole units, the interpole coil (Fig. 56) is tapered off in steps. Little insulation between turns is needed; insulation for the full machine voltage must be used between coil and pole core.

182. Field Coils.—With the full-laminated type of main pole, the pole core section is rectangular; the coils, therefore, are rectangular. Shunt coils are of round or square wire, depending on the size. They are always vacuum impregnated. If no other insulation between coil and pole is provided, the coil is heavily taped; or taping may be omitted in favor of the use of an insulating coil spool, or some other scheme of insulation applied to the pole surface before the coil is put on (see Figs. 65, 67, and 68).

Series coils, perhaps in the most widely used plan, surround the shunt coils, being placed out near the yoke where there is the

most available space (see Figs. 56 and 65). Unless made in the one-layer strap style, they are compactly built of round or square wire, the coil-wall section often being approximately square.

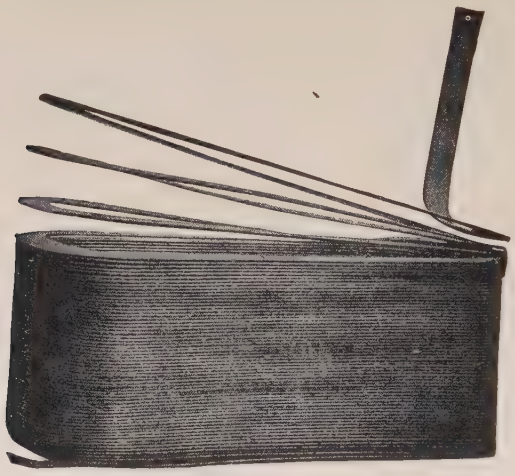


FIG. 64.—A strap coil. (*Courtesy the Allis-Chalmers Company.*)

183. The Yoke.—Formerly, cast-iron yokes were greatly used, because cast iron is easily obtained in complex shapes and is easily machined. Lately, the art of making dependable steel castings has developed; pound for pound, steel has about twice

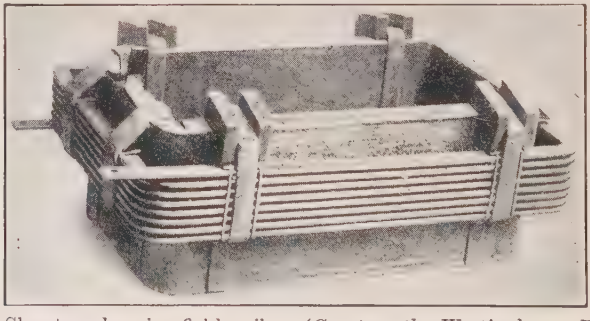


FIG. 65.—Shunt and series field coils. (*Courtesy the Westinghouse Electric & Mfg. Co.*)

the permeability of cast iron. Hence, steel yokes (Fig. 56) are widely used at present. Also, special machinery for bending heavy rolled-steel slabs has been developed, leading to the use of rolled-steel yokes in all but the largest sizes.

The axial length of the yoke is nowadays made about 1.50 to 1.75 times the armature core length. This gives enough yoke overhang beyond the field coils to give them good protection

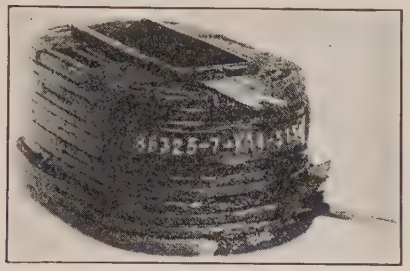


FIG. 66.—Interpole coil. (*Courtesy the General Electric Company.*)

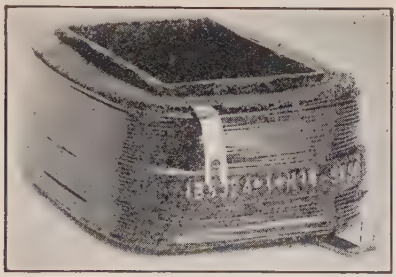


FIG. 67.—Shunt field coil. (*Courtesy the General Electric Company.*)

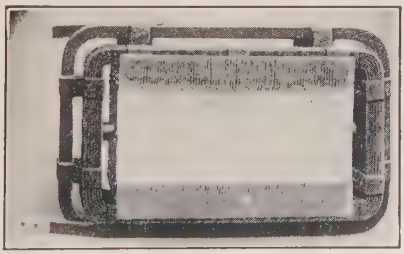


FIG. 68.—Face view of assembled pole unit. (*Courtesy the Westinghouse Electric & Mfg. Company.*)

against damage and yet leaves enough thickness, as based on necessary magnetic section, to insure the proper stiffness of yoke.

184. End Bells or Brackets.—The end bells or bearing brackets generally are cast iron. They fit the yoke closely, yet not so tightly but that the one at the commutator end can be shifted

slightly to allow of accurate setting of the brushes. Quite recently some pressed-steel brackets, with welded stiffening ribs, have appeared.

185. Non-open Types.—The usual motor or generator has open end bells to permit of air access and exit. A *semi-enclosed* machine has a netting of expanded-metal screen placed over the openings, the openings usually being about ½ inch. *Enclosed* machines have tight (removable) cover plates which close the machine entirely, so far as dust is concerned. *Forced-ventilated* machines are enclosed machines which are piped up to a fan or a ventilating system. Some open-type machines actually are to a certain degree of the forced-ventilated type in that there is a fan placed on the armature itself; but this does not class them in the forced-ventilated type, as that type is defined.

CHAPTER XIX

ARMATURE WINDINGS

Inasmuch as whole books have been written on the subject of windings, it cannot be hoped to cover the field very effectively in one chapter. Fortunately, outside of the double-layer lap (multiple) and the double-layer wave (series) winding, there are very few kinds having any practical significance; and these few are seldom used, in comparison with the lap and wave. This book will remain within the field covered by these two types. Even so, by no means can all of the facts and relations concerning lap and wave windings be set forth here. Enough information is given to furnish the background upon which the student can work in handling the more usual problems arising in direct-current machine design; for further facts or data, the reader is referred to the bibliography at the end of this chapter.

186. The Simple Lap (Multiple) Winding. I. *Styles of Representation.*—Figure 69 shows, in the fanned-out style, a simple lap winding. Figure 70*a* shows the same winding according to the developed style of drawing a winding. Figure 70*b* reduces the winding picture to a mere schematic diagram showing only the connection between commutator bars, and the number of paths.

Any style of drawing a winding has its advantages and disadvantages. The first method or style shows about everything there is to see about the winding, but this is true only for very simple cases. In all usual cases, if all the wires were drawn in, it would be very hard to follow; and in any case, it takes a good deal of time to make the drawing. The second method developed is easier to draw but, for most people, is not so easy to follow through; this is because of the open-ended effect of the drawing. The third method is easy to sketch, but it shows nothing as to where the coils are situated with respect to the poles. Everyone should be familiar with all three methods and should freely draw on the advantages of each as occasion demands.

II. *Paths.*—The *paths* in an armature winding are the routes by which current flows through it from the positive to the nega-

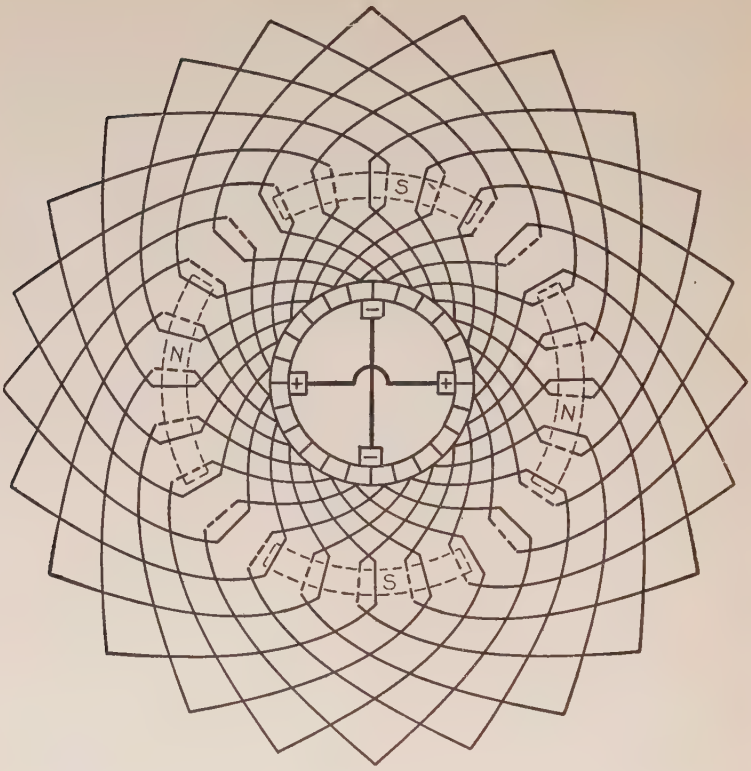
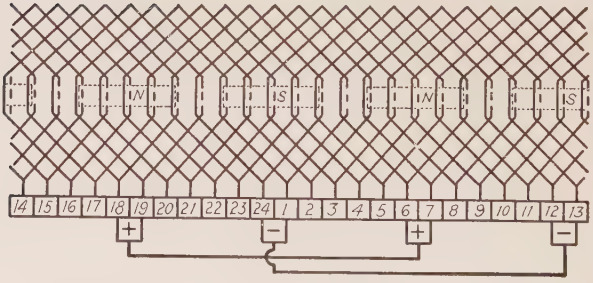
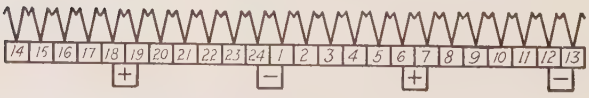


FIG. 69.—Fanned-out scheme, lap.



(*a*) Developed scheme.



(*b*) Circuit-only scheme.

FIG. 70.—Lap winding schemes.

tive sides of the line. Adjacent brush sets are opposite in polarity; from Fig. 70*b* it is evident that the lap winding has as many paths as there are poles.

The position of a path with respect to the pole fluxes is an important factor to make definite note of. From Fig. 69 or 70 it is seen that one path takes in (omitting commutated coils) the upper coil sides in one pole pitch, and the lower coil sides in an adjacent pole pitch. The voltage generated in a path then depends directly on the flux of two adjacent poles.

III. *Brushes.*—A full number of brushes (or brush sets) must be used; for if a brush position is omitted, the current will not be distributed under the poles correctly, and either the capacity of the machine to carry load is reduced, or if fully loaded at the shaft (as a motor), armature currents must be excessive to produce the required torque.

IV. Number of Coils.—In small machines any number of coils may satisfactorily be used. But as soon as the unit becomes large enough to warrant the use of equalizer connections (Chap. XX), the number of coils must be divisible by half the number of poles; for an equalizer spans two pole pitches and must connect to the winding at two points which should be at the same potential. In all large machines where the armature punching is built of two, four, six, or more sectors, the number of slots is divisible by the number of poles; and as the coils per slot must be a whole number, the number of coils is also divisible by the number of poles.

187. The Simple Wave (Series) Winding. I. *Fanned-out Style.*—Figure 71 shows a four-pole wave winding drawn in the fanned-out style. This winding has 23 coils and commutator bars. It is a *progressive* winding, in that, beginning at a bar and going, say, clockwise, following through enough coils (two, in the four-pole case) to come back near the starting point brings us out on the bar *ahead* of the one started from. Figure 72*a* develops a winding for an identical case, except that here a *retrogressive* winding is given. Here, in making one trip around the armature, the bar *behind* the bar started from is dropped upon. There is no practical difference between the two types, retrogressive or progressive, so far as performance is concerned; but as the retrogressive uses a little less copper in the end connections, it is the one usually used.

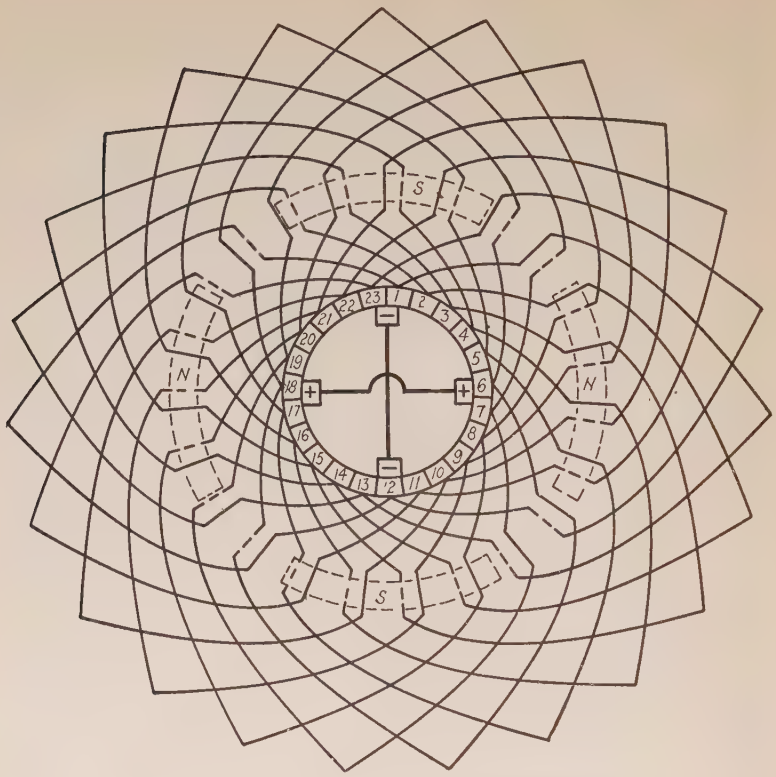
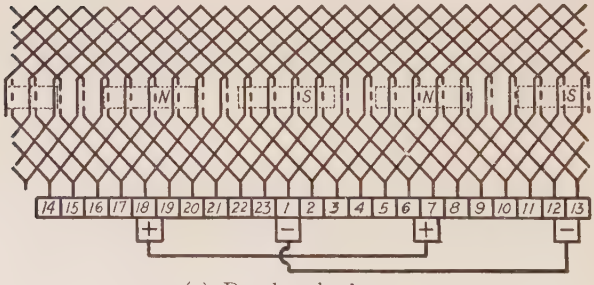
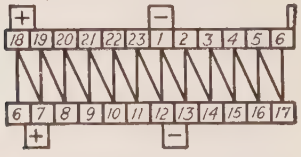


FIG. 71.—Fanned-out scheme, wave, progressive.



(*a*) Developed scheme.



(*b*) Circuit-only scheme.

FIG. 72.—Wave winding schemes.

Figure 72*b* shows the winding in a way corresponding to the simplest way for drawing a lap winding (Fig. 70*b*). A somewhat similar scheme was devised by one of the author's students, which suggested to the author the particular form used here. The wave winding is notoriously difficult to understand; whereas, this form, coupled with the use of the other two styles, very much helps to clear up the two or three difficult points involved in number of paths, use of less than the full number of brushes, and commutation.

II. *Paths.*—The student's greatest difficulty with the wave winding lies in seeing that no matter how many poles there are, or how many brush sets are used, there are but *two* paths through the armature. To establish this important fact, first compare Fig. 71 with Fig. 72*b* and then Fig. 72*a* with Fig. 72*b*. It is seen that bar 1 in every case is connected to bars 12 and 13, and likewise for any other sets of bars. Then Fig. 72*b* represents, as concerns electrical connections between bars, either the progressive or the retrogressive case.

Having established the fact that Fig. 72*b* accurately shows the electrical paths, it is at once evident from this drawing that, in a four-pole machine, there are but two paths from the positive brushes through the winding to the negative brushes. That there are but two paths in a multipolar machine is as easily shown.

Another way of getting at the two-path demonstration is to see in Fig. 71, for instance, how many ways there are for active current to leave the positive brushes. The reader is advised, at this point, to seek out all the coils being commutated by either positive brush acting alone and also by both acting together, and to color the coils so as clearly to identify them. Now, the left-hand brush makes contact with two bars, which have a total of four coil leads leaving them. But three of these leads simply go to the two bars contacting with the right-hand brush and hence are leads of commutated coils. There are remaining only two leads in all, or one away from each positive brush, which, if followed, will carry through the active coils and end at the negative brushes.

While making this study, the ability of the machine to operate with less than the full number of brush sets should be investigated (see next article).

As to the position of a path under the poles, a close study of the winding drawings will demonstrate that one path (omitting

reference to commutated coils) fills the upper halves of the slots of one pole pitch, then the lower halves of those of the next pitch, then the uppers in the next, lowers in the next, etc., until the path has been found under all poles. The other path uses the remainder half-slots. Thus, the generated voltage of a path is contributed to by the flux of each pole. No matter how individual pole fluxes may vary, the two path voltages will be balanced.

III. *Brushes.*—In Fig. 71, let the right positive brush be removed. One coil which was commutating due to both plus brushes is thereby removed from commutation, and this coil now assumes active path current; that is, both active paths start from the left positive brush. Note that the only change is to add a coil to one path, and to affect commutation to some degree; *there are still two paths left.* Likewise, either negative brush may be removed. A multipolar wave-wound armature can operate on any number of brush sets, provided a positive and a negative set are used. If a short commutator is desired more than the cost of extra brush sets is disliked, as is usually the case in industrial machinery, the full number of sets is used. If it is difficult to inspect and change brushes, as in railways motors, only two sets are used.

188. Number of Coils.—Textbooks invariably give the rules for the number of coils to use in a wave winding in terms of a formula; but unless the derivation of the formula is evident, little understanding of the subject is arrived at. Only *retrogressive* windings will be presented here.

Consider the four-pole case. It is first noted that all coils must span the same number of bars, else any attempt to make a regular winding will only result in a tangle, or in omitted bars or coils. Next, begin with the lowest bar (Fig. 71) and go each way. The two coils started on must arrive on adjacent bars at the top of the commutator, and for a retrogressive job, the two top leads must not overlap each other. (This particular winding is progressive.) Now, as these two coils must span the same number of bars, a mica segment at the top of the machine must be diametrically opposite a bar at the bottom, and furthermore, between the two bars connected to either coil, there must be the same whole (any) number of bars. Then all that is required of this machine is that it must have an odd number of bars and coils.

Consider the six-pole case. First, let three bars be placed at *equal* spacings around the commutator periphery. Three coils connecting these bars would reach completely around the machine and would span equal amounts; but the winding is closed in three moves and is therefore worthless. Instead, let four bars be placed, two being adjacent bars (Fig. 73). The bars are so placed that the three coils shown span equal amounts. All that is then necessary is to fill in the vacant arcs with the same number of bars each and to connect to them coils exactly like the three already drawn. The general plan now becomes clear. In any

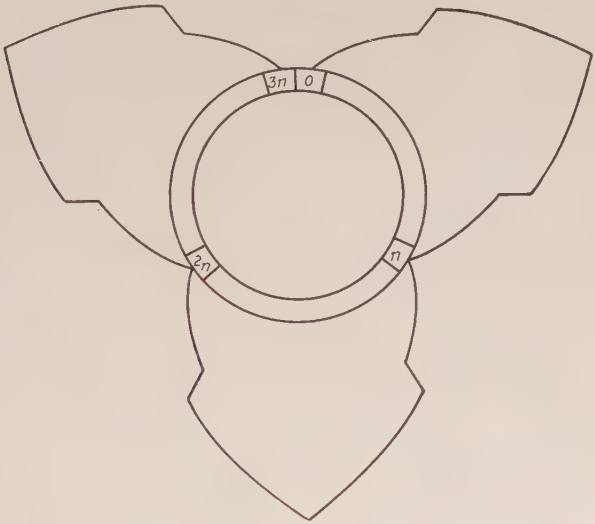


FIG. 73.—Start of 6-pole wave winding, retrogressive.

case, a coil may span n bar pitches; in going once around the armature there must be as many of these spans as there are pairs of poles; and there must be one bar pitch left over.

For a retrogressive wave winding, the number of bars is a multiple of the number of pairs of poles, plus one.

As the number of coils and the number of bars is the same, the above rule holds for the determination of the number of coils to be used.

189. The Frogleg Winding.—The Allis-Chalmers Company has recently developed and patented what is called the *frogleg* winding, so called because of the appearance of a combination coil as shown in Fig. 74. This is a four-layer winding. Suppose

the two middle layers in the slots comprises a complete lap winding; then the outer layers, that is, upper and lower, comprise a complete wave winding, or *vice versa.* The two complete windings are connected to the one commutator; in order to make this possible, a proper number of coils and bars must be used.

This company claims to have had much success with this type of winding as applied to heavy-duty steel-mill motors, applica-

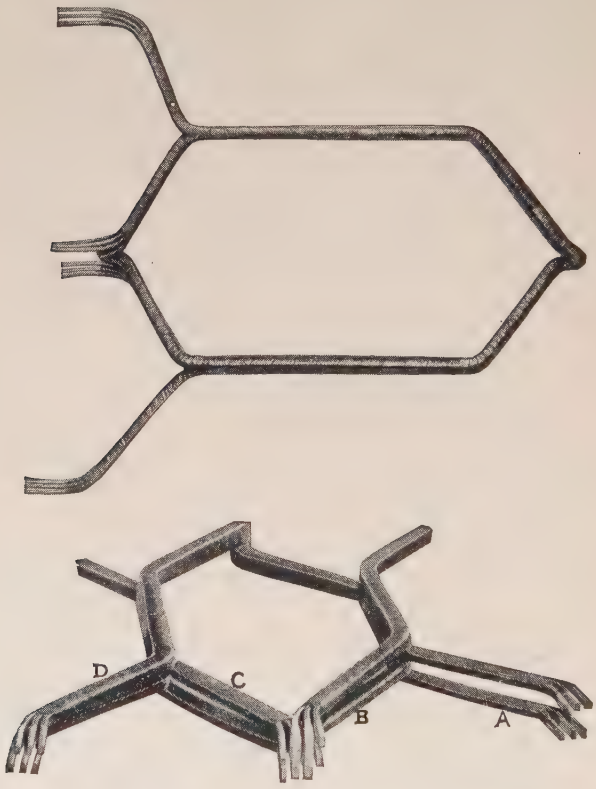


FIG. 74.—Frogleg winding coils. (*Courtesy the Allis-Chalmers Company.*)

tions in which bad commutation troubles are likely to materialize. The winding will be referred to in this book only in one other place—in the chapter on Equalizer Connections—where it may properly be discussed.

190. Choice between Lap and Wave Windings.—It is interesting to note that in the case of a two-pole machine, the two types of winding degenerate into one and the same thing.

The great majority of four-pole industrial motors are made for 115- or 230-volt operation; in many cases, there would be little choice between lap or wave windings; but the simplicity desired causes the choice to be made largely from the wave, because the path voltages are inherently balanced, no matter how pole fluxes may differ.

For multipolar units, the choice depends on several factors. The wave winding gives fewer and heavier conductors; beyond a certain section area, eddy currents due to slot flux variation becomes undesirably high, or the conductor may be too stiff to be easily or practicably handled in the processes of coil forming and armature winding. In general, high voltage or low speed will require many conductors per path, which requirement is better met by the wave winding. On the other hand, the fact that the path voltages in a wave winding are always balanced reacts to the disadvantage of the machine so wound. This is because the wave winding does nothing to correct flux unbalance; whereas the large lap-wound machine, equipped with equalizers, corrects flux unbalance, hence corrects unbalanced magnetic pull on the armature and shaft deflection resulting therefrom.

Small wires cost more per pound than big ones, and in general, it costs more to wind and insulate several small wires than one big one. As the lap winding always calls for more wires than does the wave, it might seem that, with the exceptions of objection noted above, the wave would always be preferred. Yet the wave can give *too few* conductors in certain cases: There is a limit to the minimum number of slots to use, and there must at least be two conductors per slot, a fact that sometimes forces the use of the lap winding without respect to other considerations.

191. Relation of Coils to Slots.—Aside from railway motors and special units, usual machines are economically built with a number of slots ranging from about 7 per pole in small sizes to about 15 per pole in the largest sizes. If commutator bars equaled slots in number, the bars would range from 7 to 15 per pole. Now the whole voltage is applied to the bars spanning the arc from one brush to the next (or to where the next *might* be, in the case of a wave-wound armature with some brush sets omitted), and this span corresponds to a pole pitch. Safe practice dictates that the volts between adjacent bars should average not over 15. For a 230-volt machine, this figure requires that at least 15.3 bars per pole shall be used. And usually, the average

number of volts per bar is made nearer 4 to 8, instead of 15. In the usual case, then, there must be more bars than slots.

Now a coil has two sides—the two straight slot parts—and there must be at least two coil sides in a slot, or one coil per slot. By bunching the coils into groups of two coils, twice as many coils as slots are obtained, with twice as many bars as slots. Nearly all ordinary machines have one, two, or three coils per slots.

192. Dead Coil.—Certain wave windings are made possible only by the use of a dead coil or its equivalent, the jumper. Suppose a 33-slot, four-pole armature is to be wave wound. One coil per slot will work out all right, or three coils per slot; but with two coils per slot, 66 coils and bars will not work into a wave winding. Yet, for other reasons, it may be necessary to use this combination. Then all 66 coils are put on, but one coil has its leads taped up and it is left unconnected; 65 bars are used to which to connect the 65 live coils.

Or another way around may be used. Before explaining it, it should be stated that in an ordinary machine with many coils per path, if one coil is burned out or grounded, temporary repairs may be made by cutting the defective coil loose and closing the circuit by a wire or jumper which is soldered in to the bars to which the coil went. In building the above armature, all 66 coils may be used with 67 bars, and the jumper connects to the two places where the absent coil 67 otherwise would be. A slight gain in economy is thus made, in that all coils are placed in use.

It usually occurs to the student to ask, If a dead coil is to be used, why not save that coil for another machine? It is less expensive to make all coils alike than to make a dummy to fill the space left if the dead coil is not put in; and the dummy would have to have the right weight, else it would unbalance the armature; moreover, a dummy might not shrink and expand with temperature changes like other coils in the same slot, thus tending to cause either excessive tightness or excessive looseness in the slot.

193. The Diamond Coil.—The usual term by which the formed two-layer coil is known is the *diamond coil*, because of its shape. One slow way of forming the coil is to wind its wire directly onto the armature, using the armature as the coil mold. It takes skilful work to do this, and even then individual coils are likely to vary in shape and length. This hand method was at first super-

seded by the invention and use of wooden molds, upon which each coil was formed. Many special coils still are made up in this manner.

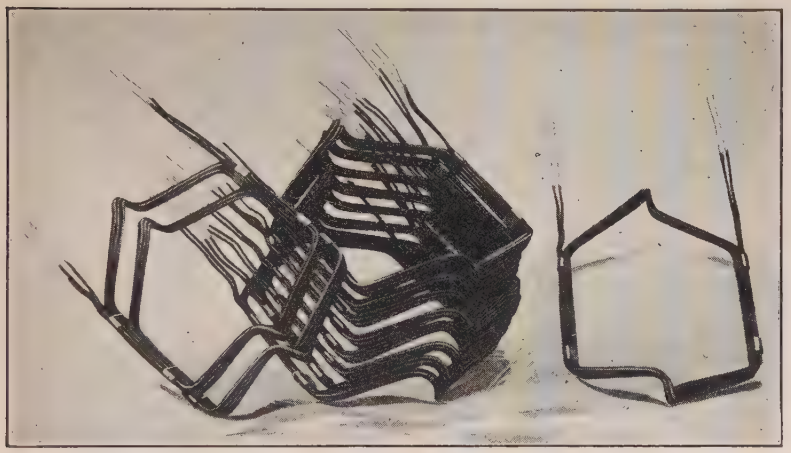


FIG. 75.—Pulled coils. (*Courtesy Reliance Electric & Engineering Co.*)

The *pulled* coil is nowadays the usual coil used. It is easily seen that any ordinary diamond coil might be closed up on itself; the lower side would swing neatly under the upper side, and the end connections would straighten out to be in line with the now

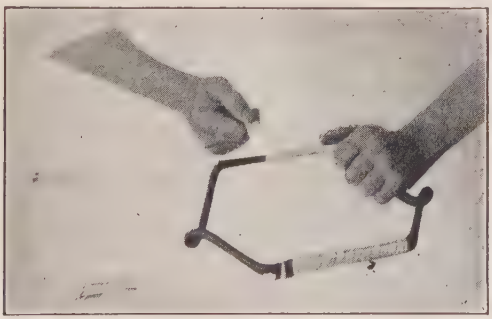


FIG. 76.—Taping a formed group-coil. (*Courtesy the Reliance Electric & Engineering Company.*)

parallel sides. The coil is first made in the closed-up form; that is, a long, closed-hairpin shape of coil is wound on a simple adjustable mold. It is then placed in the coil puller, which has adjustable jaws. The jaws grab the middle parts of the long

coil, so as to keep straight what are to be the coil sides; the jaws are then swung apart on a proper radius. Meanwhile, cheeks and pins placed around the curls at the ends keep the curls in line with the coil center. These end devices move inward as the sides swing out. The finished shape is like any of the coils of Fig. 77. One mold and one coil puller can be adjusted to handle a large number of different coils; the operations are fast. Thus exactly similar coils are formed with a low labor cost (see Figs. 75 and 76).

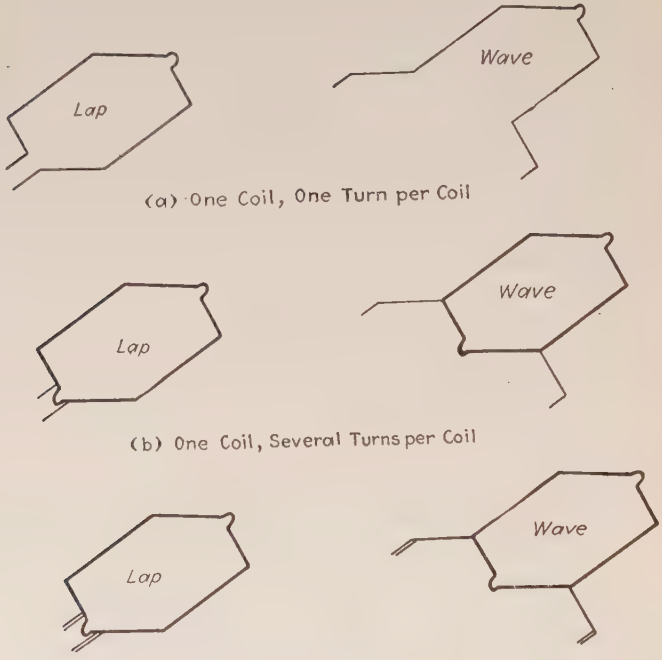


FIG. 77.—Evolution of lap and wave group-coils.

194. Definition of a Coil.—Before proceeding further to discuss coils, the exact definition of a coil should be framed. The usage varies with different writers and different manufacturers. Throughout this book, the following definitions will be followed:

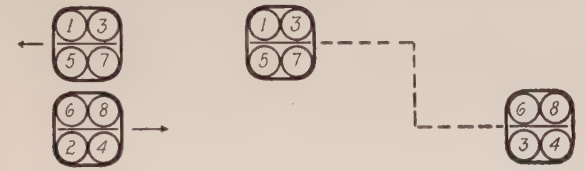
In leaving a commutator bar, going through a part of the winding, and first returning to another bar, one **coil** *is traversed.*

When two or more coils are grouped together under the same tape, so that the assembled unit enters the armature as a unit and

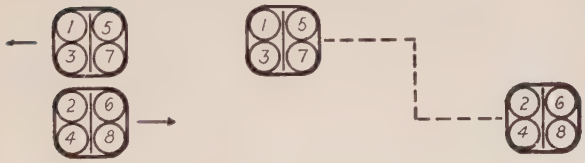
thereby fills an upper and a lower slot half, the unit is called a **group coil.**

There are as many coils as bars, for a coil has two leads, and each bar has two leads soldered to it. The number of coils in a group coil is the same as the number of coils per slot. The presence of two coil sides in a slot means one coil per slot, and the group coil has degenerated into a single coil. The presence of four coil sides in a slot gives two coils per slot and also two coils in the group coil. There is always one group coil per slot.

Figure 77*a* shows lap and wave coils of one turn each. Figure 77*b* shows single coils of several turns. Figure 77*c* shows group coils containing two coils, as evidenced by the four leads.



(*a*) Horizontal division.



(*b*) Vertical division.

FIG. 78.—Position of coils in a group-coil. (Left views—coil side sections before pulling.) (Right views—coil side sections after pulling.)

195. Arrangement of Coils in a Group Coil.—Refer to Fig. 78, which shows the sections of the two sides of a coil. At the left (Fig. 78*a*), the two sides are one below the other, as they occur before being pulled. After pulling, indicated by the arrows, the upper and lower sides are separated, as at the right. This is the case of two coils in a group, two turns in a coil.

Imagine that we are standing at the commutator end, which is the front of the armature, and looking towards the back. Let one coil be made so that (Fig. 78*a*) beginning at a commutator bar, we go to the back by conductor 1, return by 2, back by 3, return by 4, and go to another bar. The other coil is traversed by starting from a bar next to the other beginning bar, going back

by conductor 5, returning by 6, back by 7, returning by 8, and to a bar next to the other finishing bar. This placing of conductors gives a horizontal or peripheral division of coils, which has two bad shortcomings. The two coils lie in different positions in the slots, which is not a good feature as concerns commutation. The other disadvantage relates to what are known as *cross-overs*. The leads of the inner coil (5-6-7-8) must get above the upper row or coil sides, or below the lower row or layer, in order to connect to the commutator. In bringing these leads out, they cross or pass the wires of the outer coil, thus requiring extra insulation at the cross-over point.

The arrangement of Fig. 78*b* is much better. A vertical or radial division of coils is attained by winding one coil so as to take in the 1-2-3-4 conductors, and the other one likewise, 5-6-7-8. Note that the front ends of conductors 1 and 4 become leads and also those of 5 and 8, that the leads are all at the extreme inner or outer surface of the double layer, and also that they are in regular order for connection to the bars. Yet cross-overs are not eliminated; they are left to occur at the extreme ends, or curls, of the coils, where the wires are carried past each other in the open air; thus the crossed wires are separated both by an air space and by the tape which is applied.

196. Coil Pitch.—The pitch of a coil may be related to or stated in terms of the number of commutator bar pitches it spans, or the difference between the numbers assigned to its coil sides, or the number of slot pitches spanned. The last usage will be followed throughout this book; that is, if the slots are numbered, and if a coil occupies parts of slots 1 and 15, the coil pitch or span is $15 - 1 = 14$ slot pitches. A *full-pitch coil* spans exactly a pole pitch. Unless the number of slots is divisible by the number of poles, full-pitch coils cannot be had. In direct-current machines, it is usually best, for reasons pertaining to commutation, to use coils of as nearly full pitch as possible.

197. Armature-winding Insulation.—There are any number of insulation combinations used on armature coils. No one class of insulation, or combination of insulations, is best. Most machines of ordinary temperature rises and ordinary voltage use class A insulation (treated or impregnated cellulose materials) having a deterioration limit of 105°C. Large machines with strap or bar conductors usually have a combination of classes A and B insulations.

TABLE VI.—SLOT-INSULATION ALLOWANCES FOR VOLTAGES UP TO 500

Both width and depth allowances include about 15 mils for irregularity of stacking of laminations.

Depth allowance includes 0.115 inch for band, or 0.205 inch for wedge, as the case may be.

Allowances are in mils.

Number of conductors across slot		1	2	3	4
For double-cotton-covered round or square wires, and double-cotton-covered strap not wider than 0.15 inch., nor deeper than 0.35 inch.	Width	90	100	110	120
	Depth, banded	250	250	250	250
	Depth, wedged	340	340	340	340
Add allowance given to total width or total depth of double-cotton-covered conductors, to get punched slot dimensions.					
For bare strap wider than 0.15 inch., or deeper than 0.35 inch.	Width	100	125	150	175
	Depth, banded	280	300	300	300
	Depth, wedged	370	390	390	390
Add allowance given to total width or total depth of bare straps, to get punched slot dimensions.					

Figure 79 shows a slot-and-coil insulation combination suitable for 115- or 230-voltage windings, although when round wires are shown, it is more than likely that vertical paper or treated cloth spacers would be included between adjacent coil sides.

Table VI gives suitable space allowances for representative arrangements of conductors in slots, both for banded and wedged retaining methods. Depending on the kinds and combinations of materials used, these allowances would be a little high or low.

The A.I.E.E. Rules call for a 60-second test of new machine circuits between each circuit and the grounded metal armature or frame and between each circuit and every other circuit at an effective value of sine-wave voltage equal to twice the rated voltage plus 1,000, the frequency to be not less than that of the normal machine frequency. Thus a 115-volt unit is tested at 1,230 volts; a 230-volt unit, at 1,460 volts. Practically, both units would likely be tested at 1,500 volts, and again practically, both machines would be equally well insulated as a general rule.

In fact, it is customary to insulate ordinary machines operating below 500 volts as they were 500-volt machines.

It would seem that the above tests call for enormous factors of safety; and if the various insulating coverings were taken at their face values of volts per mil, the computed voltage-breakdown strength of any ordinary coil would be in the thousands of volts. However, after the coils have been handled several times before being wound onto the armature, and then have been hammered into the slots and wedged or banded down, the factor of safety has in many spots been greatly reduced.

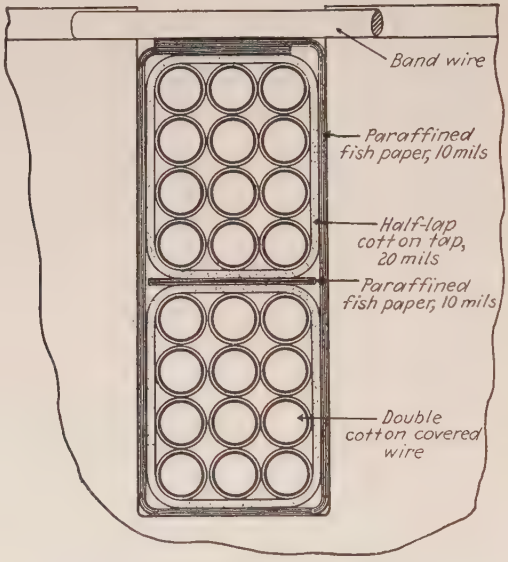

Band wire
Paraffined fish paper, 10 mils
Half-lap cotton tap, 20 mils
Paraffined fish paper, 10 mils
Double cotton covered wire

FIG. 79.—Slot insulation, 230 volts.

The insulation shown in Fig. 79 is typical, so far as group-coil taping and slot lining are concerned. Double-cotton-covered conductors are used, except in the case of large bar or straps, which are either cotton taped or wrapped with a mica paper wrapper. Insulation between adjacent turns of a coil is usually amply taken care of by the cotton covering, which is varnish dipped or impregnated. This also may sometimes be considered enough to insulate between adjacent coil sides within the group coil; if not, paper, treated cloth, or mica paper spacers or wrappers are interleaved between coil sides. The group coil is wrapped with cotton tape, half lapped, the end connections being suffi-

ciently taped at this time. The taped group coil is then given several varnish dippings, being heated and dried before and after each treatment, or else is vacuum impregnated. By this time it is more than likely that the coil section has swelled out of the rectangular, and the final operation before winding onto the machine is to hot press the coil back to shape. The paraffined-fishpaper slot lining lubricates the passage of the coil into the slot and acts thereafter as an excellent mechanical protection for the coil insulation.

The slot insulation or lining, and the slot part of the coil insulation, must project from either end of the slot by an amount varying from ½ to 1 inch; the lower amount is for small, ordinary-voltage machines, and the larger is for large, high-voltage units. This is to prevent creepage breakdown from conductors back to the armature iron.

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CHAPTER XX

THEORY OF THE ACTION OF EQUALIZER CONNECTIONS IN LAP WINDINGS

The usual explanation given for the action of equalizer connections (cross-connections) in lap windings of armatures is inadequate. In this chapter the writer's theory is set forth.[1]

198. Circulating Currents Due to Inequalities.—Small direct-current machines can be built with lap-wound armatures, which operate satisfactorily without equalizer connections. When large sizes were first attempted, it was found to be next to impossible to keep them in operation for any length of time. Air-gap variations due to manufacture, shaft deflection, or bearing wear caused inequalities in main-pole fluxes to appear. In the lap winding, a particular armature path has its voltage generated only by the flux of two adjacent poles. If pole strengths are unequal, the voltages of the various parallel paths become unequal. This leads to the circulation of large currents between paths, irrespective of whether the machine is loaded or not. These currents must pass through the brushes, the resulting overload on some of the brush contacts proving to be a serious limitation on the machine's capacity to carry current.

The equalizer cured the difficulty. An equalizer connects points on the winding that should be at the same potential. It is usually stated that the equalizers simply furnish a much easier path for the circulating currents than is found through the brushes and that thereby the brushes are spared the duty of carrying the overloading currents. A little thought will show that this idea is superficial. Without equalizers, the currents circulating through the brushes and the regular or ordinary armature paths furnish a direct-current machine phenomenon; whereas equalizers amount to alternating-current taps, and whatever current flows by virtue of their presence furnishes an alternating-current machine phenomenon.

[1] Moore, A. D., "Theory of the Action of Equalizer Connections in Lap Windings," *Elec. Jour.*, December, 1926.

199. Two Kinds of Unbalance.—There are two kinds of unbalance possible in the machine, and it is not usually recognized that, depending on the method used, either one or both kinds of unbalance may be corrected. Flux unbalance occurs first, this then causing unbalance of generated voltage in the parallel paths. Without equalizers, the circulating currents enable the paths to have the same terminal voltage by virtue of the path IR drops occasioned by the current flow, and the magnetic unbalance remains uncorrected. With equalizers, the currents flowing by virtue of the presence of these taps act primarily to correct the magnetic unbalance; this accomplished, no voltage unbalance can remain.

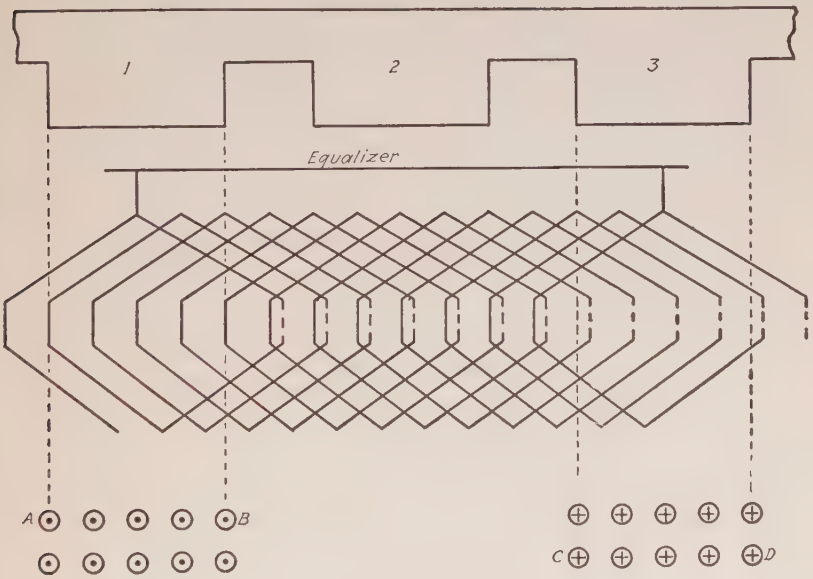


Fig. 80.—An equalizer circuit.

200. The Equalizer Circuit.—In Fig. 80 is shown part of the lap winding of a multipolar machine; one equalizer connector is shown here, spanning exactly two pole pitches. The part of the winding tapped or short-circuited by this connector, together with the connector, form a closed path which will be called the *equalizer circuit.* The problem is to find what kind of current will flow in this circuit due to unequal pole fluxes, when and where it occurs, and what its effect will be magnetically.

Under pole 2, there are seven slots, and the equalizer circuit fills these slots entirely. When one of these slots passes through the flux of a pole, voltage is induced in both upper and lower coil sides. But this voltage, being in the same direction along the slot wires, acts in the circuit to nullify itself, because *in this circuit*, an outward voltage in an upper wire is opposite to an outward voltage in a lower wire. Then no wires in these seven slots can act to produce equalizer circuit current.

There are five slots under pole 1, and the circuit occupies the upper halves of them; the circuit also occupies the lower halves of the five slots under pole 3. If poles 1 and 3 are equal and identical north (or south) poles, their fluxes will induce equal, but in this circuit, opposing, voltages. It follows that with a machine in perfect magnetic balance, no equalizer circuit current can flow. But if poles 1 and 3 are unequal, there will be a net induced voltage that will establish current.

201. Analysis of Magnetic Unbalance.—It is of interest to see how unbalanced cases may be analyzed. The primary rule governing all cases (except those in which unusual flux leakage occurs) is that total pole flux entering the armature must equal total pole flux leaving it. In a four-pole unit, such consecutive pole strengths as 6-9-10-6 cannot happen, for here total north flux does not equal total south flux; a condition such as 6-9-10-7 is possible, as inspection will show.

A six-pole unit can be analyzed as follows:

Pole numbers	1	2	3	4	5	6
Pole signs	*N*	*S*	*N*	*S*	*N*	*S*
Pole strengths	10	9	8	6	6	9

These pole strengths may be resolved into a fundamental or underlying machine having equal pole strengths, plus several components:

Underlying machine	6	6	6	6	6	6
Component machine 1	3	3	0	0	0	0
Component machine 2	0	0	2	0	0	2
Component machine 3	1	0	0	0	0	1

The underlying machine, having equal pole strengths, is balanced. It can set up no circulating currents of any kind and therefore

presents no problem. Component 1 consists of equal *N*-*S* poles 1 and 2, the remaining poles having zero components. Two such flux components will create equalizer circuit currents.

Or the above machine can be analyzed differently:

Underlying machine	8	8	8	8	8	8
Component machine 1	2	0	0	0	−2	0
Component machine 2	0	1	0	−1	0	0
Component machine 3	0	0	0	−1	0	1

In this scheme, a component machine consists of four zero-component poles, together with two north or two south poles carrying the component flux, one of the two being reversed.

Equalizer theory could be developed by studying the train of events induced by a component machine derived by either analysis; the difficulty is that in one component machine, the active poles may be adjacent; in another, far apart. A single analysis would not be general. The theory will be derived, therefore, in terms of phenomena set up by *one* component pole.

202. Cancellation of Part of the Equalizer Circuit.—It is already shown that the central conductors of the circuit under pole 2 (Fig. 80) have no net voltage effect. Further note that any current flowing in the *equalizer circuit* will, if it flows inward in the upper halves of these slots, flow outward in the lower halves. Equalizer circuit currents in these slots, therefore, have no general magnetizing effects, and these central conductors may as well not be there. The remaining active conductors of the circuit are found in the upper-half span *AB* under pole 1, and the lower-half span *CD* under pole 3, all being in series with themselves and the equalizer connector.

203. Voltage Induced by One Component Pole, in One Circuit. In Fig. 81*a* is shown an eight-pole machine, wires *AB*-*CD* representing the active conductors of one equalizer circuit.

Let the zero of time be taken when the circuit is in the position shown. Let the positive direction in the circuit be taken as indicated by the conventional marks on the conductors.

The flux component to be used is that of pole 4, a north flux component being used. Conductors *CD*, moving to the right, will travel from their present zero position for two pole pitches while having voltage induced in them by the flux of pole 4. With usual flux distribution, the wave of voltage induced will be

roughly triangular or trapezoidal when the voltages of all *CD* conductors are added up.

After this travel is accomplished, conductors *AB* will be in the former position of conductors *CD*, and during the next two pole pitches of travel, another wave of voltage will be induced in them, opposite, however, to the former *CD*-voltage, when taken around the circuit. When the first voltage induction stops, the second begins. This statement is made under the assumption that, for

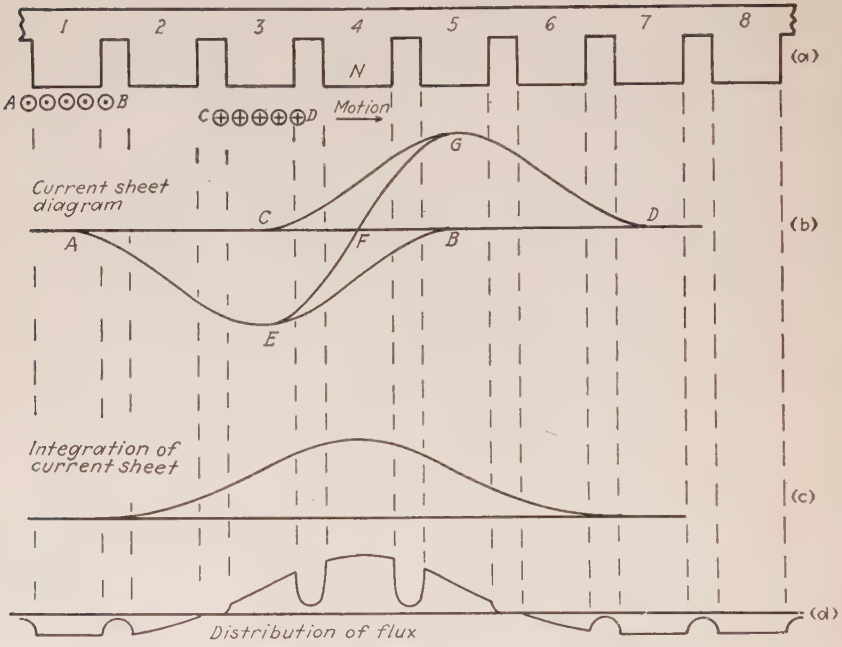


Fig. 81.—Equalizer circuit effects.

usual designs, the *AB-CD* spans would very nearly cover a full pole pitch each.

Thus one full cycle of circuit voltage is induced. Assuming temporarily for ease of study that the wave is harmonic, the wave is shown by curve *e* (Fig.82).

204. Current in the Equalizer Circuit.—The reactance of the equalizer circuit, for all usual frequencies, is high compared with its resistance. When a sinusoidal voltage is suddenly impressed on such a circuit, the switch being closed when the voltage passes through zero (which corresponds to the case here), the first cycle

of current is like that shown by curve i (Fig. 82). This is a sine wave, displaced 90° behind the voltage and elevated until it rests upon the axis. In this case, the current transient ends here; in the usual circuit case, when the voltage is continued, the current wave lowers gradually until it becomes symmetrical with the axis, when stable conditions are reached.

This current result can be seen directly, if it is agreed to neglect iR drops, for if e is the induced voltage, and L the henries inductance,

$$e = -L\frac{di}{dt},$$

or

$$edt = -Ldi,$$

$$\int_0^t edt = -L\int_0^i di,$$

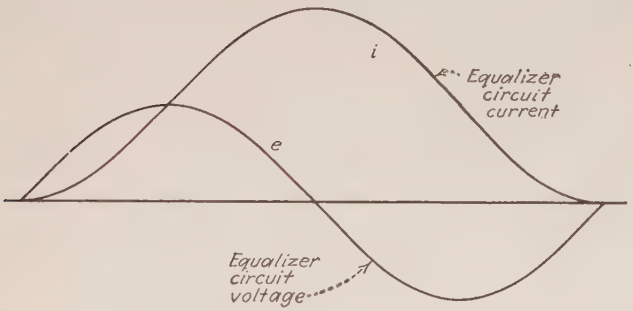


FIG. 82.—Equalizer circuit voltage and current.

which shows that the area under the voltage curve, integrated from the point where it begins, represents at time t a figure in proportion to the current built up by that time. Thus, it does not matter whether the voltage wave is sinusoidal or not. In any case, its two half-cycle parts are equal in area, which means that the current builds up to some maximum value and then comes down to zero again, at the time when the voltage ends its cycle.

Plotting current in a space diagram (Fig. 81*b*), curve *CGD* shows how the middle conductor of the *CD* span varies its current in traveling four pole pitches, while the current variation of the middle conductor of group *AB* is given by curve *AEB*.

205. Current Sheet Due to Many Equalizers.—Every equalizer circuit that comes along will have a middle-conductor variation like that shown. Assume for the time being that all

possible equalizer connections are installed. Then every conductor is at all times a middle conductor in some span. This means that on the average, there will be middle conductors distributed along the four pole pitches under consideration, and that curves *CGD* and *AEB* then represent current sheets distributed in space, the currents being middle-conductor currents. For magnetic effect, net current sheets are more important; so adding up these two sheets, the curve *AEFGD* represents the net current sheet due to middle conductors.

Remembering now that a span has conductors to the right and left of the middle, these other conductors would add current sheets in a manner similar to that prevailing if this curve *AEFGD* is duplicated several times at equal spaces, the duplicate curves reaching as far to the right and to the left as the distance over half a span, or one-half pole pitch. The summing of all these component curves would give a resultant curve beginning somewhat to the left of point *A*, shaped much like *AEFGD*, crossing the axis at *F*, and ending to the right of *D*. For the magnetic argument to follow, *AEFGD* may as well represent the whole net current sheet as produced by all equalizer circuits superimposed.

206. Magnetizing Effect of Current Sheet.—Given a current sheet, what distribution of magnetomotive force follows? As is easily shown, in moving to the right, every time a certain quantity of current (in amperes) is crossed, the m.m.f. (in ampere-turns) changes by an equal amount. The *shape* of the m.m.f. wave, therefore, is found by beginning at the left end of the sheet integrating. This gives a curve, all above the axis, as shown in Fig. 81*c*.

This distributed m.m.f. wave must set up flux according to the familiar criterion that all flux entering the armature must leave it. The general effects follow easily if it is assumed that all reluctance resides in the air gaps, it being further assumed that gaps are equal and constant. It follows that the flux densities would be like those indicated by the curve (Fig. 81*d*), and this curve becomes proper when its axis is shifted until the areas on either side of the axis have total equal quantities.

207. Discussion of Magnetic Effects.—It is at once seen that the main effect of the current sheet is to reduce the pole 4 north component flux, which is itself the cause of the sheet. All other poles are affected, but to a minor degree. It can be properly argued that, with the troublesome component pole mostly elim-

inated, any secondary component poles created, as for instance, the components for poles 3 and 5, will in turn proceed to set up secondary current sheets that will eliminate them (nearly) and so on until a close balance is obtained.

Any very close study of the problem is rather out of order, for the simple reason that the individual machine will have its own peculiarities of original flux distribution; also because load currents will cause flux distribution distortions to different degrees in different machines. The principal point is that, wherever an undesirable flux component exists, there will be found equalizer circuit currents of sufficient quantity nearly to equalize it out of existence.

208. The Special Four-pole Case.—If an unbalanced four-pole unit is analyzed by the second of the above flux-analysis schemes, the component machine consists of two opposite dead poles and two opposite poles which, though normally north and south, now have components of equal value, one being opposite in sign to the other.

This amounts to a two-pole generator, and the equalizer circuits amount to short-circuited, alternating-current windings, since each circuit amounts to a double-pitch, distributed coil. As is well known, an alternating-current generator operated short-circuited carries such lagging currents as greatly to reduce its own field flux. It should be noted that the single cycle of voltage induced by one pole in one circuit here lasts for a complete revolution; hence the current will occur as a stable succession of cycles; in short, the transient conditions of the multipolar machine degenerate in this case into ordinary alternating-current phenomena.

209. Degree of Equalization Possible.—B. G. Lamme cites the case of a 14-pole generator, equipped with equalizers, which was operated for a considerable time; commutation was not good, but it was not bad enough to cause shutdown. It was eventually found that field coils numbered 6 and 9 had been interchanged in assembly, due to the numbers being read upside down; and that the excitation of these two poles was therefore completely reversed. In spite of this extreme condition, equalizer currents acted to balance up the fluxes to a remarkable degree.

The writer has made approximate calculations on the excess losses occasioned by a 50 per cent unbalance in a four-pole unit. Two opposite poles were assumed to be normal, the other two

having either a 50 per cent excess and deficit of flux due to unequal gaps, or a 50 per cent excess and deficit of field ampere-turns. For a usual machine, the copper losses at full load would be increased by about 30 per cent due to equalizer circuit currents.

In practice, one-half or one-third the full possible number of equalizers are installed. All that is necessary is to get enough overlapping of equalizer circuit conductor spans, so that the net current sheet will be quite steady. Early experiences with equalizers showed that if only one or two equalizer rings were put on, the commutator developed black spots. This is to be expected, for then the spans would not overlap, and the desired magnetic reactions would occur spasmodically, corresponding to the pulsating armature reactions obtaining in the single-phase generator. This would cause pulsations in pole fluxes, resulting in pulsating inequalities in generated voltages; and the brushes would carry pulsating circulating currents.

Equalizer currents are damping currents, not circulating currents in the ordinary sense. If, as in the usual explanation, equalizers worked by simply furnishing an easier path than the brushes do, then we would not dare to install them. It would be similar to turning out the lights in a room by short-circuiting the line. The lights would go out, and so would the machine.

210. Equalizer Action of the Frogleg Winding.—This winding was mentioned in the chapter on Armature Windings. It is a combination of a lap and a wave winding, each complete in itself, connected to one commutator. Each winding is designed to carry half the load. Although these windings are used for large, heavy-current, multipolar machines, and although the lap winding has nothing that would ordinarily be recognized as equalizer connections, the claim is made that commutation is perfect, even at great overloads.

The statement that excellent commutation is obtained is easily believed, for, although the manufacturers claim that the machine has no equalizer connections, yet such an armature is as perfectly equalized as it can be. There are no external equalizers, but every wave coil acts as an equalizer tap.

The ordinary equalizer tap or connector should span exactly two pole pitches of lap winding. Now suppose that the tap is brought from the back of the machine; but before actually tapping it into the lap winding, it is led down two slots that are about one pole pitch apart and then is spread so that it taps the

commutator at points nearly two pole pitches apart. This, it will be recognized, describes an ordinary wave coil. Still considering it as an equalizer tap or cross-connector, it is now peculiar in that it will have voltage generated in its two sides, which no external equalizer connector has. This factor must in some wise be compensated for; and it is, exactly, because of the fact that to be a wave coil (retrogressive), it spans, at the commutator, one less than two pole pitches of lap coils, thereby reducing the equalizer circuit voltage and compensating for it by the voltage induced in the wave coil. The compensation is perfect.

The literature on this winding implies that the winding is superior in that it has no copper waste in external equalizers, and that perhaps circulating currents due to magnetic unbalance are so reduced that circulating current losses are much lower for this winding than for a lap winding equalized in the ordinary way. The circulating currents are still there, of course, being found in the lap winding just where they would be and to the same extent as if a full number of equalizers were used; the difference is that instead of flowing through external equalizers as a return path, they return through the wave coils. It is doubtful if circulating current losses are much lower. As the lap winding, however, is as perfectly equalized as can be, the winding will give as perfect a correction to magnetic unbalance as can be attained. Correcting magnetic unbalance not only reduces circulating currents to the minimum needed to maintain the armature m.m.fs., which correct the unbalance but, in addition, unbalanced magnetic pull on the armature is nearly eliminated.

CHAPTER XXI

MAIN-POLE AND INTERPOLE LEAKAGE FLUX

It is customary in books on design either to give tables or curves for leakage coefficients for various types of machines, or else to make arbitrary assumptions as to leakage-flux path conformations by which the amount of leakage flux can be solved for mathematically. In this chapter the actual flux conformations in the interpolar spaces are brought out by resorting to the technique of field mapping. Enough of a beginning is made to indicate how to handle cases not covered here.

211. Main-pole Leakage.—As in many similar cases where there are several symmetrically placed flux-producing windings, the leakage field of a multipolar machine can be analyzed in two ways: (*a*) A field winding can be considered alone in its effects, after which the other fields as caused by all other fields acting alone are superimposed on the first one, and the total effect found; or (*b*) advantage is taken of the fact that the radial planes separating pole pitches, as due to all field windings acting together, are bound to be equipotential surfaces. The second analysis is used.

Figure 83 shows an interpolar space in a four-pole unit, without the interpole. By the second analysis mentioned above, the vertical line shown represents a radial plane which, together with the similar plane beyond the left-hand pole shown, blocks out or encloses all flux and potential lines set up by this pole's winding.

In order to begin the job of mapping the leakage field, it was assumed that the yoke uses one-fourth, the teeth one-fourth, and the gap one-half of the m.m.f. of the field coil. The single conductor is shown with four quarter-potential lines (dotted) emerging from it, there being one-fourth of the m.m.f. of the conductor between any two adjacent lines. The armature core m.m.f. drop is neglected.

All iron parts except possibly some teeth under the pole will have such high permeability that flux must, as usual, be about normal to whatever iron surface it approaches. There will be

the usual singular points. Near the central plane of the machine, the field will be nearly two-dimensional, and it will be so treated until later.

Near the conductor there is bound to be a singular point on the pole side, and a quarter-potential line S_1 is drawn thereto. The next, the upper, equipotential line S_2, since the yoke is to have 25 per cent of the m.m.f., must cut through the yoke near the place shown; and it must hug the inner yoke surface nearly all the way out to this point, for the yoke is roughly an equipotential surface also. Immediately below S_2 will be another

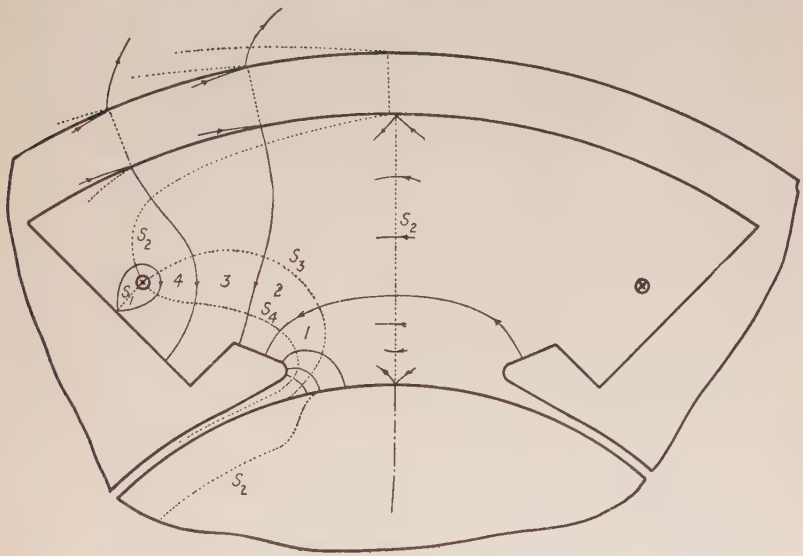


FIG. 83.—Main pole leakage.

equipotential, not shown, which bends downward at the radial plane and follows it closely; for all practical purposes, the radial plane may also be called S_2. This line then touches the armature, follows its surface to the left, and upon reaching the pole-tip region where the teeth become saturated, enters the teeth and is shown floating along their bases.

S_3 drops down through the interpolar space and touches the armature surface about under the pole tip, following the surface to the left.

S_4 enters the gap and for a *smooth gap* would follow it midway between armature and pole face. S_1 splits at its singular point

and therefore is found coming down the pole side to follow the pole face. All conditions for m.m.f. distribution are satisfied. Tube 1 is three squares long, tube 2 is about three squares long, tube 3 is over three squares long, tube 4 about 3.5, and the tube next to the conductor has all four squares. This is as it should be. This field was not drawn for high accuracy; in fact, in any case where the iron cannot be assumed to have infinite permeability, high accuracy in field mapping is very difficult. Yet it is likely that, on the two-dimensional assumption, the leakage flux as found by this map would be correct to within 10 per cent of the actual value.

212. Evaluating Main-pole Leakage.—Continuing the above case, consider tubes 1, 2, 3, and 4 to constitute the leakage flux. Below tube 1 is useful fringe flux; in fact, a small part of tube 1 is really that. With the exception of tube 4, the m.m.f. across this portion of the field is equal to the field coil m.m.f. minus the yoke drop, approximately. It will be remembered that in the two-dimensional field, the flux per axial inch is 3.2 times the ampere-turns difference of potential times number of tubes divided by squares per tube. As the leakage is desired for the whole pole, there is as much on one side as the other, and the value must be doubled. Therefore, the leakage flux per ampere-turn per axial inch is

$$\phi_{ml} = 3.2\left(\frac{4}{3}\right)2$$

in this case. The ampere-turns should be the total minus yoke drop.

The lower part of the pole is increased in flux by the four tubes, whereas the upper part is reduced in flux (and so is the yoke) by as much as leaves it. It will be safer to count only on the increase and forget that a decrease in upper pole core or yoke flux occurs.

A central conductor only is shown in the drawing; maps drawn by the writer but not given show that the central conductor of the field coil section, for all practical purposes, very well represents the average of all conductors in effect.

213. Main-pole Leakage Flux, Interpoles Present.—Formerly a radial equipotential plane ran down the center of the interpolar space. The interpole has very high permeability, and so its two sides will be good equipotential surfaces. Hence, it is still found that the main-pole leakage jumps over to a plane equipotential surface, but this surface is nearer the main pole,

unless the main pole is made to keep its distance. As shown in Fig. 84, main-pole leakage has increased to something over five tubes, it having been four tubes without the interpole.

It is significant to note that the main-pole leakage goes across the iron of the interpole, while the interpole flux itself will mainly be along the interpole core. As the two fluxes are at right angles within the interpole core, they will not be additive in saturation effect, as is definitely established by fairly recent research. But particularly near the surface of the interpole, the two fluxes will be somewhat in line; therefore main-pole leakage will help to some unknown degree in saturating the interpole circuit.

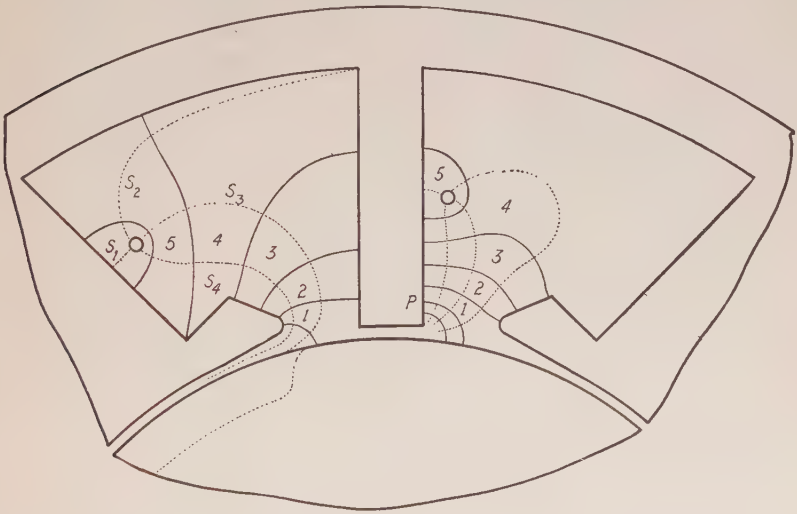


FIG. 84.—Main and interpole leakage.

214. Interpole Leakage.—In the right-hand part of Fig. 84 is shown the leakage-flux field of a central interpole turn. Let us discuss the field for a moment under the assumption that armature reaction does not exist. The flux entering the interpole face is added to by such tubes as 1, 2, 3, and 4, the addition being rapid at tube 1, which is narrow, and dropping off somewhat in addition rate as the conductor is approached. The maximum pole core density occurs at the singular point, next the conductor.

Taking up separately the field due to armature reaction, which is not shown, flux from the armature will mainly curve to the left and enter the interpole, or to the right and enter the main pole tip. Very close to the interpole corner this field will conform

closely to the pattern followed by the interpole flux itself. As the two fluxes are opposite, there will be a partial cancellation for the region where conformity exists; that is, from about the point P down, the net flux will be simply that due to the net interpole gap ampere-turns, which amount to something like 50 per cent or less of the total interpole ampere-turns.

But above point P the reaction field rapidly becomes extremely weak, as even the roughest kind of armature reaction field map will show; then above P, the leakage flux entering the interpole will be determined almost entirely by the total interpole m.m.f.

This map shows that leakage flux is very weak above the conductor, and may, so far as saturation effects go, be neglected; this weakening above the conductor corresponds exactly to the very low densities below a conductor in a slot; it is that case upside down.

The lower the conductor is placed, the lower the leakage becomes. That is why the interpole coils in synchronous converters are placed around the foot of the pole; converters have a quite low reaction, and the total interpole m.m.f. required is much less than is needed in the direct-current machine.

215. Evaluating Interpole Leakage.—Such tubes as 1, 2, 3, and 4 are leakage tubes. As in the main-pole leakage case, it is the total leakage per pole that is desired. Considering the leakage on one side in this map to be five tubes, each four squares long, the leakage flux per axial inch per ampere-turn is

$$\phi_{il} = 3.2\left(\frac{5}{4}\right)2.$$

Investigation for the effect of turns higher up and lower down shows that, for a coil uniformly distributed over approximately the whole interpole core, the leakage as determined from a study of the field of the central conductor will be sufficiently close.

216. Effects of Interpole Leakage Flux.—If a full number of interpoles is used, a main pole receives as much interpole leakage flux of one kind on one side as it does of the opposite kind on the other side; there is then no net change in main-pole core flux due to interpole flux leakage. If the interpoles are of half the full number, the effects are along the line of those noted in Chap. XXII, where it will be seen that, although the interpole flux actively concerned in commutation does reduce main flux somewhat, it is of negligible amount in good designs. In that study, leakage fluxes were neglected; as interpole leakage may amount

in some cases to much more than the active interpole flux, main flux reduction may be as great as 1 per cent. Of a much more pronounced degree is the effect of the interpole leakage in adding to the excess turns needed on the interpole to take care of yoke saturation.

217. General Leakage-flux Values.—Field-map studies, which cannot be given here, were made by the writer for two-, four-, and six-pole machines of usual proportions. Results are given in Table VII to cover the following conditions: (*a*) slots per pole, 10 or 15; (*b*) main-pole arc, 70 per cent of the pole pitch, with no interpoles; (*c*) main-pole arc, 70 per cent, interpole arc, 10 per cent, and 10 slots per pole; (*d*) main-pole arc, 66.6 per cent, interpole arc., 13.3 per cent, and 15 slots per pole.

Reviewing the conditions for the cases with interpoles, 10 slots per pole, with main-pole arc of 70 per cent, means the main pole covers 7 slot pitches; interpole arc of 10 per cent means the use of a unit-arc interpole covering effectively 1 slot pitch, and there is a slot pitch between interpole and main-pole tip. The other case has 15 slots per pole, with main pole covering 10 slot pitches and the interpole 2; here there is a distance of 1.5 slot pitches between main-pole tip and interpole.

TABLE VII.—LEAKAGE FLUX PER AXIAL INCH PER AMPERE-TURN, MAIN AND INTERPOLE LEAKAGE

ϕ_{ml} is flux per axial inch per ampere-turn, including both sides of main pole.

ϕ_{il} is flux per axial inch per ampere-turn, including both sides of interpole.

Number of poles	Two		Four			Six		
Slots per pole	....	10	....	10	15	....	10	15
Slot pitches, main-pole arc	....	6	....	7	10	....	7	10
Per cent of pole pitch, main-pole arc	70	60	70	70	66.7	70	70	66.7
Slot pitches, interpole arc	none	1	none	1	2	none	1	2
Per cent of pole pitch, interpole arc	none	10	none	10	13.3	none	10	13.3
ϕ_{ml}	13	9.6	8.5	10.6	10.6	5.6	7.5	7.5
ϕ_{il}	....	6.4	....	7.5	7.5	....	11	11

218. Leakage from Pole Ends.—There is no large amount of iron near the ends of the poles as is found near the sides in the interpolar spaces. The field around the ends is three-dimensional and hence very difficult to investigate. Suppose the core of a

main pole is square in section (as is often the case). According to the writer's judgment, the end leakage would be roughly one-half the side leakage, granted usual proportions and no unusual cause for excessively high or low leakage values anywhere. This would also likely be somewhere near the truth for an interpole so short that its section is square. As the side leakage is given in terms of flux per axial inch, and the figures in the table include both sides, then, if the width W_p of the pole core is stated in inches, the end leakage would be W_p times the flux per axial inch. This amounts to finding the flux per axial inch for the sides and multiplying it by the sum found by adding half the pole-core width to the pole-core axial length.

Problems

44. Check some of the values of unit leakage flux as given in the table, by making a scaled sketch of one or two of the given machines and mapping the fields.

45. It is common practice to limit the production of machines to certain fixed armature diameters; various ratings are gotten out of each diameter by making the armature and poles longer or shorter. Suppose several machines are built on the same diameter but with varying lengths; and suppose they are identical as to field and interpole ampere-turns. Which would have the higher leakage coefficients, the longer or the shorter machines?

46. A four-pole unit has an armature diameter of 12 inches, the armature core axial length being 6 inches. The main-pole arc covers 70 per cent of the pole pitch, while the interpole arc covers 10 per cent. The gap flux per main pole is 0.27×10^6. The pole axial length is 5.25 inches. It is desired to keep the pole core density to 90,000. Find the other dimension of the rectangular pole core section.

CHAPTER XXII

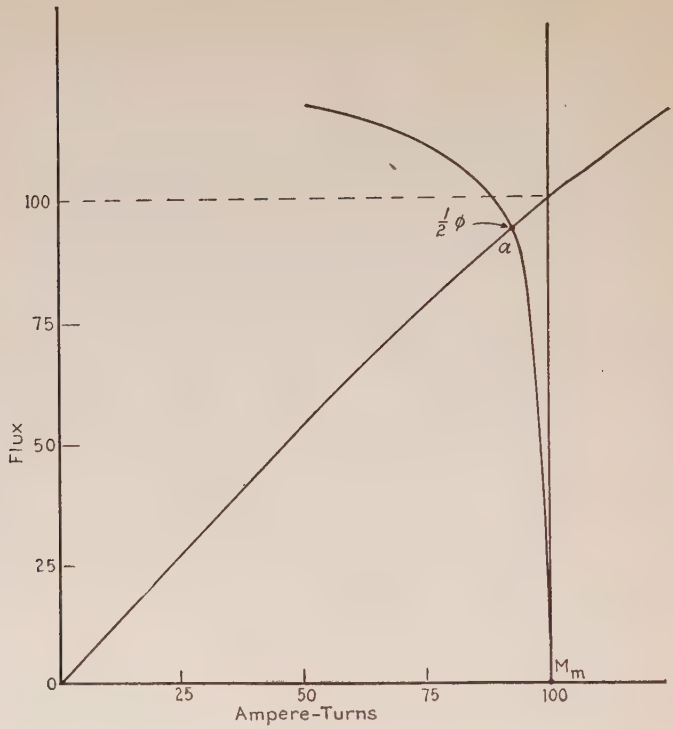
INTERPOLE FLUX AS AFFECTED BY SATURATION OF MAIN FLUX PATH

By the flux analysis in common use, the flux of a north interpole is considered as returning by way of the adjacent south main pole. It is commonly recognized that the saturating of teeth, main-pole core, and yoke will, due to resulting reluctance increases, thereby diminish the expected interpole flux; that the reduction in this reversing flux may become serious, is also well known. As far as the writer knows, no solution method for finding the amount of the reduction has been published. There are two general cases, involving the use of the full number of interpoles, or half the full number. By virtue of its symmetry the former case is not difficult, and the writer gives herewith his graphical solution. The second case, with half the full number of interpoles, may be solved by the cut-and-try method, but a close solution by that method is hopelessly long and involved. The graphical solution given is due to one of the writer's students, Stephen L. Burgwin, and is a highly ingenious device.

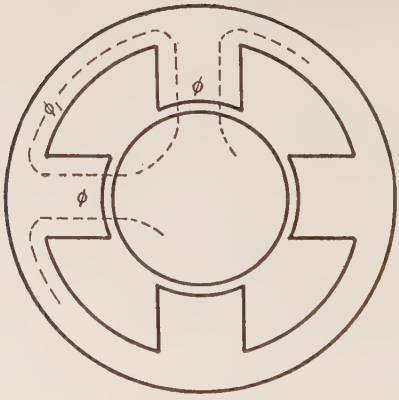
219. No Interpoles Present.—To help fix terminology and usage, this case will be considered first. In all cases a four-pole machine is dealt with. Figure 85*b* shows the machine. The flux of each pole is Φ, and Φ_1 represents the flux in the yoke. *Leakage of main and interpole flux will be neglected for the present.* Then the yoke flux Φ_1 is simply equal to one-half the pole flux Φ.

Taking up the graph in Fig. 85*a*, the abscissa as usual has the m.m.f. scale, and flux is given by vertical ordinates. The curve rising to the right represents *half* the main-pole flux, or $\Phi/2$, plotted against the ampere-turns drop for one pole core, one gap and one set of teeth; this same curve will appear in the graphs for the two succeeding cases.

The excitation, or m.m.f. of *one* main field coil, marked as M_m, is also marked 100. This value is taken as standard, interpreting all m.m.fs. or drops in terms of it; the 100 means 100 per cent. At this excitation, if the yoke had no reluctance, the flux per



(*a*) Graphical solution.



(*b*) Magnetic circuits.

FIG. 85.—Graphical treatment of machine flux circuits.

pole would be *twice* the value as read from the curve; the value marked, flux = 100, is $\Phi/2$. Thus this half-pole flux value is also going to be used as a standard of reference for other flux values, this one being considered as 100 per cent.

A saturation curve for the yoke must be drawn, but it is now convenient to plot it *backward* against the ordinate at M_m; and it is plotted so that (*a*) its ordinates are values of Φ_1, the flux in any section of the yoke, and (*b*) the horizontal scaled distance between it and the M_m ordinate line gives the m.m.f. drop for *one-eighth* of the yoke total path length.

The intersection marked (*a*) solves for the fluxes. Whereas, if yoke reluctance were zero, $\Phi/2$ would be 100 per cent., the effect of yoke reluctance, for these curves, is to drop it to about 94.

As will be seen, the introduction of interpoles will further reduce the main-pole flux, although often to a negligible degree.

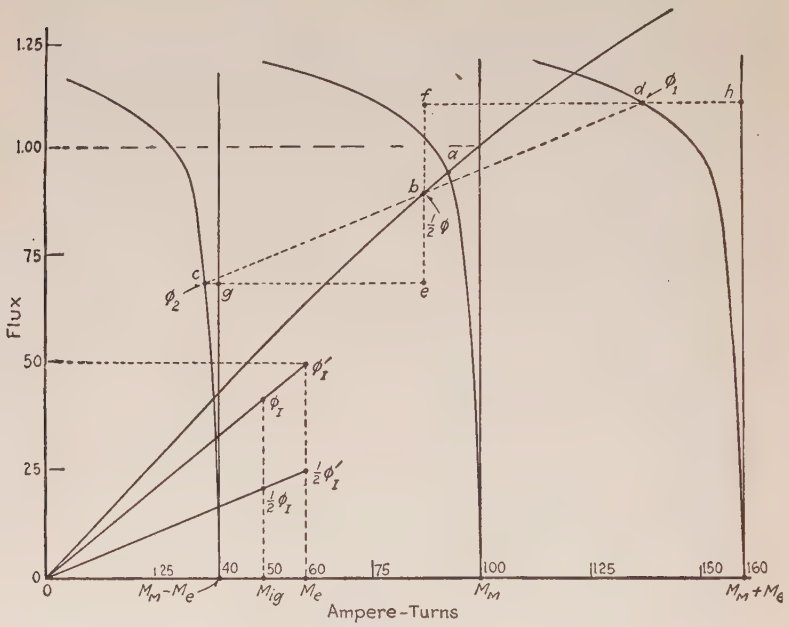
220. Interpoles Present: Full Number. I. *The Solution.*— Referring to Fig. 86*b*, the upper left north interpole is shown with all of its flux returning in the left south main pole, and thereby causing the intervening one-eighth of the yoke to carry a flux Φ_1 that is greater than that of the next yoke part, carrying Φ_2, by the amount Φ_i, which is the interpole flux; that is, $\Phi_i = \Phi_1 - \Phi_2$.

Two of the differing closed magnetic circuits to be found here are: (*a*) the Φ_1 circuit, including the Φ_1 yoke part, the north interpole path, and the south main-pole path; (*b*) the Φ_2 circuit, comprising the Φ_2 yoke part, the north main-pole path, and the north interpole path.

If this were an electric-circuit problem, equations would be written expressing for each closed circuit the fact that, in going one way around, the summation of the e.m.fs. must equal the summation of the IR drops. Identical relations prevail in the present case; and if the equations of the saturation curves involved were readily obtainable, the magnetic case could likewise be solved for. In general, it is simpler and easier to solve magnetic problems graphically.

In the Φ_1 circuit, going clockwise, the m.m.fs. are each positive and add; the ampere-turns for the south main-pole coil equal M_m; let the *excess* ampere-turns of the north interpole coil be denoted by M_e. (A large portion—usually at least 60 per cent —of the total interpole m.m.f. is equalized by the armature reaction m.m.f.; the *excess* installed over and above the equalized

m.m.f. is the only active component available for setting up the interpole flux.) Then, acting around the circuit, the summation of m.m.fs. is $M_m + M_e$.



(*a*) Graphical solution.

(*b*) Magnetic circuits.

FIG. 86.—Graphical treatment of machine flux circuits.

The summation of *drops* around the circuit consists of the sum of three positive drops: in the yoke part, in the interpole core

and gap, and in the main-pole path (consisting of pole core, gap, and teeth).

In the Φ_2 circuit, going clockwise, the m.m.f. summation is $M_m - M_e$. The drop summation is the yoke drop plus main-pole path drop, *minus* interpole path drop.

Refer now to the graph (Fig. 86*a*). Two saturation curves for the interpole path (interpole core and gap) are given: the upper one giving *total* interpole flux plotted against the resulting m.m.f. *drop* in the path, and the lower curve giving half-flux values. The upper curve shows that if the interpole flux is 50, the drop in the interpole path is 60. Assume that the interpole excess ampere-turns M_e are 60. Then, if no further investigation is made, but it is simply assumed that the available flux-creating interpole ampere-turns were all available to act on the interpole path reluctance, Φ_i of 50 would be expected.

The graph is now prepared for action, by having ordinates drawn at $M = 40$ and $M = 160$, these being respectively $M_m - M_e$ and $M_m + M_e$. The saturation curve for the one-eighth part of the yoke is drawn backward against each, as shown. Now for the solution.

Consider the lower (the $\Phi_i/2$) interpole saturation curve: by the use of two triangles, the slope of this curve is carried up into the region above so that the eventually obtained line *cbd* will have its slope. The object is to juggle the moving triangle until, as verified with a pair of dividers, $cd = db$. This furnishes the solution.

II. *Proof of the Solution.*—Observe that the intersections *c*, *b*, and *d* give the ordinates for Φ_2, $\Phi/2$, and Φ_1. *cf*, scaled, gives the interpole flux Φ_i, and this flux must equal the difference between the two yoke fluxes, which it does on the graph. $eb = bf$ by construction, each being $\Phi_i/2$, which, due to the slope of line *cbd*, means that *ge*, which equals *fd*, scaled, gives the interpole path drop.

In the Φ_1 circuit, scaling from the flux axis over to *b* gives the drop in the main south pole path; *fd* is the interpole path drop, added thereto; and *dh*, further added, is the drop in the Φ_1 yoke part. The drop summation must equal the m.m.f. summation, which it does, or a total of 160.

In the Φ_2 circuit, from the flux axis to *b* is as before; *subtracting* the interpole drop *ec* and *adding cg*, which is the Φ_2 yoke part

drop, gives the drop summation, which should equal the m.m.f. summation, as it does.

Note first that the value of $\Phi/2$, for main-pole flux, has dropped from about 94 to about 89.

Next, transferring *ce*, the interpole drop, to the m.m.f. axis of the interpole saturation curve and calling this drop by M_{ig}, it is seen that the drop here is 50, whereas M_e is 60. Then only five-sixth the interpole flux is obtained that would have been counted on had there been no investigation made for saturation effects; or interpreting it in the more logical way, the drop of 50 indicates that the interpole flux will be 41.7, and this is the flux that should have been designed for; having insisted upon obtaining it, it is found that the M_e ampere-turns must in this case be made six-fifths as much as are needed for the interpole path alone.

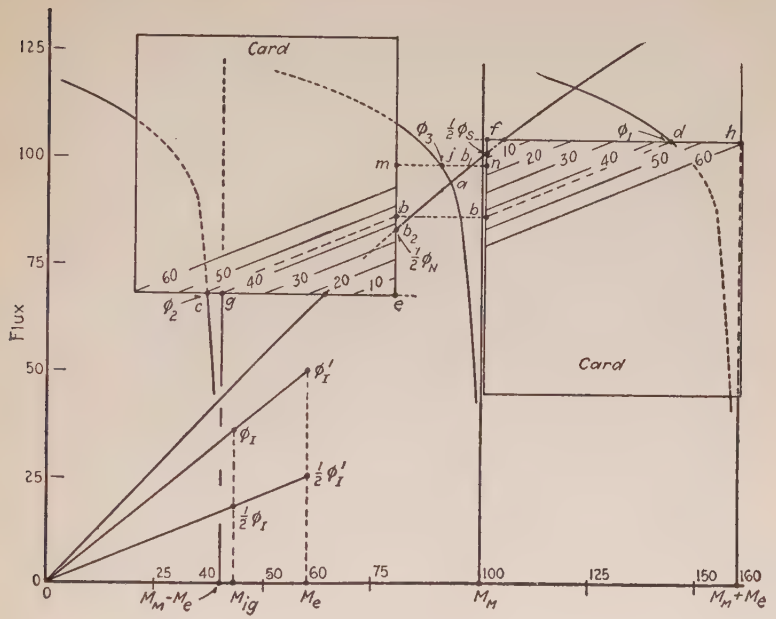
A third and very important conclusion is this: that instead of interpole flux varying in proportion to the armature current as the load is increased from zero, it has reduced from that proportionate value, for this load, by one-sixth; at greater loads, an increasingly greater reduction will occur. Commonly, at four or five times full load, flux in the interpole gap actually goes through zero and becomes reversed.

221. Interpoles Present: Half Full Number. I. *The Solution.* As seen in Fig. 87*b*, the north pole flux now becomes less than the south pole flux, and so the symbols Φ_n and Φ_s are introduced; and besides the two differing yoke fluxes already on record, there is a new one, Φ_3, to be found. The two closed magnetic circuits as above considered are again present, with the addition of a new one, the Φ_3 circuit.

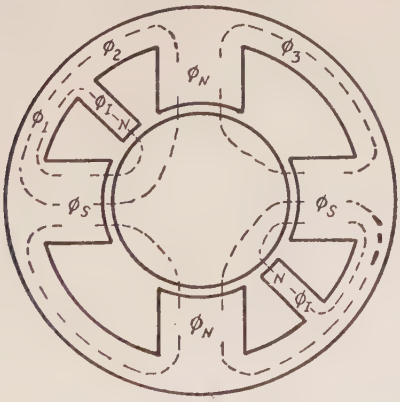
The graph is prepared just as for the preceding case, but instead of using simple tools such as triangles and dividers to obtain a solution, further equipment must be made. Two cards, exactly alike, are prepared. The cards are included in the drawing in Fig. 87*a*. The uniformly spaced sloping lines on the left-hand card are drawn so that, with the card lined up with the axes, the lines have the slope of the $\Phi/2$ saturation curve. The line marked 60, for example, is identical with this curve, provided this curve is stopped at $M_e = 60$. The operations for solution will be taken up in order.

1. The cards are placed about as shown, and each lined up with the axes at all times. Whatever line is tentatively chosen for trial on the one card must be duplicately used on the other card.

Interpolate for lines not drawn. The line used here (value, about 44) is interpolated for. Considering line *cb*, *c* must lie on the yoke



(*a*) Graphical solution.



(*b*) Magnetic circuits.

FIG. 87.—Graphical treatment of machine flux circuits.

curve as shown, and *b* must lie horizontally across from *b* of *bd* on the other card.

2. Then *d* must lie on its yoke curve as shown.

3. The edges of the cards intersect with the main saturation curve at b_1 and b_2. Fix n so that fn is twice fb_1; fix m so that em is twice eb_2. Then m and n must be horizontally opposite.

4. mn gives the intersection j. This point must be midway between m and n.

If, by sufficient juggling of the cards, these four conditions are fulfilled, the solution is obtained. The best way for one to gain respect for the juggling necessary is to try it; even with the aid of the cards, by the method outlined, a close solution is difficult enough to obtain. Without the card method, a cut-and-try solution is a great deal more difficult.

II. *Proving the Solution.*—The proper intersections, as marked with the flux symbols, give the various flux ordinates. eb and bf are equal, and each is equal to $\Phi_i/2$. Their sum gives Φ_i, which equals $\Phi_1 - \Phi_2$, as it should by reference to the requirements of the magnetic circuit.

If these lengths eb and bf are $\Phi_i/2$, then, as shown in the preceding case, ce is the interpole path drop; so is fd. Checking circuit Φ_1, if intersection b_1 gives $\Phi_s/2$ as marked, then the drop in the south pole path is scaled from the flux axis over to this intersection; fd adds to it, as it should, and dh, the yoke drop for Φ_1, adds; the total drop equals the total m.m.f., as it should. b_2 intersection gives $\Phi_N/2$; this pole drop *minus* ec plus cg (for the Φ_2 drop) gives 40, the total m.m.f. for that circuit, as it should.

The magnetic circuit requires that $\Phi_s/2$ shall be the mean of Φ_1 and Φ_3; having fixed mn so that b_1 is midway between f and n, the requirement is satisfied, for intersection j gives Φ_3. Likewise, $\Phi_n/2$ must be the mean of Φ_1 and Φ_3, which likewise it is.

Perhaps the hardest feature is to see why j must be midway between m and n. This is seen by checking around the Φ_3 circuit. The drop in one-eighth of the yoke for Φ_3, plus the mean of the abscissa-scaled distances to b_1 and b_2 (which gives the mean of the drops for the north and south main-pole paths) is just equal to *half* the total drop of the whole circuit. This should equal the m.m.f. of one main field coil, M_M, which it does.

III. *Discussion.*—Transferring ce to the interpole saturation curve, it is seen that the interpole flux is reduced more here than in the case of the full number of interpoles, even though identical basic figures and curves were used.

The average value of main flux has suffered likewise, but to a rather negligible extent, as before. The important effect on

the main flux is that a series of strong and weak poles are created, causing unbalanced magnetic pull; also, unless equalizers are used for a lap-wound armature, circulating currents would enforce the abandoning of any attempt to do with half the full number of interpoles (unless interpole fluxes were very weak).

Leakage flux effects have been omitted, for the problem was difficult enough; it is easily seen that leakage flux, by increasing yoke and pole core densities, would make all bad effects still worse. Values of Table VIII below include interpole leakage effects.

The reduction in interpole flux (or increase in required ampere-turns) is increased by making the yoke curve bend over. It is always easily possible to keep yoke reluctance down; that it is important to do so is very evident.

Main-pole path reluctance has nothing to do with the reduction of interpole flux.

222. Approximate Design Values.—Table VIII gives the results of a series of studies made on the saturation curves used to illustrate the work of this chapter. It was found that, as based on the saturation curve for *cast steel* as given in this book, the nominal yoke density should be allowed to go above 85,000 only in case the interpole flux is very weak. The *nominal yoke*

TABLE VIII.—EFFECT OF YOKE RELUCTANCE ON INTERPOLE AMPERE-TURNS AND EFFECT OF INTERPOLES IN REDUCING MAIN FLUX

For nominal yoke density of 85,000

Interpole flux (including interpole leakage flux) in per cent of main-pole flux	Half-number interpoles			Full-number interpoles		
	M_{ig} in per cent of M_m	k_i	Per cent reduction in main-pole flux	M_{ig} in per cent of M_m	k_i	Per cent reduction in main-pole flux
10	30	1.2	Negligible	30	1.04	Negligible
	50	1.1	Negligible	50	1.02	Negligible
20	30	1.3	0.5	30	1.08	Negligible
	50	1.15	0.5	50	1.04	Negligible
30	30	1.45	0.5	30	1.12	Negligible
	50	1.3	0.5	50	1.08	Negligible

density is the yoke density uncorrected for change due to presence of interpoles. The coefficient k_i as given in the table is used to multiply M_{ig}, the reluctance drop for each interpole core and gap, to get the total excess ampere-turns of the interpole winding. The excess m.m.f. is that part of the total unbalanced by armature reaction m.m.f.

223. Variation of Interpole Flux Compensation with Load Change.—As a machine is loaded, the yoke saturation effect in reducing the interpole flux below the proportionate value it should attain for a given load increases. Thus, if the interpoles are adjusted to give perfect commutation at no load, they will be too weak at full load. The machine would then be said to be *undercompensated.*

The usual procedure is to overcompensate the machine slightly at full load. Then it will easily carry reasonable overloads, even though then undercompensated; and at light loads, when overcompensation greater than that at full load would occur, the brush current load is so light that no commutation trouble occurs.

CHAPTER XXIII

MAIN-POLE FRINGING FLUX; CARTER'S GAP COEFFICIENT

The three kinds of useful fringing flux of the main pole each represent a type of design problem having difficulty out of all proportion to the values involved.[1] The designer does not wish to neglect fringe fluxes entirely; neither does he care to make the same percentage allowance for fringing in every machine designed; nor again, remembering the relative sizes of the quantities involved, does he wish to make a careful solution for each, in every case.

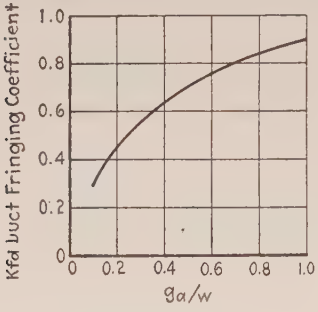
The treatments offered here are entirely satisfactory for the intended purposes of the book. Each flux is adequately taken care of in the easiest possible fashion; yet each allowance is made in the proper direction and approximately to the proper extent for the individual design.

224. Definition of Fringing Flux.—It is not only desirable but strictly necessary that calculation and recording practices used in carrying a design through shall be kept to simple and understandable form. The machine, unfortunately, refuses to make such practice easy to invent. Suppose that all machines had constant air gaps, smooth armatures, no ducts, and no flux beyond the confines of the air gap. The flux in the gap would be radial, and for short gaps, it could be considered as rectilinear. The section area of the gap would be simply the pole face area. In spite of actual conditions, the machine's quantities must still be referred, so far as is possible with good accuracy, to some such simple gap area.

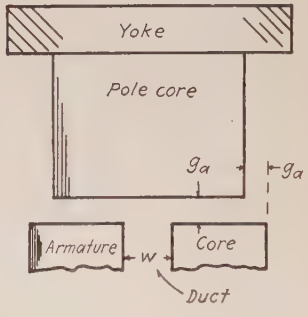
The armature is slotted, so that the principal portion of the gap flux is two-dimensional; and this main part of the useful flux approximately extends over an area equal to the pole arc T times the gross iron core length, L_g, or L_gT. L_g does not include the radial vent-duct axial widths.

[1] Douglas, J. F. H., "The Reluctance of Some Irregular Magnetic Fields," *A.I.E.E.*, June, 1915.

The two-dimensional field becomes three-dimensional at and near the pole tips, the axial ends of the gap or pole ends, and the vent ducts. *As a matter of easy calculation*, it can be *assumed* that the two-dimensional field exists right up to the limits of the area or areas going to make up the arbitrary area $L_g T$; and in terms of this assumed condition, all extra useful flux actually existing above this calculated value can be called *fringing flux*.

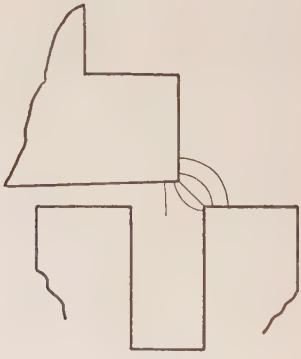


(a) Duct Fringing Coefficient



(b) Armature Overhang

(c) Pole Tip Design



(d) Uncorrected Pole Tip

FIG. 88.—Fringing flux.

225. Fringing at Axial Ends of Gap; End Play.—To favor the maintenance of a good, long-wearing cylindrical surface on bearing, brushes, and commutator, the armature assembly should be allowed some end play in the bearings. But if the pole face is made exactly as long as the armature core total length, L_a, magnetic pull will tend always to center the armature under the poles

and prevent the desirable axial oscillations from taking place. The armature can be made the shorter member, or the pole; it makes little difference in most designs. Suppose the armature is made the longer of the two and is extended, as shown in Fig. 88*b*, beyond the pole at each end by an amount equal to the gap clearance.

The writer finds, by approximate three-dimensional field analysis carried out in isometric perspective that, with such proportions, the actual flux in that neighborhood can be well approximated for by assuming the two-dimensional field of the gap to exist right out to the armature ends.

Thus, if this assumption is made, and the design at the ends of the armature is made as suggested, the necessity for making allowance for the fringing flux there is entirely eliminated. Incidentally, it usually is true that such allowance of armature overhang is entirely practicable for taking care of end-play requirements.

In many direct-current machines, the gap length or clearance varies over the pole arc; in these cases, the overhang at each end is made equal to g_a, the average air-gap length or clearance.

226. Pole-tip Fringing; Elimination of Noise.—The pole arc is nowadays almost invariably considerably wider than the pole core. The extension of the pole face toward the interpolar space creates the pole horn or pole tip, as it is variously called. Figure 88*d* illustrates the simplest and worst design of pole tip. Compared with the design of Fig. 88*c*, the square tip would raise the leakage flux of both main poles and interpoles, even though it would also slightly raise the useful fringe flux reaching out into the interpolar space.

Another defect of the square-tip design is the noise it may cause. As the teeth pass under the pole face, or one tooth passes another in an induction motor, the magnetic pull on them is radial, neglecting distortion due to armature reaction. As the tooth begins to draw away from the pole tip, the pull on it acquires a suddenly applied tangential component, which almost as suddenly disappears as the tooth gets out of range of the pole's influence. The forcible jerk thus applied is sufficient to make some machines (notably, in the case of induction motors) howl so badly that no one knowing about it will buy them.

The most practical cure, which at best reduces this particular noise to a reasonable minimum, is to round off the corner on the

pole tip, at the same time gradually reducing the total tooth flux as it emerges, by increasing the gap length near the end of the pole arc. There are no published data, experimental or otherwise, showing what changes in gap and what roundings of tip are best, as far as the writer knows. Now, barring the effect of differing vibration periods of light or heavy teeth and of various pole-tip periods, it would seem that if the most economical way to treat one machine could be found, that same way should apply to many machines, provided the method involved certain general characteristics.

It seems reasonable to believe that the shading off in tooth flux would be accomplished in one slot pitch of pole arc, and that the rounding of the tip should be fixed with a radius bearing a fixed proportion to this distance. With these factors in mind, the treatment indicated in Fig. 88*c* is proposed. The dotted line represents the surface of the last slot pitch of pole arc, as determined by the electromagnetic design of the air gap (treated elsewhere in this book). This surface, fixing the value or values of gap length for the region it covers, is determined without reference to noise prevention and under the assumption that there is *no pole-tip fringing.* Now, to prevent noise, the gap will be arbitrarily and uniformly increased toward the extreme tip, where the electromagnetic design calls for a gap value of g_t. Next, the tip will be rounded off with a radius equal to 0.1 times the slot pitch, or $0.1(s + t)$, and finally, the surface will be cut back at an angle of 45° with the armature radius through the tip.

The increase in gap at the tip will be related to tooth saturation and the ratio of tooth ampere-turns to gap ampere-turns. Let the following values be considered:

Maximum Tooth-root Density B_{rm}	Increase g_t by Per cent Indicated
140,000 or less	40
150,000	50
160,000	70

In order to shade off the tooth flux in all cases at roughly the same rate, it can be seen that the greater the tooth saturation becomes, the greater the gap increase must be to compensate for it. The table makes allowances in the right direction.

Now, with this accomplished, it happens that the fringing-flux allowance is automatically (even though approximately) made.

The gap flux is calculated as for the previously designed gap which ends with a tip value g_t, as it is assumed that no fringing occurs; but fringing does occur; but again, increasing the gap in the above manner to eliminate noise reduces all flux densities in that region, so that the total useful flux is approximately brought to the above arbitrarily computed value. This treatment assumes ordinary proportions; with unusual proportions, field mapping of the fringe should be resorted to.

227. Vent-duct Fringing.—The vent duct is illustrated in Fig. 88*b*, only one duct being shown. If the slots are assumed to be absent, simply in order to enable the duct fringing field to be mapped as two-dimensional, it is found that the total gap flux, including that flux in the ducts going up to the pole face, is greater than if the real two-dimensional field of the gap proper is calculated as if it existed right up to the duct edges and stopped there.

There are four ways of interpreting duct effect for calculation purposes: to consider it as increasing gap flux, as increasing effective gap area, as decreasing gap ampere-turns, or as decreasing gap length, it being understood that whichever variation is supposed to take place, all other variations do not.

It seems preferable to use the first of these interpretations, for that process will preserve intact the simple arbitrary designation of gap area L_gT and will also preserve simplicity in other relations which follow later. Let B_{ga} stand for the average density of the regular two-dimensional field in the part of the gap not near to any edges of the field. Then L_gTB_{ga} is the simplest statement of pole face flux or gap flux, but it is too low, as duct fringing is not included; it can be most simply included by assuming the B_{ga} density reaches out part way over the duct and then finding what part of the duct width is effectively covered. This was done by the field-mapping method (neglecting effect of slots) with the curve of Fig. 88*a* as the result. This curve, for instance, shows that if the ratio of gap clearance to duct width (g/w) is 0.2, the coefficient k_{fd} is about 0.45, which means that the B_{ga} density is extended 45 per cent of the way across the duct.

To illustrate, suppose L_g is 10 inches, with three ducts each 0.375 inch wide; and let the gap length be 0.125. The ratio $g/w = 0.333$, for which k_{fd} is about 0.57. Then the useful flux is

$$\Phi = (L_g + nk_{fd}w)TB_{ga}$$

where n = number of ducts; and in this case,

$$\Phi = (10 + 3 \times 0.57 \times 0.375)TB_{ga}$$
$$= 10.64\ TB_{ga}.$$

Very often, the design is started off in such a way that Φ is known before B_{ga}; the order is then inverted to find B_{ga}.

Let K_f be the fringing coefficient, such that $L_g T B_{ga} K_f$ gives the flux per pole. Then

$$K_f = \left(\frac{L_g + nK_f w}{L_g}\right).$$

If the gap under the pole arc varies, use g_a, the average gap, in finding the ratio g_a/w.

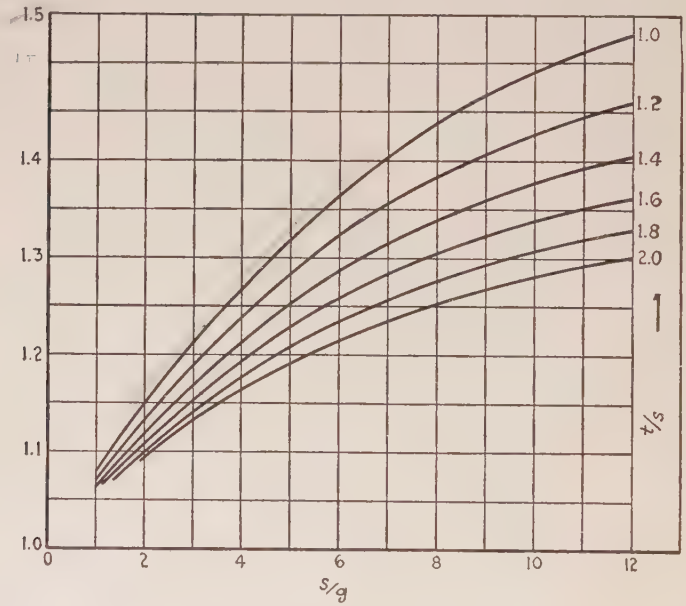


Fig. 89.—Carter's coefficient.
(g = gap length)
(s = slot width)
(t = tooth (top) width)

228. Carter's Coefficient.—If again the armature were smooth, gap density would be uniform, and the gap m.m.f. would be simply

$$\text{Ampere-turns} = 0.313BL = 0.313Bg$$

where g is the gap length in inches.

The actual slotted armature turns the gap field into a two-dimensional field, and if it were not for the mathematical work

of F. W. Carter,[1] it would be necessary to map the field in each case in order to evaluate ampere-turns against average gap density, or, as certain commutation flux fields are later handled, advantage could be taken of the fact that in this sort of field the flux per ampere-turn per axial inch is fixed as soon as *dimensional ratios* are fixed. On this basis it is only necessary to make a judicious selection of representative ratios of the dimensions, map, say, ten cases, and plot curves covering the entire usual range.

Carter assumed a developed case (infinite radius), parallel slot sides, and slots infinitely deep, and worked out the formula for finding Carter's coefficient by which the value $0.313Bg$ is multiplied to give actual gap m.m.f. drop. Anyone familiar with field-mapping technique can easily show that the practical departures from these simplifying assumptions are not worth bothering about.

Figure 89 gives a family of curves which are used by interpolation to find the coefficient for any usual case.

It should be noted distinctly that the density to be used is not the actual density—there is any number of actual densities—but the average density. Furthermore, if the gap varies over the pole arc, the density to use becomes still more specialized: Over a slot pitch the gap will never vary much; so using the average gap length for a slot pitch and the coefficient for that gap length, the density entering the equation is the average density for that slot pitch. If C represents the coefficient,

$$\text{Gap } NI = 0.313CB_g g$$

where B_g now is the density in the gap region being treated, and g is the gap length thereat.

Problem

47. Select three widely differing sets of proportions for gap, slot, and tooth dimensions, and check Carter's coefficients by mapping the magnetic fields.

[1] CARTER, F. W., *Elec. World and Engineer* (English), Nov. 30, 1921.

CHAPTER XXIV

ARMATURE REACTION

The subject of armature reaction is not easily understood. If any one factor is to be held responsible for the usual difficulties, it is that the science of field mapping or of field studies as approached through mapping has not had sufficient attention paid it in undergraduate courses. One who cannot intelligently discuss m.m.f. division of a current pair acting upon the gap-and-iron magnetic circuit of a machine cannot confidently handle an armature reaction problem. A part of the work on field mapping leads directly to a clear understanding of reaction effects.

229. Definition of Armature Reaction.—As used by various writers, the term *armature reaction* is applied so variously that it hardly can be said to have a generally accepted definite meaning. It always means this at least: It stands for the effect the armature currents have in changing the machine flux from what it would be like if the currents were zero. The term will be used in two senses herein: (*a*) as the general title standing for the train of phenomena for which armature currents are responsible, or (*b*) more specifically, as meaning the m.m.f. in ampere-turns exhibited across some part of the magnetic field, due to armature currents.

PRELIMINARY REACTION STUDIES

230. Simplest Case of Reaction.—The simplest case it is possible to devise is that of a two-pole motor which is so degenerated as to be unrecognizable as a motor. The yoke is a smooth iron cylinder; the poles are absent; the armature is another smooth cylinder, concentric with the yoke; and there is but one coil of one turn, full pitch. The entire setup is symmetrical. There is no main field, only the "reaction" field of the armature turn, which is located in the gap. If this machine is split open and unrolled or developed, it becomes as shown in Fig. 90*a*.

As shown in Art. 59, the m.m.f. of the current pair is equally divided in a symmetrical case. The graph shows the distribution of m.m.f. by a dotted line, and distribution of flux density along the armature surface by a solid line.

There will be singular points above and below each wire, assuming infinite permeability in the iron. The m.m.f. curve shown is correct for *only one interpretation:* An ordinate on it represents the m.m.f. drop existing along a line of flux going from one iron surface to the other. This curve passes through zero immediately below the conductor center, which center is immediately above the lower or inner singular point where the flux direction reverses, as does the m.m.f. drop. This curve does *not* relate to flux about the wire *inside* the singular point lines, unless it is given another special interpretation.

The solid-line flux curve gives the flux density along the armature surface. As the field approaches the singular-point regions, the density falls off, whereas, a short distance from such regions, the constant air gap used here would give nearly a constant density.

231. Simple Case of Reaction, with Distributed Coils.—Let two more full-pitch turns be introduced into the above machine, so that all are at equal spacings as shown in Fig. 90*b*. Separate m.m.f. and flux-density waves can be drawn for each turn; or what is easier, the m.m.f. blocks can be added algebraically and the resulting flux found by use of it. The value of one step in the m.m.f. wave must be clearly understood. Returning for a moment to the previous case, if the turn carried 10 amperes, the m.m.f. around a closed line of flux inside the singular points is 10 ampere-turns. Outside the singular points where true gap flux occurs, the m.m.f. drop from one iron surface to the other along a line of flux is 5 ampere-turns. Now in moving along the gap from the left, if a conductor is passed, 10 ampere-turns are passed; also, the m.m.f. wave changes its ordinate value by the same amount. The same thing happens no matter how many turns there may be; so in the distributed-coil diagram, the change in ordinate scale from one step to the next in the m.m.f. wave is equal to the number of amperes in the conductor which is passed at that point.

232. Reaction Effects with Varying but Symmetrical Gaps.—As shown in Fig. 90*c*, the poles have now been put back into the machine, but the gaps under the pole are made to vary; however,

the kind of symmetry has been preserved by which the m.m.f. division of a turn is still 1:1, or equal. The m.m.f. wave is just as it was before the poles were introduced, but due to greatly different air-path lengths at various points, the flux-density curve (for density at the armature surface) becomes the odd-looking cyclic wave as shown.

233. Reaction in Varying Gaps Due to Smoothly Distributed Currents.—Imagine now that the two conductors have been

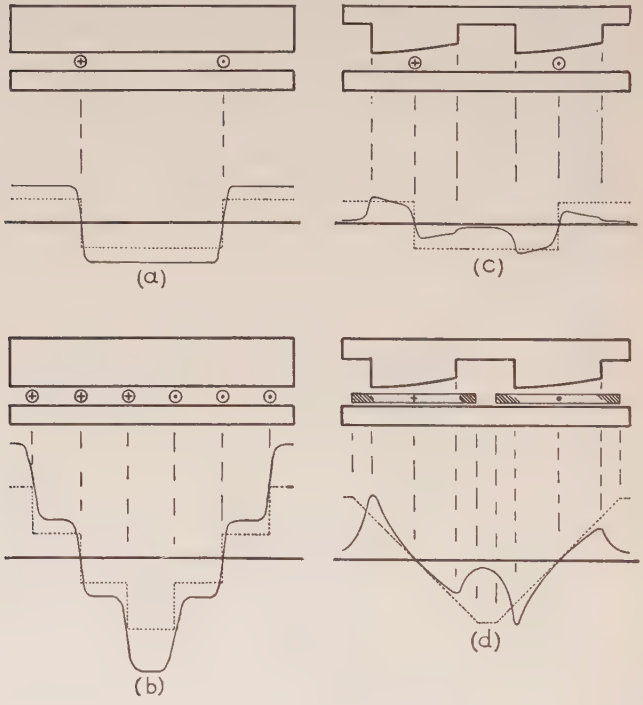


FIG. 90.—Reaction studies.

flattened out into sheets, each working at constant current densities, so that the current under a pole is smoothly distributed as in Fig. 90*d*. Consider a differential element of current in the one sheet as being paired off with the homologous differential element of opposite current in the other sheet; the two elements act as an infinitesimal current in a full-pitch turn; therefore the m.m.f. division of the turn will be again 1:1. Beginning anywhere in the gap and moving to the right, the amperes of a conductor which is passed will equal the change in ordinate-value

of the m.m.f. wave, as shown above. The current will be passed at a uniform rate until the end of the sheet is reached; therefore the m.m.f. wave has a uniform slope as shown, with constant values between ends of opposite current sheets. The *maximum reaction ampere-turns* will equal *one-half the current in the sheet*, as is perfectly evident. The flux wave must rise to peak values of density at the pole tips but must fall rapidly for the interpolar spaces; and the flux density has zero value at the centers of the sheets.

234. Magnetic Circuit Not Symmetrical; Unequal M.m.f. Division.—In the case of Fig. 90*C*, there are places for the introduction of two interpoles (commutating poles). If both are put in, the circuit remains symmetrical, and the m.m.f. division of the turn remains unchanged. But let only one interpole be introduced into the center space. On the assumption that the m.m.f. wave is still correct, there would be added to the flux wave a large flux hump under the interpole; this would put the flux wave in the absurd position of trying to declare that the flux into the armature is greater than that which comes out; this is impossible. The case simply demands that the m.m.f. be redistributed in accordance with the change of path reluctances; and this is accomplished by raising the whole m.m.f. wave above its present position and shifting the flux wave accordingly, until areas above and below the axis for the flux wave become equal.

This shifting of the m.m.f. wave with the use of half the full number of interpoles is a feature disregarded in the literature on the subject of reaction effects. In the work of this book, this effect will in most cases be disregarded, not because it is of negligible value, for it is not, but because the subject matter will be found difficult enough without adding this factor to it.

235. Detailed Reaction Effects in the Actual Machine.—Referring to Fig. 91, the discussion of this article will refer only to the *right-hand half* of the drawing and graph. Compared with the previous simple reaction cases, a new fact must be here admitted, that the iron cannot everywhere be assumed to have infinite permeability. The principal effects of iron saturation are noted in the teeth of the armature, where the m.m.f. drop in a tooth may sometimes be as much as there is in the air gap for that tooth pitch. Thus the surface of the armature no longer is an equipotential surface. The ampere-turns per inch, however,

will be quite low both in the iron of the pole face and in the armature core below the teeth. Therefore, the two equipotential surfaces which were hitherto coincident with the iron surfaces now become (*a*) one which very nearly follows the pole face, as before, and (*b*) one which very nearly follows the armature surface in the interpolar space in which tooth saturation is very low, but which sinks to the level of the tooth roots in running under the pole, where tooth saturation is high.

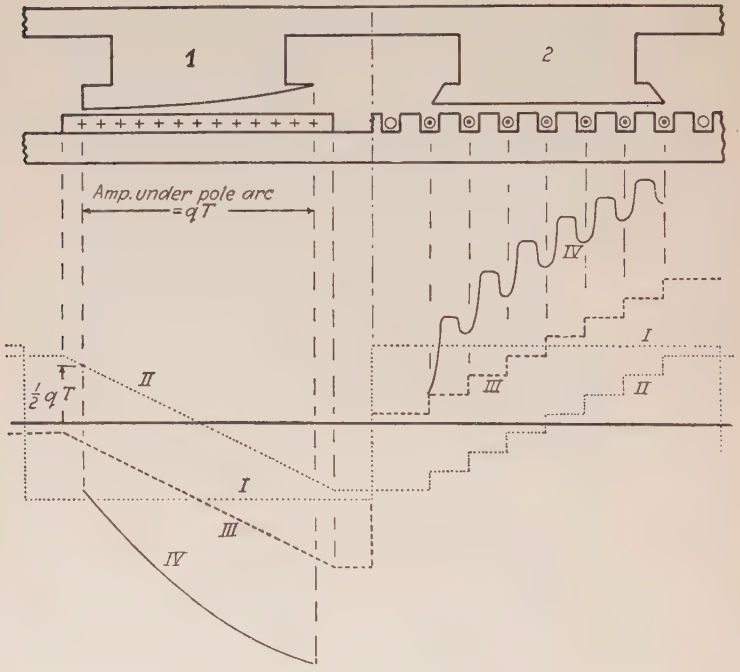


Fig. 91.—Armature reaction treatment.

The m.m.f. wave I represents the distribution of the main pole *gap-and-tooth ampere-turns*, that is, the portion of the total field m.m.f. that exists as m.m.f. drop over the gap-and-tooth part of the main field flux magnetic circuit.

The stepped-off m.m.f. wave II is the reaction wave due to active slot currents, of which there are 7 units or slotsful per pole pitch; the currents in the two slots located centrally in the interpolar space are excluded as belonging to commutation considerations.

M.m.f. wave III is the sum of I and II; and wave III, applied to tooth-and-gap distributed reluctances, gives flux-density wave IV; this last curve represents densities as found along the pole face, but referred to the armature radius. Note the high densities above the teeth and low values above the slots; also, tooth saturation increase towards the right prevents tooth flux from increasing in proportion to increasing m.m.f.

236. Conventional Reaction Treatment.—In carrying through the figures for a given design, it would be entirely impracticable to calculate the data for and plot the detailed curves just discussed. Fortunately, a much simpler and equally good treatment is available, which is shown in the *left-hand half* of Fig. 91. The current in the active slots is ironed out into a smoothly distributed sheet of current which is as wide as the total of the slot pitches involved; along with this device, there comes into being a smoothly sloping m.m.f. wave instead of the actual stepped wave. The m.m.f. maximum is the same as the actual value. M.m.f. wave I is that of the main field, II is that of armature reaction, and III is the sum of the two.

The resultant m.m.f. at any point is applied to the saturation curve for gap and tooth for that point. This saturation curve consists of B plotted versus NI for gap and tooth, the B being the average density over a tooth pitch. The development of this curve is taken up in the next chapter.

The smooth flux-density curve IV really runs through the average heights of the tooth-pitch humps in the actual flux wave, and the average height of this curve is the same as the average height of the actual wave.

The teeth are omitted in the left-hand pole pitch, but to simulate the condition of increase in tooth-pitch reluctance towards the high-density side of the pole arc, the pole face is shown shaved back to give an increase in gap.

Note that the maximum reaction m.m.f. in ampere-turns equals half the current under the pole face expressed in amperes; the current is qT, where q is *ampere-conductors per inch of periphery*, and T is the inches of pole arc; the reaction NI at the pole tips is $qT/2$.

Note further that reaction m.m.f. is zero at the pole face center, for the usual case when brushes are on neutral (and disregarding possible lack of magnetic circuit symmetry), and that reaction m.m.f. increases in proportion to distance from the

center, until the current sheet edge is reached. This pair of facts will be used in the next chapter in the discussion on *gap design.*

Problems

48. A four-pole machine has two interpoles, each of full axial length. The main-pole arc actually covers 6.5 slot pitches, but with fringing effects included, the effective arc may be considered to be 7 slot pitches in this machine. Slot $= s = 0.4$, tooth width $= t = 0.6$ inch. Main-pole gap is constant at 0.12 inch.

The interpole arc actually is a little less than a slot pitch. Nearly all flux from the face end of an interpole, where it enters the armature, will be included in a span of 2 slot pitches. The value for this, in terms of flux per axial inch per interpole gap ampere-turn, is given by the ϕ_i curves (Fig. 106). The interpole gap is 0.18 inch. For main gap, use Carter's coefficient.

Assume that all iron in the machine has infinite permeability. If armature reaction, under the assumption that reaction m.m.f. is symmetrically distributed, figures at 4,000 ampere-turns at the interpole position, how much is it actually at a neutral point where there is an interpole?

49. This problem will be confined to the main-pole air gap only. The machine is a two-pole motor, with 24 slots, without interpoles. All iron will be assumed to have infinite permeability. The gap is constant at 0.09 inch. $s = 0.3$, $t = 0.45$. Main-pole arc covers 8 slot pitches. First, find the gap ampere-turns needed to give an average gap density of 35,000.

With the above-found value of field excitation on, the armature is loaded until each slot carries 200 amperes. Plot the curves of m.m.f. distribution due both to field and the armature reaction. Plot the resulting gap-density distribution.

With above conditions, except now that brushes are shifted against rotation by an amount equivalent to 2 slot pitches, plot all curves as required in the preceding paragraph.

Throughout this problem, neglect all flux in the interpolar spaces. Note that under the assumptions made, reaction does not change the total flux per pole until brushes are shifted.

CHAPTER XXV

GAP-AND-TOOTH FLUX RELATIONS; GAP-AND-TOOTH SATURATION CURVES; GAP DESIGN

The most important part of the main flux path of a machine consists of the combination of teeth and air gap, and it is the most difficult to handle. The problems to be solved are simple enough when tooth densities are low; but for the higher densities frequently used, the relations become involved, and at best, only fairly good approximate solutions can be attained with the expenditure of a reasonable amount of labor.

The deliberate, close design of the air gap is not a process ordinarily given much stress in direct-current machine design; but it is possible to gain specific advantages by paying a not unreasonable amount of attention to details, as is brought out by the treatment below. Air-gap design is, of course, a very important phase of design in producing all kinds of alternating-current machinery; the principles and methods outlined here apply to all such cases.

237. Tooth Saturation Curves.—Notation: t is width of tooth top; t_r is tooth-root width; $k = t/t_r$, the tooth constant; d is tooth depth; B_r is tooth-root actual iron density; M_t is the tooth m.m.f. drop in ampere-turns.

Designers use various approximate methods for obtaining the graph of B_r *versus* M_t, which is the tooth saturation curve. In the usual trapezoidal tooth the density varies over a wide range from top to bottom. One old rule is to find, for a given B_r, the density at a point one-third of the tooth depth d from the bottom or root, and use the corresponding NI per inch as the average value over the whole depth d. This rule often gives good results, but not always.

A more accurate but more laborious way is to divide the tooth depth into 10 or 20 equal increments, and find the NI drop in each. The sum gives M_t; when this is done for enough values of B_r, the saturation curve can be plotted.

To facilitate the work of the student in using this text, as well as to illustrate the methods of good designers who wish to save

valuable time and yet be accurate, the curves of Fig. 92 are given. Various values of k serve as parameters, and B_r is plotted against M_t, all for a tooth depth of $d = 1$ inch. These curves were made by dividing the tooth into 20 increments, and by use of the curve for the electrical sheet given elsewhere.

It will be evident that for a given K and given B_r, if $d = 2$ inches, for instance, each of the 20 increments into which the tooth might be divided would be twice as high as in the 1-inch

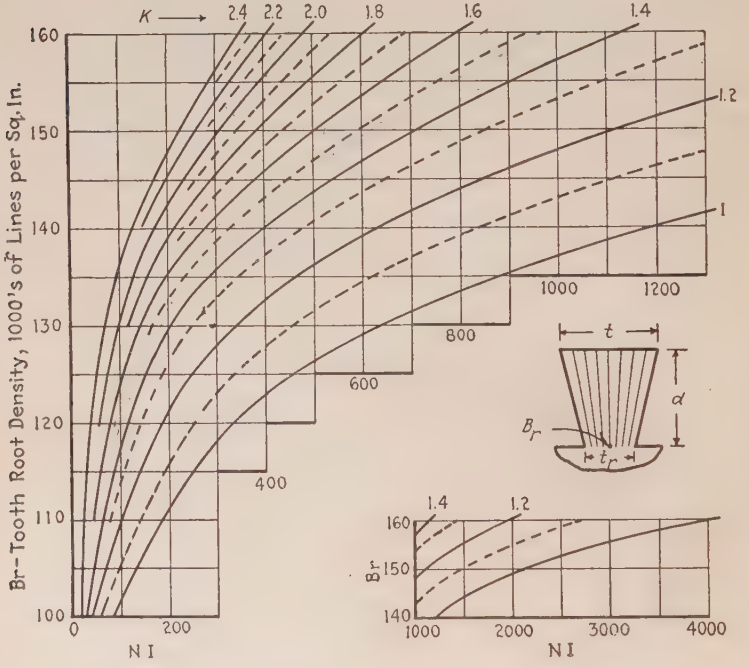


FIG. 92.—Tooth saturation curves for $d = 1$ inch ($k = t/tr$).

tooth, and an increment would have the same densities but twice the ampere-turns as has the homologous increment in the 1-inch tooth. Thus M_t becomes proportional to tooth depth under the restrictions of using given values of k and B_r.

238. Gap-and-tooth Quantities: Low Densities.—Notation: B_g is air-gap average density over a tooth pitch; M_g, gap m.m.f. in ampere-turns; s, slot width; k_1, ratio of tooth-root iron section to gap section for 1 axial inch of a tooth pitch; other symbols as above.

At $B_r = 120{,}000$ or less, M_t is quite low as compared with M_g. Nearly all of the flux coming through the tooth-root stays in the tooth until it emerges near or at the top. The gap field is distributed very closely like that assumed for the work leading to the derivation of Carter's coefficient; that is, as concerns gap flux, the tooth-and-slot iron surface is virtually an equipotential surface. The flux out of the slot bottom, and the flux in the air, varnish and oxide between the iron of adjacent laminations are each negligible compared with the flux in the tooth-root iron.

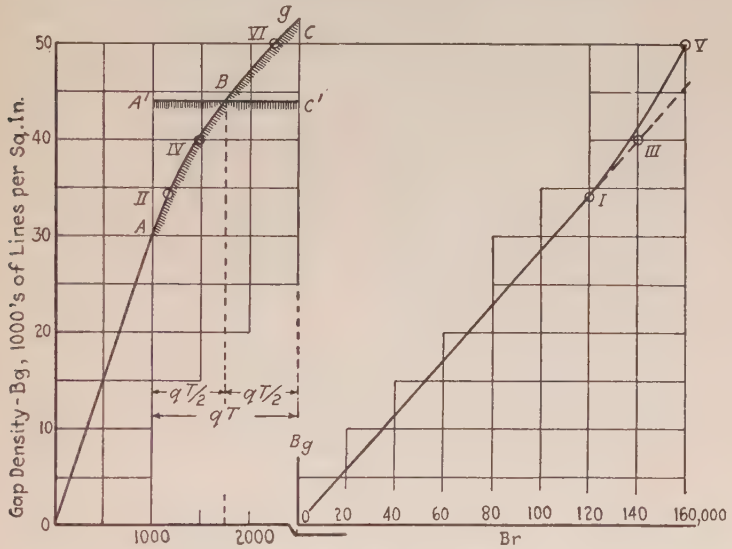


FIG. 93.—Gap and tooth saturation curve.

The ratio of tooth-root iron section to gap section for 1 axial inch of a tooth pitch is k_1, and

$$k_1 = \frac{(0.93t_r)}{(s + t)}$$

where 0.93 is the inches of iron in an axial inch of tooth laminations. 0.93 is called the *stacking factor*. Then

$$B_g = B_r k_1,$$

or

$$B_r = \frac{B_g}{k_1}.$$

A curve of B_g vs. B_r is started by plotting the point I (Fig. 93) and drawing a straight line from there to the origin; B_r is here taken at 120,000.

M_t is determined from the tooth saturation curves for this root density. B_g having been calculated, M_g, the gap NI, is calculated by use of Carter's coefficient, and point II of the gap- and-tooth saturation curve is plotted.

Similar calculations are carried out for $B_r = 140{,}000$, giving the data for plotting points III and IV, but as is shortly seen, the two curves are not put exactly through these two points.

239. Gap-and-tooth Quantities: High Densities.—Notation: M_4, m.m.f. for root quarter of tooth; k_4, constant for same; B_s,

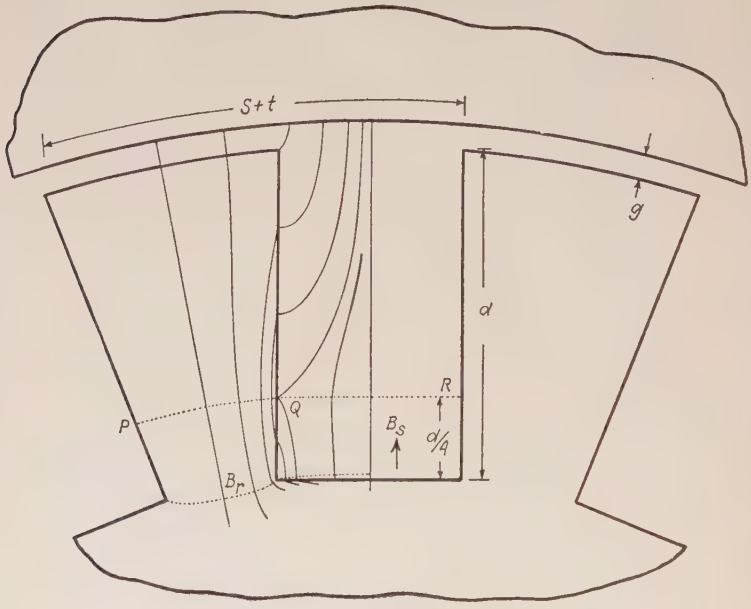


FIG. 94.—Gap and tooth field: high density.

density in bottom region of slot; k_2, coefficient by which to multiply $B_r k_1$ to correct B_g for increase due to slot flux.

Throughout this article, the condition of $B_r = 160{,}000$ will be dealt with. At this density, the flux conformation takes on the aspects depicted by Fig. 94. Hydrodynamical experiments in analogous cases and other investigations clearly show that some certain flux line comes from the slot bottom, touches the tooth side at point Q, and reenters the slot to rise through it to the pole face. An equipotential line PQR floats through tooth and slot at an approximately constant radius.

It is assumed that this line PQR lies above the slot bottom at a distance equal to $d/4$. The slot bottom, being always at low density, furnishes another approximate equipotential surface, and between it and QR the slot flux will be roughly rectilinear. The m.m.f. acting over this $d/4$ length of flux path is M_4, which is found by calculating k_4 for the lower quarter of the tooth's depth, and using the tooth saturation curves. B_s, the approximate density in the slot-bottom region, is then figured. Neglecting the flux between iron of adjacent laminations, the flux per axial inch at the tooth root level, for a tooth pitch, is

$$160{,}000(0.93\ t_r) + sB_s.$$

Whereas, for low densities,

$$(s + t)B_g = B_r(0.93t_r),$$

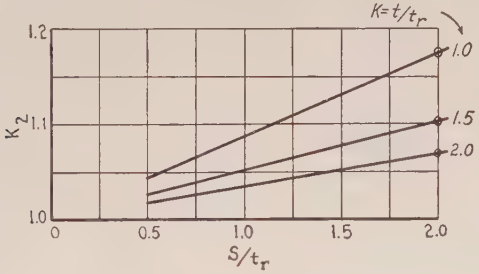


FIG. 95.—Curves of K_2.

it is now seen that B_g is increased due to the sB_s factor. This factor will influence B_g to an extent of 20 per cent or less, varying with relative slot width and tooth constant k. To avoid the necessity of making the uninspiring calculation leading to correcting B_g for slot-flux effect, the curves of Fig. 95 are given. At $B_r = 160{,}000$, B_g now becomes

$$B_g = B_r k_1 k_2.$$

Point V is now plotted on the $B_g - B_r$ curve.

To get M_t it is assumed that the tooth saturation curves apply as before. This is not entirely correct, for flux coming up through the tooth root is increased up to the level of PQ, and is appreciably reduced from there up.

To get M_g it is assumed that Carter's coefficient still applies to the kind of flux distribution now obtaining in the gap; this also is not quite correct. No approximate treatment for this saturated-tooth case can ever be wholly satisfactory; but to

attempt to produce a nearly perfect solution would entail so much work that the method would become wholly impracticable for design purposes. It is believed that the method suggested is good enough and simple enough to deserve a place alongside of other methods now in use.

It is assumed that M_g plus M_t as just found, will give correctly the gap-and-tooth NI for the root density used, 160,000. Then point VI is plotted.

240. Drawing the Curves.—The tendency for B_g to be increased by addition of slot flux increases slowly as $B_r = 120{,}000$ is exceeded, rising faster as 160,000 is approached. The B_g vs. B_r curve is therefore passed through point I, made to run smoothly near to but above point III for 140,000, and through point V. Likewise, in the gap-and-tooth saturation curve of B_g vs. $(M_g + M_t)$, this curve follows very closely an air-gap line up to near point II, passes slightly above point IV, and smoothly on to go through point VI.

241. Gap-density Distribution, No Load and Full Load: Constant Gap.—Notation: M_{gt}, gap-and-tooth m.m.f. furnished by main field excitation; M_{rt}, armature reaction ampere-turns at the pole tip; B_{ga}, gap density averaged over the pole arc; B_{gm}, maximum value of B_g; B_{rm}, maximum value of B_r; q, armature ampere-conductors per inch of periphery; T, pole arc in inches; g, gap clearance.

Before passing to the discussion of design of the air gap, it will be better to take a given constant air-gap clearance g and by fixing load and excitation conditions, find how the gap flux behaves as to distribution and total quantity.

Let the scale values as given for the curves of Fig. 93 be used; let the main field supply out of its total m.m.f. a major portion M_{gt} equal to 1,750 ampere-turns, to act over the gap-and-tooth path under one pole. And let $qT = 1{,}500$ amperes.

By q, the ampere-conductors per inch of periphery, is simply meant the amperes per peripheral inch of current sheet; it may be found by dividing the amperes per slot by the slot pitch. If qT is 1,500, then M_{rt}, the ampere-turns reaction m.m.f. at one pole tip, is 750. The net m.m.f. at the strong tip is 2,500, and at the weak pole tip, 1,000.

The constant gap clearance g used under the pole arc has been used to obtain the gap-and-tooth saturation curve II-IV-VI as outlined above. The net m.m.f. acting on this gap-and-tooth

path varies from 1,000 at one tip, through 1,750 = M_{gt} at the center, to 2,500 at the other tip. The gap density B_g then varies as shown by the curve arc ABC, from about 30,000 to 52,500.

Furthermore, since the reaction m.m.f. to be added to or subtracted from M_{gt} varies in proportion to the distance one goes away from the center of the pole arc (granted brushes on neutral) the arc ABC gives to exact scale the distribution of gap density along the pole arc, if the horizontal span marked qT under the arc is considered to be the pole arc itself drawn to the scale that would fit it into this space.

Now, assuming that the excitation remains unchanged (or at least, that M_{gt} is unchanged), let reaction become zero by removing the load. The gap density everywhere becomes that due to M_{gt} = 1,750, which gives B_g = 44,000. This density is also shown distributed over the same imaginary pole arc as was used to interpret the curve ABC. ABC is then full-load gap-density distribution, and $A'BC'$ is no-load distribution.

It is perfectly evident that the full-load flux per pole is less than the no load, due to tooth saturation effect. The reduction of flux by reaction as load increases often has its serious effects.

B_{ga} for no load is 44,000; for full load, it is less. And B_{gm} at full load rises to 52,500, which causes root-density maximum, B_{rm}, to go well over 160,000.

Tooth loss is principally due to the highest value of tooth flux attained while passing under the pole and not to the way in which it is attained; this fact needs remembering when gap design is taken up.

242. Per cent of Pole Arc, X, with a Constant Gap.—The usual gap design has characteristically a certain portion at the center in which a constant gap length occurs, the gap length tapering to greater values towards the two pole-arc tips. It is also usual to make the two sides of the gap *symmetrical* with the pole-arc center.

Let X represent the per cent of the pole arc taken up by the constant gap-length region. This constant gap is always the least gap occurring under the pole arc. In some designs, X may be zero; that is, the gap begins tapering at the center; in others, X may have any value between zero and 100 per cent. In designs with low armature reaction or when the gap is made very long, X may be made 100 per cent, in which case the gap length is constant everywhere.

243. Comparison of Various Gap Designs.—In order to complete certain arguments, Fig. 96 has been prepared, which is basically like Fig. 93, except that two new gaps have been used; that is, calculations have been carried out by which the saturation curves for gap-and-tooth for three gap lengths, g_1, g_2, and g_3, were drawn. Furthermore, these three gap clearances are selected so that g_2 is the mean of the other two. The three clearances might for instance be 0.10, 0.20, and 0.30 inch. To aid in the work, the intermediate dash-line curves are drawn in

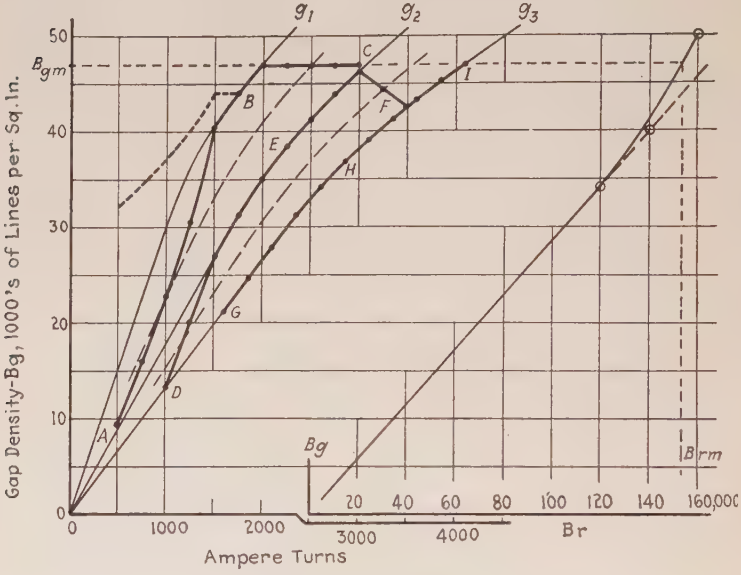


Fig. 96.—Gap design.

by interpolation, so that there are available gap-and-tooth saturation curves for five gap clearances, all equally spaced as to equal incremental increase from g_1 to g_3.

To better the discussion, let qT be changed to 3,000 amperes, so that M_{rt} is 1,500.

Gap design 1 is indicated by arc GHI. Here, the largest gap clearance g_3 is used throughout under the pole arc. $X = 100$ per cent. The point H is horizontally midway between G and I; hence it marks the needed value of M_{gt}, which is 2,850 ampere-turns of main field excitation required for gap and tooth. B_{gm} is 47,000, giving B_{rm} of about 153,000. Average gap density for

this distribution, which is proportional to main-pole flux at full load, is about 35,000.

Gap design 2 is indicated by arc *DEC*. Note that the black dots scattered along the line *DEC* are equally spaced horizontally, which virtually divides the pole arc, which is imagined to be spread underneath, into 10 equal widths. Then by use of the black-dot markers and their positions on the saturation curves, the gap clearances that have been used along the pole arc can be found out. It is seen that at the left pole tip—point *D*—g_3 is used; at the other tip—point *F*—g_3 again is used. The gap clearance midway between g_2 and g_3 is indicated at the two dots first inside the pole-tip extremes of the pole arc; and all the central part of the pole arc, amounting to 6 of the 10 arc widths, has the gap g_2. $X = 60$ per cent. In short, the gap is symmetrical about the center point, is constant at g_2 value out to where a uniform increase begins, with the ends of the arc swelling to g_3 at the tips. Point *E* marks the value of M_{gt}, which is now 2,250; B_{gm} is slightly below the former value, being 46,500; B_{rm} is about 151,000; and B_{ga} is about 38,000.

Gap design 3 is indicated by *ABC*. The gap is again symmetrical about the center. A constant clearance g_1 covers the central two-tenths of the pole arc; and the gap is tapered so that, at the strong-flux side, B_{gm} remains constant, the tip gap being nearly g_2. Note that the gaps used at the weak-flux side copy those at the strong side of the pole arc. The point *B* shows that M_{gt} 1,750. B_{gm} is 47,000; B_{rm} is 153,000; and B_{ga} is 37,000.

All three designs give about the same full-load pole flux. They all cause about the same tooth loss, as B_{rm} is about the same in the three cases. In the order taken up, field excitations for gap-and-tooth are 2,850, 2,250, and 1,750.

244. Factors Affecting Gap Design.—The shorter the gap length, the lower field excitation becomes, thus reducing field copper and field loss, as well as field leakage flux.

But short gaps cause higher pole face losses, which are caused by the tufted tooth flux sweeping across the pole face and varying the density therein. Short gaps result in high distortion of gap flux. High distortion means that pole flux will reduce more as load increases. High distortion means a large ratio of maximum to average gap density, which raises the maximum voltage between adjacent commutator bars. Short gaps require better manufacturing to avoid unequal gaps under different poles,

both with new and worn machines. As high distortion raises the flux reduction due to reaction, the reduction from no load to full load may become great enough to give the effect of a differentially compounded machine; as a motor, the speed may become unstable, requiring the use of a series stabilizing field winding on a shunt motor.

245. Process of Designing the Air Gap.—A given manufacturer, for a given class of machinery, taking such factors as expected bearing wear, unbalanced magnetic pull and pole face losses into account, would fix a reasonable *minimum gap clearance* for every armature diameter. Such clearances are given in Fig. 116.

This graph also gives gap clearances one may reasonably expect to have as *maximum* values, which may or may not be attained in a particular design. The minimum gap we shall call g_1, and the possible maximum g_3.

Data will be worked up for three gaps, the minimum, the possible maximum, and the mean of the two, for a given armature diameter. The three gap-and-tooth saturation curves and the B_g *versus* B_r curve, as in Fig. 96, are prepared.

It is assumed here that the armature design is already completed, as it must be before gap design is tackled. The tooth loss allowed for cannot materially be changed without partial redesign of the armature. Therefore, B_{rm} has been fixed. Also, B_{gm} is thereby fixed, and the pole flux needed fixes B_{ga}. M_{rt} is, of course, fixed.

A line corresponding to the dash lines (Fig. 96) is drawn vertically up from B_{rm}, and one over to B_{gm}; the latter line establishes the *roof* of the graph. No values can go above the roof, else tooth loss allowed for will be exceeded.

M_{gt} is assumed so that B on ABC falls on the g_1 curve; this curve is followed up until it strikes the roof, whereupon the roof line is followed to the right until the end of the pole arc span is reached.

This process fixes all gaps on the strong side of the pole arc. In general, the two sides of the pole arc must be duplicates, so that no one will ever have any difficulty operating the machine in reverse rotation. By fixing the gaps for the left side of the arc to duplicate those to the right, the entire distribution curve is obtained. The question now is, Does B_{ga} equal the required value? The average height of the curve ABC is found, and,

B_{ga} as required to furnish the needed pole flux being known, we know at once if this gap design gives too much or too little flux. If too little, a greater M_{gt} is assumed and the process is repeated, until B_{ga} is satisfactory.

By establishing the roof and following it as above outlined, the most economical design of field coil is assured. This is going on the principle that once B_{gm} is *attained*, it may as well be *maintained* until the pole tip is passed, for it will do tooth loss no good to bring tooth flux up to a peak value as in design *DEF* and then drop it before leaving the pole arc limits.

A half wing of the *ABC* no-load distribution curve of gap density is shown in the dash line ending at *B*.

CHAPTER XXVI

COMMUTATION: CURRENT REVERSAL AND CONTACT DENSITIES

Although many writers have handled the subject of commutation, the beginner in commutation theory is not much helped thereby. The assumptions made by different theorists often disagree. The different treatments differ radically in usage and nomenclature. The simpler treatments often are misleading, in that, in order to produce an easy treatment, untenable assumptions create, first, a too artificial problem and, second, lead to untenable conclusions.

A good deal of valuable talent has gone to waste due to the tendency of some writers to treat commutation mathematically. The author feels that, as soon as a treatment goes much beyond the use of arithmetic and elementary calculus principles, treatment becomes useless. The subject is really so extremely complex that it is beyond the ability of anyone to handle it mathematically.

A considerable part of the following treatment is original with the present writer.[1]

246. The Purpose of Commutation.—The first and principal function of the commutator and brushes is so to handle a moving group of coils that the winding will be divided up into two or more paths that have nearly stationary positions with respect to the pole fluxes. This involves the sudden shifting of a given coil from one path having one current to another path carrying the opposite current. The problem of commutation is to effect this shift without getting into trouble about it.

247. Ideal Commutation.—Ideal commutation has two basic requirements. First, commutation must be successful; that is, it must not at any time, due to flash-overs or whatnot, interrupt the service the machine is rendering; and brush heating, brush or commutator pitting, and deterioration due to sparking must

[1] Lamme, B. G., "A Theory of Commutation and Its Application to Interpole Machines," *A.I.E.E.*, October, 1911.

be reduced to the point where long life of commutator and brushes is attained. Second, the wattage losses due to the function of commutation must be very low. It is evident that high losses will cause overheating and hence unsuccessful commutation in one way or another; and on the other hand, unsuccessful commutation as due to sparking inherently involves high losses. The two basic requirements are interdependent.

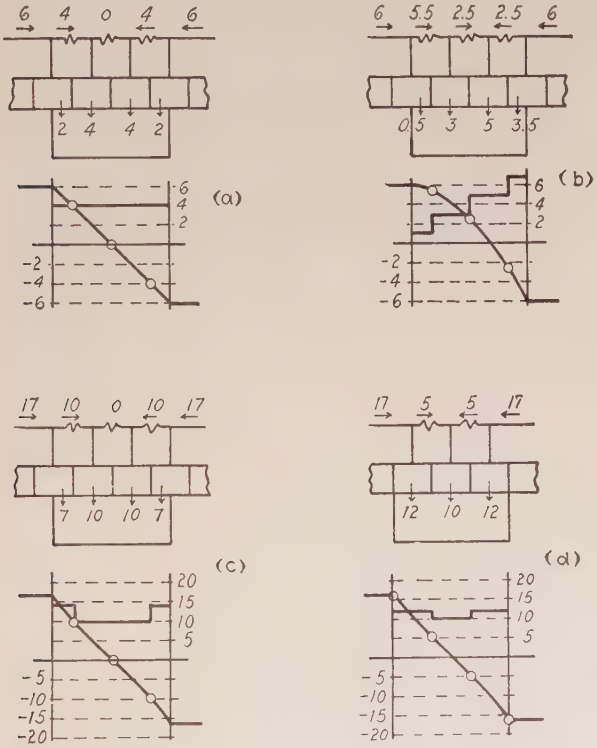


Fig. 97.—Study of current reversal and contact densities.

248. The Explanation of Sparking and the Cause of the Beginning of Sparking.—Refer to Fig. 97*a*. Consider the coil to the right that is being commutated. This coil, before entering commutation, carried the left-hand path current, which is 6 amperes to the right. It is now nearly through commutation, and its current is 4 amperes to the left. In a fraction of a second it will be out of commutation and must then carry the right-hand path current of 6 amperes to the left. Suppose that its current

does not completely reverse during commutation, so that at the instant of leaving commutation, it does not have 6 amperes to the left. The 6 path amperes from the right must then divide between the coil and the right-hand lead; if there is current down this lead, it must be going somewhere; it must be jumping across a spark or arc, from the bar which has just left the brush, back to the brush. All this is a long-winded statement of the *usual statement that failure of the coil current to reverse is the cause of sparking;* that is, it has just been demonstrated that if coil current does not completely reverse, there is sparking at the brush tip. This demonstration, or explanation, or statement of fact, which expresses a *condition,* is so frequently written into elementary textbooks and so often given by teachers that nearly every student has been influenced to believe that it also is the *cause of sparking.* And what is worse, the cause of sparking and the factors tending to make sparking *begin* are so frequently omitted from the discussion that much confusion has resulted.

It is not nearly so necessary to know why sparking continues, once it has begun, as to know what began it. Likewise, we are not nowadays nearly so much concerned with sparking commutation as with sparkless commutation.

Sparking begins when the temperature at the brush contact rises above a certain limiting temperature. The temperature rises due to the excessive loss of energy at the contact. Excessive energy losses are due to excessive current densities at the contact. *The cause of sparking, therefore, is high current density.* If, therefore, high current density can be prevented, sparking will not occur.

249. Current Reversal and Contact Density.—Figure 97*b* shows a case in which the brush arc covers three commutator bar pitches, the brush now being in contact with four bars. In order to make a start on the subject, path currents and commutated coil currents have been *assumed.* By use of arithmetic, currents in the coil leads were then found. For instance, if 6 amperes enter from the left, and 5.5 go on to the first coil, 0.5 ampere must go down the first lead and flow from the first or left-hand bar to the brush.

Immediately below the coil centers are plotted (circular points) the coil currents in the graph. A smooth curve passed through these points and running from +6 at the left to −6 at the right is the curve of coil current or curve of *current reversal.* This

curve holds for every coil, provided all coils attain the successive values of current as given over the respective coils. As is shown later, in the usual case the current of one coil does *not* reverse like that of the next.

Instead of plotting actual current density in amperes per square inch of brush contact, for the present a much easier representation is used. The average current density under a bar is in proportion to the bar's current divided by the width of the bar in contact with the brush; let this value be called the *current per whole bar*. For instance, the left-hand bar is only half on the brush, and its current is 0.5 ampere; the current per whole bar is then $2 \times 0.5 = 1$, and the current density is proportional to 1. Thus the stepped-off curve shown in the graph was obtained.

It is at once evident that at the left, where the current reversal is beginning slowly, the contact density is low; at the end of commutation, or to the right, a steeper reversal curve accompanies a high contact density. The reason is simple, but very important. Consider any step in the density curve. The step is one bar wide (except the end ones). The difference in ordinates of the reversal curve is simply the difference in currents of two adjacent coils, which difference must flow down the common lead and into the bar between them. This bar current, divided by the bar width, gives a value proportional to density. But this amounts simply to dividing the change in coil current by the space covered by the change, which in turn amounts, in approximate terms, to the derivative of current with respect to area or width. At a given point, the density is proportional to the slope of the reversal curve.

It is seen that the density changes from 1 to 7 for extreme values. Now the density represented by the 7 value might, if maintained for minutes or hours, cause so much loss at the brush tip as to start sparking there. What will reduce this density? Anything will reduce it that will straighten out the reversal curve. To get the best results, all densities should be constant, so that heating everywhere will be the same. But constant densities will demand a constant slope of the reversal curve.

Figure 97*a* illustrates this condition. Coil currents were so chosen as to give a straight-line reversal curve, and as seen from the graph, the density values become constant.

Straight-line reversal would seem, then, to be ideal. Whether it is or not is answered partly in the next article.

250. Resistance Commutation.—Consider a two-pole motor, symmetrically built, so that when one brush is positioned as in Fig. 97*c*, the other is placed likewise. Between the positive and negative lines there is a circuit consisting of two paths in parallel, their ends terminating in identical sets of commutated coils, leads, bars, and brushes, one set being shown in this drawing. Suppose now that the commutated coils are not affected by any flux change of any kind. Then the currents in coils, leads, bars, bar contact surface, and brush body would be completely determined by the resistances of these various parts.

The resistance of brush body, bars, and leads is negligible; hence, if 17 amperes enter from either side, as assumed in this case, current will distribute to the various branches in accordance with the resistances of coils and bar contact areas.

1. If contact-area resistance is zero, the outer half-bar contacts will carry 17 amperes each; no current will flow through the resistance of the commutated coils, and none will get to the two inner bars. Then current density for the outer half-bar contacts will be very high.

2. If there is contact resistance and zero coil resistance, it is clear that the current will divide among the bar contacts in inverse proportion to the resistances of these contacts; if contact resistivity is constant in ohms per square inch, then contact current density will be constant. As brought out in the preceding article, constant contact density means straight-line coil current reversal.

3. If coil resistance is small but appreciable compared with resistance of the contact area of a bar, the current distribution will be something like that shown in Figs. 97*c* and 97*d*, where reversal is nearly straight line, and density is nearly constant.

In the usual case, with carbon or graphite brushes, coil and contact resistances bear such relations to each other that these results are typical.

In conclusion, a very important statement can be made: If all fluxes affecting commutation can be eliminated, nullified, or equalized, or if their effects can be rendered negligibly small, resistance commutation will practically take place, and commutation will be entirely satisfactory.

Very high contact resistance would give nearly straight-line reversal and nearly constant contact density; but the means would defeat the end, for high contact resistance means great

(even though uniformly distributed) loss at the contact, and the whole brush surface would begin to arc and burn.

251. Study of a Simple Case. I. *Object.*—By carrying through in fairly complete style a simple case of commutation, a great deal of value is derived; many of the factors entering into more complex cases are found and evaluated.

II. *Data.*—A two-pole machine, with 24 slots, 1 coil per slot, 12 conductors per slot, 6 turns per coil, full-pitch winding, will be studied. The speed is 125 r.p.m. The armature current is 40 amperes, or 20 amperes in each of the two paths. The brush arc is equal to one bar pitch, and brush area is 1 square inch for each brush. There are no interpoles (commutating poles). Resistance per coil is 0.002 ohm; coil resistance will be neglected. The brushes are on neutral.

III. *Fluxes.*—There are two sources of flux which affect commutation.

1. The commutated coil currents have a flux component that changes with the currents; this flux has an inductive effect, setting up a voltage in the commutated coil which tends to prolong the original current; thus, it delays the reversal of current.

2. The active (uncommutated) armature coils set up a flux through which the commutated coils pass, and this flux also tends to delay reversal.

There are two identical coils undergoing commutation at the same time; the upper side of one coil occupies the upper half of the same slot in which the lower side of the other coil lies. Thus the flux due to one coil will have a mutual inductance effect on the other. It is assumed that commutated coil flux is such that effectively reversing the 20-ampere current of a coil means reversing 10,000 maxwells linking with that coil. Thus one coil turn has a flux-linkage change of 20,000 maxwells or lines during its current reversal, or the flux per coil ampere is $10{,}000/20 = 500$ lines. The presence of the other commutated coil will double the effect, as is seen in the equation below, where a factor of 2 is used.

It is further assumed that the flux due to active coil currents amounts to 20,000 lines cut by the wires in one slot during a travel of one slot pitch.

IV. *Voltages Induced in the Coil.*—A coil travels one slot pitch during commutation. Its six conductors on one side will cut 20,000 lines due to active currents, and the six on the other side will cut the same. The time of commutation is

$$\frac{60}{125 \times 24} = \frac{1}{50}, \text{ or } 0.02 \text{ second.}$$

Assuming that this flux is passed through uniformly, there will be a constant induced voltage in the coil of $Nd\Phi/dt$ which is

$$2\,\frac{20{,}000 \times 6}{0.02 \times 10^8} = 0.12 \text{ volt.}$$

The commutated coil flux as found above, is 500 lines per coil ampere; hence, $\Theta = 500i$. The voltage induced by this flux is

$$2 \times 6 \times 500\,\frac{di}{10^8 dt} = 6 \times 10^{-5}\,\frac{di}{dt}.$$

V. *Current Reversal; Graph.*—The above voltages will delay the current reversal, making it lag a possible straight-line reversal. A reversal curve will be assumed (Fig. 98). Here the various quantities are plotted in the upper left figure against time as abscissa. It is assumed that in the first quarter of time lapse, or 0.005 second, the coil current changes only 4 amperes; in the next equal increment of time, 8; next, 12; and last, 16. The sum of these four changes equals 40 amperes, the total change required to reverse.

The slope of the reversal curve (curve I) is di/dt. By measuring the slope at various points and using the relation found above for evaluating voltage induced by commutated coil-current flux change, these voltage values are found; adding the constant 0.12 volt found above, the total induced-voltage curve is plotted in curve II.

Curve III is obtained most easily by raising curve I by 20 amperes. The reader will easily verify the fact that curve III represents the current entering the brush from the *exit*, or leaving bar; and reading from the horizontal line at 40 amperes, down to this curve, gives the current entering the brush from the entering bar.

The next step is to find out what conditions would force the current reversal to take place as assumed. This means the finding of the unit resistance values at all points along the brush contact. Carbon brush contacts have this property: as current density rises, contact resistance falls. Therefore, with delayed reversal, and higher densities near the right brush tip, contact ohms per square inch there will be low.

VI. *Induced Voltage—Contact Drop Relation.*—Refer to the lower right drawing (Fig. 98). Consider the closed circuit con-

sisting of the commutated coil, its leads and bars, bar contacts, and brush body. The IR drops due to all parts except the brush contact can easily be neglected. Around this circuit in the

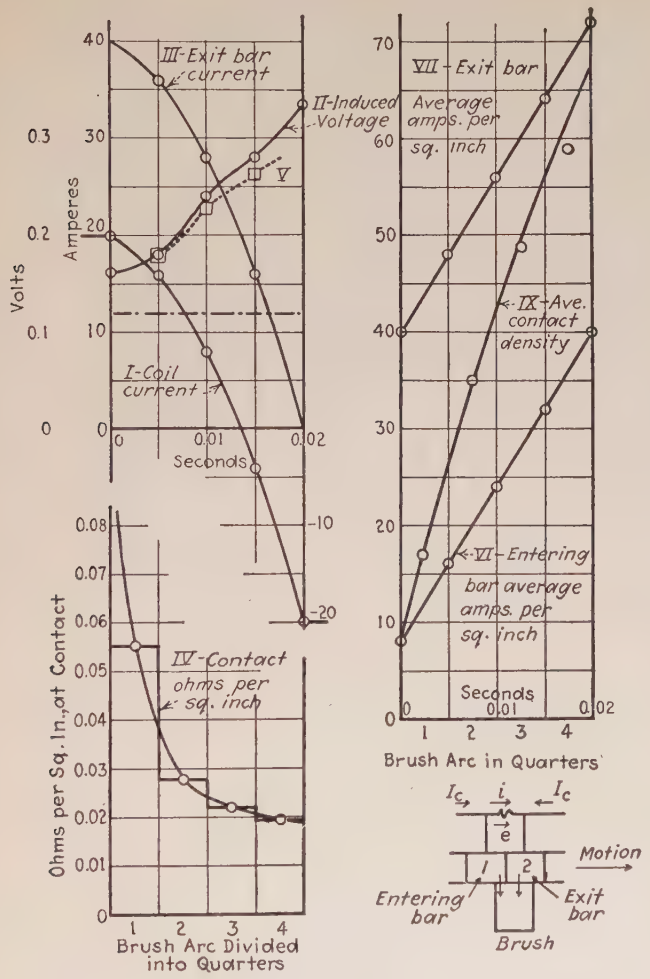


FIG. 98.—Complete study of a simple case of commutation.

clockwise direction, the summation of induced voltages must equal the summation of the IR drops. Then

$$e = \text{drop } 2 - \text{drop } 1$$

where e is the induced voltage in the coil, and the drops are the voltage drops over the contacts.

It follows that exit bar 2 must always have more contact drop than has entering bar 1, to give a positive difference all the way across; the difference being equal to the induced voltage which is varying but positive, as shown in the graph.

VII. *Finding the Contact-resistance Curve.*—Suppose the curve of required contact resistance were already obtained and plotted, as in the lower left graph. We could proceed to check it to find if it is correct. Let the instant when the coil is one-fourth of the way through commutation be worked with. At this instant, the coil carries 16 amperes; bar 2 carries 36 amperes, and bar 1, 4 amperes.

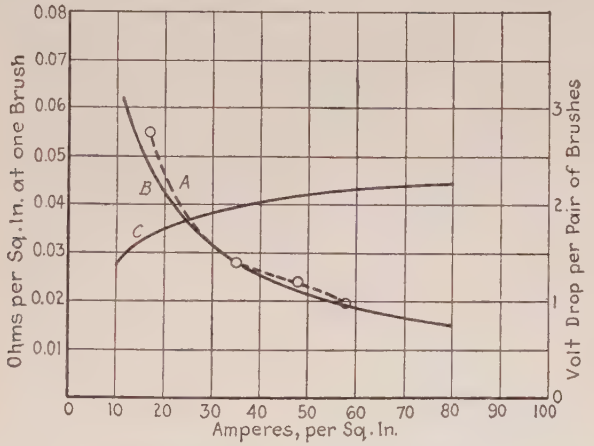
Refer to the stepped-off curve of contact resistance (curve IV) instead of the smooth one, and work by quarter-brush widths. As the brush area is 1 square inch, each quarter has a contact resistance equal to four times its ohms per square inch. Thus, bar 1 has a contact one-quarter wide, and the curve indicates 0.055 ohm per square inch. Then the bar 1 contact resistance is $4 \times 0.055 = 0.22$ ohm. The current of bar 1 being 4 amperes, $4 \times 0.22 = 0.88$ volt drop.

Bar 2 covers three-quarters or quarter widths, each having different contact resistances. These three resistances are in parallel; upon finding the total resistance and multiplying by 36 amperes, the contact drop of bar 2 is found to be about 1.06. Drop 2 − drop 1 = 1.06 − 0.88 = 0.18, which checks the value of induced voltage. Likewise, other time instants could be checked.

To *obtain* this contact-resistance curve meant the working out of one of the nastiest cut-and-try problems the author has attempted; it is much worse than any of the field-mapping jobs shown in the earlier part of the book. It was a matter of trying one contact-resistance curve, checking it through to find what induced voltages its difference in drops would require, and then trying another curve when this one failed to meet requirements. The resistance curve given does not check; the differences in drops are plotted in the dotted curve, upper left graph; but this curve is near enough to the induced-voltage curve to indicate that the contact resistances have been found pretty closely.

VIII. *Current Densities.*—The current density at any fixed point on the contact is not constant. To find its variation, the writer worked out the average densities over the quarter widths of brush arc corresponding to the times when the coil was one-

fourth, one-half, three-fourths, and four-fourths of the way through commutation. For instance, when the coil is one-fourth the way through, as seen above, bar 1 has 4 amperes, which gives 16 amperes per square inch. Bar 2 has 36 amperes, which gives an *average* over its whole contact of 48; but this average was not used; instead, the 36 amperes were distributed in proportion to the inverse of the resistances of the three widths or quarters. By this operation, the three densities in order are 38, 50, and 56. Thus each time instant gives four density values. There being



A—Resistance per sq. in. from case of Fig. 98
B—Resistance per sq. in., typical hard carbon
C—Volts drop per pair, typical hard carbon
(Graphite brush values are about ⅔ of those for hard carbon)

FIG. 99.—Brush contact characteristics.

four times instants considered, each quarter width of arc would have four values. For the last quarter width, for instance, they are about 55, 56, 59, and 64.

The average of such values enabled the middle curve, curve IX, in the right-hand graph, to be drawn. It is significant that although the density at any place varies, it does not vary much. This fact reflects on the assumption, made but not heretofore stated, that contact resistance at any fixed point would be constant. If density varied greatly, resistance might vary considerably. Thus as density is about constant, the assumption that resistance at a given place is constant is good.

IX. *Characteristic Contact-resistance Curve.*—Data are now at hand for plotting a curve of variation of contact resistance

versus contact density. For each quarter width, we have the average density and the average ohms per square inch. This curve is given in Fig. 99, and along with it a typical curve for carbon brushes is given. The two curves nearly superimpose. As the writer is not anxious to work out many of these problems, this means that it was a stroke of luck to have the contact-resistance curve for this case come out so near to being typical. It means further that if a typical brush is applied to the above case, then the current reversal will take place about in the way originally assumed, and all other values will be about as found in the study.

252. Discussion of Results.—It has been found that the induced-coil voltage equals the difference between the two contact drops. Any factor tending further to delay the reversal gives higher di/dt values at the end of commutation, which produces higher voltages and calls for higher drop differences. To attain higher drop differences, density must rise at the right brush tip and fall elsewhere. But as seen by the characteristic curve for carbon brushes, the higher the density, the lower the resistance. Due to this falling off of contact resistance toward the right (and rise of resistance to the left, or entering side), the densities will have to become still higher toward the right and lower elsewhere in order to enable the higher drop differences to be attained. Hence, a little more tendency on the part of coil current to delay its reverse means a great deal higher contact density at the brush tip.

The average contact-density curve (curve IX) indicates nearly 70 amperes per square inch at the exit tip. Ordinarily, the brush could stand such a density, could even stand 100 or 120. To see what would raise this density, each factor will be discussed separately.

I. **Speed.**—It is considered that 125 r.p.m. is a very low speed; the normal small motor runs at anything from 600 to 1,800 r.p.m. But merely doubling the speed would very likely, in this case, put the maximum contact density to well over 120 amperes. For higher speeds, other delaying tendencies would have to be reduced, or commutation would be impossible.

II. **Flux.**—Of the two kinds of delaying flux, the self-inductive and mutually inductive flux of the commutated coil is the more important; for this flux first delays the reversal and, second, causes high induced voltages when the rate of change of current

thereby becomes high. It is idle at this time to discuss ways and means of controlling the flux; this problem is more easily settled after the subject of commutation fluxes has been treated in detail.

III. **Turns per Coil.**—Suppose the turns per coil are doubled, the current per conductor at the same time being cut in half to preserve the rated capacity of the machine. The active coil flux would be unchanged, but the voltage induced by it in double the conductors would be doubled. Likewise, the commutated coil flux would be the same, but for a given di/dt, the voltage would double. Such a change would very likely make commutation without destructive sparking impossible.

IV. **Brush Contact Resistance.**—The brush contact drop is already in the neighborhood of 1 volt. To use a higher contact resistance will be to cause higher contact losses; they are already high enough to cut into the efficiency of the machine; and an increase in loss usually would mean an overheated commutator.

V. **Brush Arc.**—The effect of change of brush arc had better be left until later, when it is taken up in detail.

CHAPTER XXVII

COMMUTATION: NATURE OF COMMUTATION FLUXES

The preceding chapter developed the conditions obtaining in a simple case of commutation; however, the fluxes affecting current reversal were assumed, both as to *nature* and *quantity*. In this chapter the *nature* of commutation fluxes will be brought out with a good deal more than the usual degree of care and detail. The next chapter deals with them as to quantity.

Of all electrical subjects, commutation, perhaps, has suffered most because of improper flux analysis. Strange practices have frequently been indulged in; one of the most confusing errors relates to the description of a flux, as to its nature. Thus, writers, in dealing with transformer theory, make a specific description of leakage flux; the same writers, in dealing with commutation, will find what they call the self-inductive flux of the commutated coil and fail to call attention to the fact that the flux used is of the nature of a leakage flux; they will further fail to state that they are neglecting to take into account the *major portion* of the self-inductive flux of the coil.

253. Conditions Governing This Discussion.—Throughout the chapter the same basic case will be discussed. Figures 100 to 102 show a two-pole machine with 12 slots. It has a full-pitch winding, with 1 coil per slot. The brushes are on neutral. The brush arc is one bar pitch, so that a coil commutates in exactly one slot pitch of travel. Figure 100*a* shows one turn of a coil that is just entering commutation, rotation being clockwise. All other coils lie in active paths.

Two coils commutate at the same time and in identical ways. This is due to the fact that the winding is full-pitch, which puts the lower side of one coil in the same slot with the upper side of the other coil.

254. Commutated Coil Inductance.—The true inductance of a coil is due to all of the flux its current sets up. As indicated in Fig. 100*a* the commutated coil sets up flux across its own slots, flux in the air of the interpolar space over the adjacent teeth, and

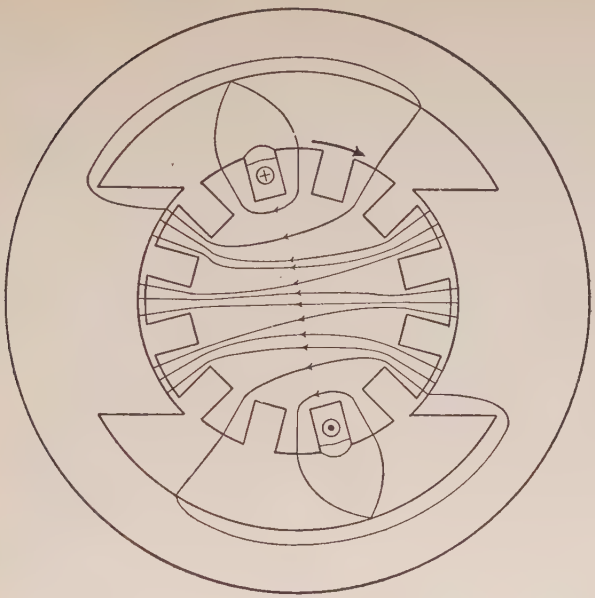


FIG. 100*a*.

FIG. 100*b*.

a very large flux that crosses the main-pole gaps and threads the main magnetic circuit. There is a complex flux also set up by the end-connection parts of the coil, which will be omitted from consideration for a while.

It is *customary* in finding the self-inductance of the coil, to take into account the slot flux and a more or less indefinite amount of flux over the slot in the interpolar space, but to neglect entirely that much greater flux which is the remaining portion following the main magnetic circuit. It is further customary to *say* nothing about this large flux portion, either to admit its existence, or to admit it and then to show why it can rightfully be neglected. It is the purpose of this chapter to credit to the account of the current of the commutated coil all flux that it produces, and then to find what part of this flux can be neglected and why.

Neglecting the slot flux, which enters into any treatment or analysis, and concerning which no question need here be raised, let the flux set up by the one turn (Fig. 100*a*) be represented by the lines with arrows on them. These lines are now 12 in number. As the current of this turn, which is just entering commutation, is still equal to the full conductor current, and as, in one slot pitch of travel when commutation is done, its current will be equal and opposite to the present value, the total *flux-linkage change* will be $2 \times 12 = 24$.

Now it is possible to see what change in linkage the remaining or active coils will produce in the commutated coil.

255. Flux Due to an Active Coil.—In Fig. 100*b* the commutated turn is again shown, beginning commutation; but the purpose of this drawing is to show what flux the turn shown located under the main pole arcs will produce. This turn has the same current as has the commutated turn, at this instant. It has the same total m.m.f., and like the commutated turn and all other turns in this case, its m.m.f. is divided equally. Its current will then set up the same numerical amount of flux in the main air gap for a given tooth pitch under the main gap, as does the commutated turn. In the large interpolar air space, its field conformation will differ somewhat from that of the field of the commutated turn. For the present, however, it may be assumed that even here a given tooth pitch will have the same numerical value of flux as it has due to the commutated turn. Thus, in this drawing and in Fig. 100*a*, 1 line is shown for each tooth pitch in the interpolar space and 3 lines for each in the main gap.

Counting the flux linkages due to this active turn, meaning *linkage with the commutated turn*, shows that there are 4 lines.

256. Flux-linkage Change Due to Motion of Active Coil.—Figure 101*a* is drawn for the time instant when the commutated turn is halfway through commutation; all coils and the armature have traveled half a slot pitch. Tooth 11 has partially left its pole tip; so the reluctance for tooth pitch 11 has increased. But approximately at least, the reduction in flux here is made up by an increase in flux in tooth pitch 2, for tooth 2 has approached its pole tip. For the benefit of those who like to think of flux lines as waving about in the air, it can be said that the flux of tooth 11 has shifted clockwise (*with the rotation, take note*) and that some of it has *shifted ahead of the upper side of the commutated turn.* The same phenomenon occurs at the lower side of the armature.

Counting the lines due to the active turn now linking with the commutated turn shows that there are 6. This is an increase of 2 lines.

Continuing the motion, Fig. 101*b* shows the field of the active turn when the travel has become one whole slot or tooth pitch, and commutation is complete. Tooth 11 flux has changed from 3 lines to 1, while tooth 2 flux has changed from 1 line to 3. There has thus been a continuous shifting forward of flux across the interpolar space, and at least some of this flux has *caught up with and passed the sides of the commutated turn* and has *linked* with this turn.

The flux now linking with the commutated turn is 8 lines. The total change in linkage is $8 - 4 = 4$ lines.

257. One Explanation.—Again considering Fig. 101*b*, suppose that the commutated turn, during commutation, had been lifted out of its slots and hastily put back into the slots just behind the ones in which it was. It would now (Fig. 101*b*) be in the slot between teeth 11 and 12, and in the opposite slot. If it were now there, the active turn flux would, as can be counted, have 10 lines linking with it, which would be an increase of $10 - 4 = 6$ lines. Now (Fig. 100*b*) before commutation started, the tooth pitches just ahead of the sides of the active turn each had 3 lines of flux, all of which were shifted forward as the armature rotated. If the commutated turn had stayed fixed in position, 3 lines would have shifted past each side of the turn, thus causing a change in linkage of 6. It can be said then, tentatively at least, that the

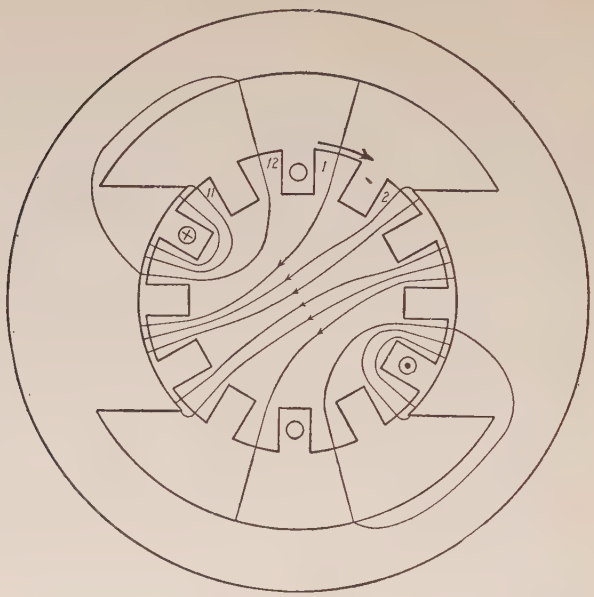


FIG. 101*a*.

FIG. 101*b*.

flux in the two tooth pitches immediately ahead of the sides of an active turn, will, when the armature rotates one tooth pitch ahead, be shifted ahead so as to change the linkage with a *fixed* turn by the sum of these two fluxes.

Now, the commutated turn is *not* fixed, and in the light of the explanation now being used, it has *run away* from the flux shift. If it is put back to where it really would be at the end of commutation, having just considered it as being fixed in space, it will *lose* linkage by the amount of 2 lines, which is exactly the total of flux originally ahead of it in the two tooth pitches its sides were about to travel through.

Instead of completing the analysis, it will now be better to agree upon plus-and-minus considerations and to use a more general system of fluxes. Then the analysis will be finished.

258. Signs.—Let the current in the commutated coil at the beginning of commutation be called positive current, and let any flux linkage like that which this current would itself cause be called positive. Moreover, if positive linkage *increases*, that will be considered as a *positive change* in linkage. As the reader can at once verify, a positive change in linkage will set up a negative voltage in the commutated turn, and *vice versa*. The thing desired is that a peaceable reversal of current shall occur. Positive voltage induced in the commutated turn will tend to maintain its positive current and hence to delay reversal; negative voltage will aid reversal. These two voltages may be considered as delaying or reversing voltages, respectively.

259. General Flux Analysis.—A new beginning is made with Fig. 102*a*, which duplicates the conditions of Fig. 100*b*, except that an active coil turn located next to the interpolar spaces is used instead of one completely under the main gaps. The field is crudely mapped in the upper interpolar space, and it is divided off into sections belonging to individual tooth pitches.

Let the flux of tooth pitch 12 be called A lines, which here are represented by 3 tubes of flux (the slot flux of the active turn being omitted, due to its near constancy). The next tooth pitch, of tooth 1, has about 2.6 tubes of flux; call this flux B, meaning B lines or maxwells. The next, tooth 3, has about 4 tubes; call this flux C lines. The next three pitches are under the pole arc; they would not have equal fluxes actually, due to variation in length of gap and iron saturation; but as will be seen, these may as well,

for ease of analysis, be called equal, each having D lines. The symbols will repeat in the other half of the armature periphery.

Beginning with tooth 1, which is immediately in front of the upper side of the turn which is entering commutation, inward and outward flux can be counted by tooth pitches. The *positive* linkage of the *flux of the active turn* with the *commutated turn* is now: Initial linkage with flux of the active turn $= B + C + 3D - A$. Letting rotation occur through one pitch, the field of the active turn is represented in Fig. 102*b*. It is at once observed that, although the flux field conformation for tooth pitches under the pole arcs is unchanged, there is a change of conformation in the interpolar air spaces. Let A still stand for the flux out of what is now tooth pitch 11, and B for flux of the central tooth, etc. Whereas A was formerly 3 tubes, it is now about 5; B was 2.6, and is now about 3.6; C is unchanged. If the changes in A and B fluxes due to change in conformation are *neglected*, the new linkage is

Final linkage with flux of the active turn $= C + 3D + A - B$. The positive change in linkage is *final* minus *initial*, or

$$\begin{array}{r} C + 3D + A - B \\ B + C + 3D - A \\ \hline -2B \qquad +2A \end{array} \text{——(subtracting).}$$

Reviewing these values for a moment, it is seen that the motion of the active turn caused its flux to produce a *negative* change in linkage with the commutated turn $(-2B)$, which is the total of flux in the two pitches just ahead of the turn when it started to commutate and thereby to pass through these two pitches; and the motion also caused a *positive* change in linkage $(+2A)$, which amounts to the flux that was in the two pitches just ahead of the active turn when it began its motion. An identical action will simultaneously occur for all other active coils, or as we are dealing with turns, there are 4 more turns to consider. They would, together with the turn just treated, give a total positive change in linkage equal to

$$(-2B + 2A) + (-2B + 2C) + (-2B + 2D) + (-2B + 2D) + (-2B + 2D), = 5(-2B) + (2A + 2C + 6D).$$

In the meantime, the commutated turn is doing things of its own. If in Fig. 102*b* the active turn shown be thought of as the

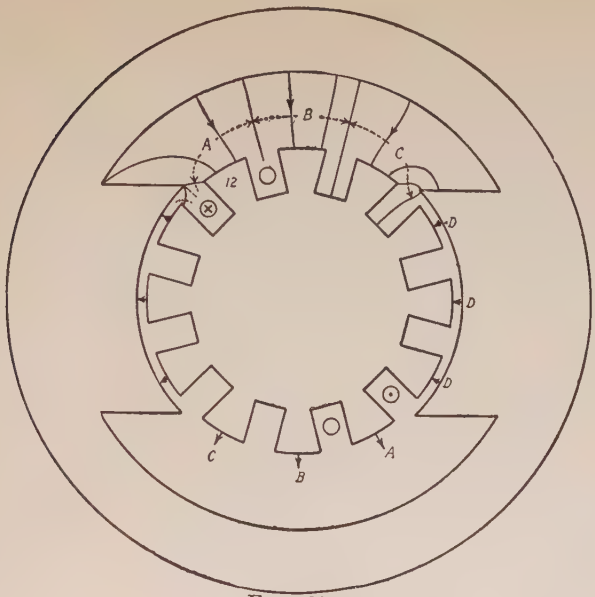


FIG. 102*a*.

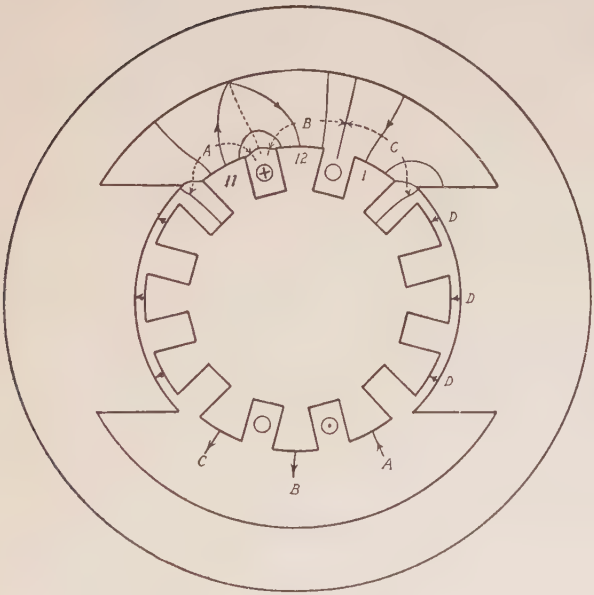


FIG. 102*b*.

commutated turn just beginning commutation, it is seen that its positive linkage of self-induction then is

$$B + C + 3D + A,$$

or

$$B + A + C + 3D \text{ by rearrangement.}$$

The commutated current reverses, so that at the end of commutation its self-inductive linkage will be this value in the negative sense. The total change in linkage due to self-induction is then

$$-2(B + A + C + 3D),$$

or

$$(-2B) - (2A + 2C + 6D).$$

The second term of this statement cancels the second term of the statement of linkage due to all active turns. The net change in linkage is then

$$\begin{aligned} &5(-2B) + (-2B) \\ &= 6(-2B) \end{aligned}$$

which, however, does not include commutated-turn slot flux.

In the *customary* flux analysis, the flux linkage corresponding to the value of $6(-2B)$ is said to be the flux in the commutation zone due to armature reaction; and the self-inductance of the commutated coil is made to include slot flux, plus some flux over the slot from tooth to tooth—just how much to include is usually a hazy point—plus the end-connection flux of the commutated coil.

260. Discussion.—Some readers who are already familiar with commutation theory will wonder why the above articles were included at all, for the same results can be found by inspection. It is just this inspection method which the writer believes has blinded most theorists to a factor which may possibly be of importance. Only by treating the flux behavior in some such way as was done above can it be seen that the change in linkage due to active coils in motion will occur irrespective of what the commutated coil current does.

It has been seen that the active coils cause a large positive linkage change, which means the induction of a reversing voltage; but that when commutation is *complete*, the flux of the commutated coil will have occasioned a large negative linkage

change, such that the two changes *on the whole* cancel out (except for the $-12B$ value noted). Now the active coil change in linkage will take place according to its own fixed laws; the current reversal may occur in any number of different ways. To neglect entirely the A, C, and D components of flux is tacitly to assume that the one linkage change *at all times* copies what the other one does. This is very far from the truth. When the subject of *pulsations* is discussed, these facts will be referred to.

CHAPTER XXVIII

COMMUTATION: QUANTITATIVE FLUX ANALYSIS; BASIC FLUX VALUES; GENERAL FLUX EQUATION; PARTIAL DESIGN OF INTERPOLE

Commutation theory is primarily a matter of flux analysis. Here the sources of commutation fluxes are divided into two distinctive groups: commutated slot currents and all other currents; the various flux fields set up by each source are analyzed in detail, and numerical results are given. This chapter is confined to the development of the flux analysis for a basis type of symmetrical machine. Variations from the basic type follow in the next chapter.

261. The Basic Symmetrical Machine.—This theory will be developed with respect to a symmetrical armature with the simplest winding and commutating in the simplest way. Figure 103 shows such a unit. It has an integral number of slot pitches in a pole pitch, so that its coils are full pitch. It has a lap winding, double-layer, one coil per slot, one turn per coil, two conductors per slot. To attain the simplest commutation phenomena, the brush arc is made equal to one commutator bar pitch. The maximum of symmetry would be attained if the machine had no interpoles, or if it had a full number; but as many machines have half the full number of interpoles, the drawing represents a four-pole unit with two interpoles.

262. Average versus Instantaneous Commutation Voltage Treatment.—The aim in handling the problem of commutation is to secure constant contact current density. From the standpoint of the *study* of commutation, comprehensive knowledge of the subject cannot be attained without paying considerable attention to actual instantaneous values of delaying and reversing voltages induced in the commutated coils while undergoing commutation. But the designer, when putting a machine together, must deal with averages; his aim is to introduce into the commutated coil as much average reversing voltage as there is average delaying voltage.

It will be well here to develop the theory with respect to the average methods and then later enter upon studies of instantaneous values.

263. Causes of Commutation Fluxes.—The interpole winding, if interpoles are used, is of course the source of the reversing flux. The delaying fluxes are in general due to active armature coil currents and commutated coil currents. In the basic symmetrical machine a commutated coil side travels one slot pitch in undergoing commutation. In Fig. 103, both conductors in slot 1 are just beginning, and both in slot 2 are just ending commutation.

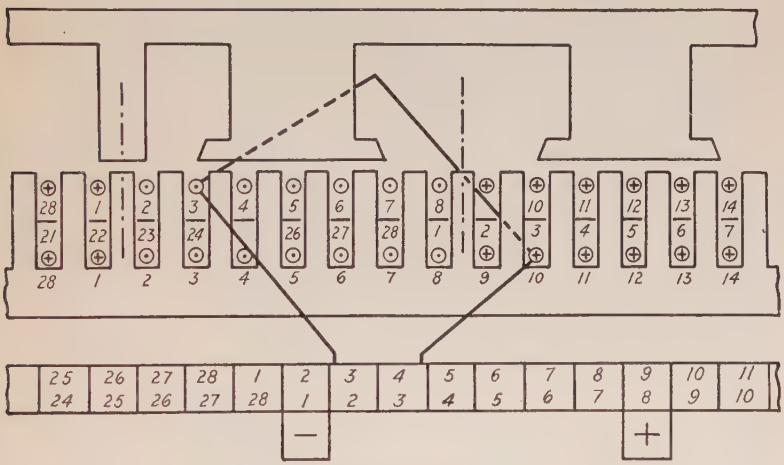


FIG. 103.—4-pole, full-pitch, lap-wound armature.

These two slots, and all others like them, which are on either side of the tooth at the neutral point, will be called the **commutated slots**; and the current in either of these slots is denoted by the symbol I_{cs}. In the basic symmetrical machine, these slots, for the positions shown in Fig. 103, carry the same current as any other slot; but in machines with more than one coil per slot, they will carry less, as is later shown.

All other slots will be called **active slots,** and their current is denoted by the symbol I_s.

By looking over the figures devoted to this chapter, the principal reason for grouping the armature currents in this way becomes evident, at least evident to one who is familiar with field-mapping processes. The commutation zone contains the fluxes of interest

(except the end-connection fields affecting commutation). The commutated slot currents lie within the flux paths; the active slot currents are all outside of the zone; hence, the currents within the region will have, in general, very different flux conformations from those due to the active currents.

264. Usage.—The total change of linkage of any kind of flux linking (or unlinking) with one commutated turn, during commutation, will be denoted by Φ with a proper subscript; and the basic unit of the same kind of flux, usually, in terms of maxwells per axial inch per ampere-turn, by ϕ with the same subscript.

265. Slot Flux.—The flux field due to a single conductor has been developed earlier in the book. As pointed out there, it can be assumed with good accuracy that the conductor sets up no flux below it and a rectilinear field across the slot above it. This rectilinear field would bulge outward near the top of the slot, but the assumption will be carried only to within one-fourth of the slot width of the top. The actual conductor grouping is seen in the lower sketch (Fig. 104). It is then assumed that the current is solidly distributed as in the middle sketch. Under the assumptions, the upper curve in the graph represents the slot densities. If B_s is the density at the top of the current mass, which is taken to be $s/4$ from the slot top,

$$B_s = \frac{3.2 I_{cs}}{s}.$$

where s is the slot width in inches.

But all of this flux is not effective in that it cannot all induce voltage in the slot conductors. An element of flux can induce voltage only in the conductors below it. For example, halfway up the slot, an element of flux affects only half the conductors; hence the density there may as well be cut in half and be then called the *effective* density. The lower curve in the graph shows *effective* densities. It is parabolic; hence the flux represented by the area beneath it is but two-thirds of the total actual flux. The *actual* flux per axial inch is (with d = slot depth)

$$\frac{B_s d}{2},$$

whereas the *effective* flux per axial inch is

$$\phi_s = \frac{B_s d}{3}.$$

Note that the expression $B_s d/3$ holds for the entire slot depth d, instead of $d - s/4$, as it apparently should be from having made the assumption that the upper limit of slot current is at a distance of $s/4$ from the slot top. The $s/4$ limit is introduced as a means of *indicating* that, at about this point, not only does

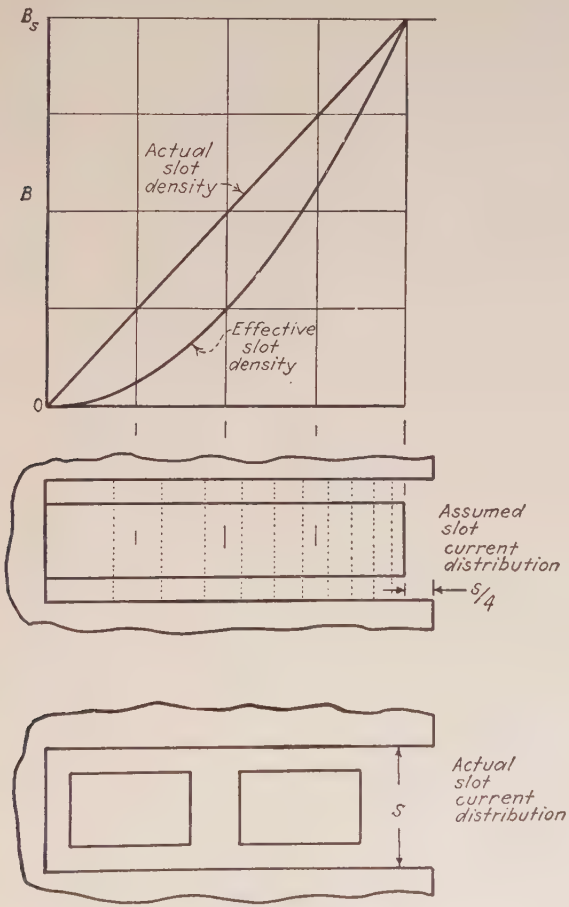


FIG. 104.—Slot flux.

slot current stop about there, but that this is approximately where the somewhat regular slot flux ends and where the irregular flux field at and above the mouth of the slot begins. These two flux fields, although being but parts of one field, are nevertheless dissimilar parts and must be separated for ease of evaluation.

The upper part—in the mouth of the slot and above it—will be evaluated as if it started from the $s/4$ point in the slot and reached upward. Now, by using for the slot flux a depth d, the slot flux is made arbitrarily a little high. This is in favor of ease of calculation and of safety.

The coil does not occupy one slot but parts of two slots; however, the above relations hold, for the upper side of the coil gets the effect of the upper half of the slot flux in one slot, and its lower side lies in the lower half of the slot flux in the next commutating zone.

A turn begins commutation linking with a total slot flux of

$$2L_a\phi_s$$

and ends with the negative of this amount. The total change in linkage is then

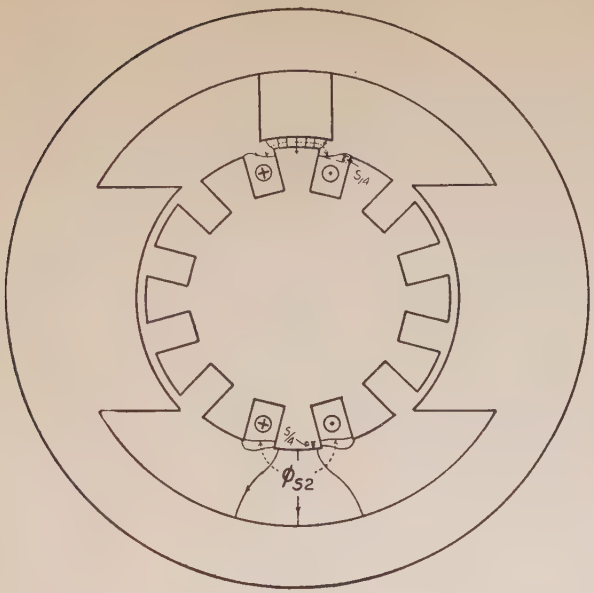
$$\Phi_s = 4L_a\phi.$$

L_a is the length of the armature, which includes ventilating ducts. The flux value is high; and a deduction for the weaker flux in the ducts can be made, if it seems desirable.

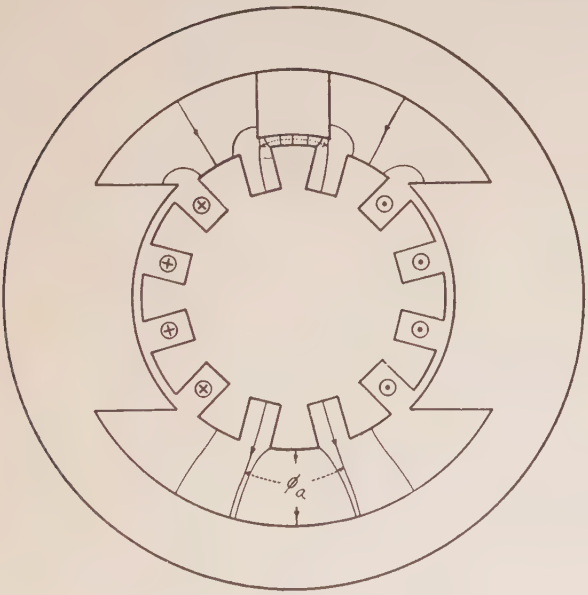
The above change in linkage is negative; so the average voltage induced by it will be a delaying voltage.

266. Flux above Commutated Slots Due to Commutated Slot Current. I. *At the Face of an Interpole.*—The flux field in the commutating zone due to I_{cs} currents is shown at the upper side of the machine of Fig. 105*a*. This type of field has been treated in Art. 60, where it was shown that the m.m.f. of a pair of conductors acting under the conditions present would divide extremely unequally; that practically all of the m.m.f. would be thrown across the air path in the immediate locality of the pair, and very little would be left to act over the return path, which has a relatively large section. The total m.m.f. in ampere-turns is equal to I_{cs} in amperes. Practically, the central tooth is a body at one potential, and the teeth on either side and the interpole base are three bodies at one other potential, the potential difference being the total ampere-turns of the pair.

Selecting various representative values of the ratios t/s and g/t, the writer mapped a goodly number of these two-dimensional fields, thereby securing data for plotting the ϕ_{s1} curves in Fig. 106. The ratio g/t is used as a parameter; the ratio t/s scales the abscissa; and the ordinates give the maxwells per axial inch per



(*a*) Commutated slot current fluxes.



(*b*) Active slot current fluxes.

FIG. 105.—Commutation flux.

ampere-turn, for the whole tooth pitch. Values of ϕ_{s1} for any usual case can be interpolated for.

Let the total axial length of interpole be L_i, this being the *total length covering the two sides of the commutated turn;* if only half the full number of interpoles is used, L_i is the axial length of one interpole; if the full number is used, it is twice the length

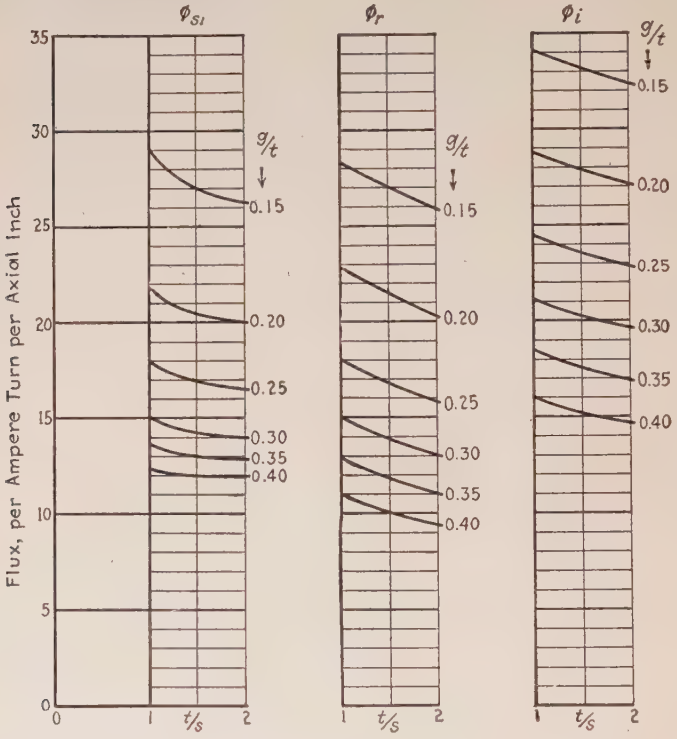


FIG. 106.—Basic values, commutation fluxes. (Interpole arc = $0.9(s + t)$).

of one interpole. Then the numerical total of the negative flux-linkage change during the time of commutation is

$$\Phi_{s1} = L_i I_{cs} \phi_{s1}.$$

NOTE.—The curves were obtained for field maps subject to two restricting conditions: (*a*) the field was assumed to exist in the top region of the commutated slots, it being extended into the slots to a depth of $s/4$ as measured along the inner slot sides, thus joining this field onto the field of slot flux; (*b*) the interpole base arc or width was $0.9(s + t)$.

II. *In the Interpolar Space Not Covered by an Interpole.*—The conditions are shown at the lower commutating zone (Fig.

105*a*). This field conformation will cover the whole L_a length if there is no interpolar here; or it will approximately cover that portion of it left exposed due to the use of a short interpole. Again, the m.m.f. of the slot current pair is nearly all exerted over the local region. It happens that for all ordinary proportions, the flux per axial inch per ampere-turn for the slot pitch is roughly constant, it being

$$\phi_{s2} = 6.$$

The total flux-linkage change for a commutated turn is negative; its numerical value is

$$\Phi_{s2} = (2L_a - L_i)I_{cs} \times 6.$$

NOTE.—The same restrictive condition prevails here, in using a value of $\phi_{s2} = 6$, as governed the preceding flux values: This field begins where the slot flux field arbitrarily was cut off—at a distance of $s/4$ from the top of the slot.

267. End-connection Flux Due to Commutated Currents. I. *In the Straight-extension Region.*—Refer to Fig. 107. The end connections of the symmetrical unit of Fig. 103 are shown in Fig. 107*a*. The coils extend straight out from the slots for a distance of from $\frac{3}{8}$ to 1 inch or more, depending on the machine voltage; this enables the slot insulation to be extended to guard against breakdown here. This extension length is denoted by X. The coil sides extending from the commutated slots are shown in section in Fig. 107*b*.

The current in each side of this pair is I_{cs}. In the usual design, the average tooth width is about equal to the width of the slot. To simplify matters, it is assumed that the pair of coil sides lie parallel to each other, and that their separation is equal to their individual insulated widths, which is equal to s, the slot width. We are interested here in finding the average flux passing between a pair of conductors. The problem could be solved by the route of finding mathematically the self-inductance of such a pair of groups of conductors or by field-mapping methods. The author used the latter method in getting the data from which the curve of ϕ_{ec} in Fig. 107*d* was plotted. It was assumed that the field due to a pair of conductors would be two-dimensional; this is not true, due to the fact that the region is bounded partly by the nearby armature iron. It is not felt, however, that the error thereby introduced is serious.

The curve gives ϕ_{ec} versus the ratio a/b, where a is the height of the single entire bundle and b is its width, the bundles being

separated by a distance equal to b which actually is s as referred to the machine. As affecting one commutated turn, the negative change in linkage would be

$$4XI_{cs}\phi_{ec},$$

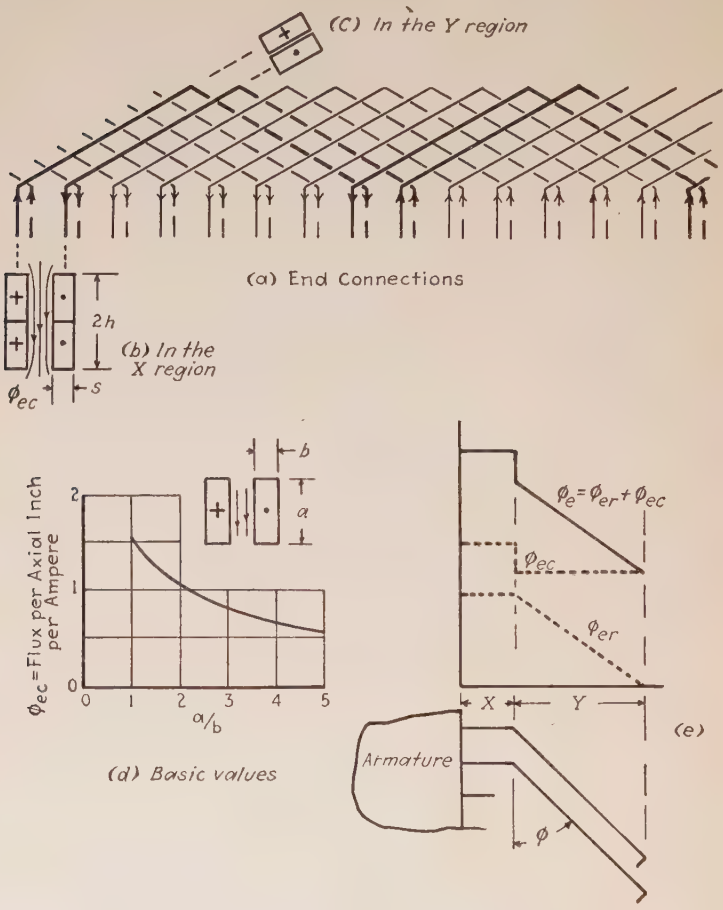


Fig. 107.—End-connection commutation fluxes.

but this form of it will not be used, as a different way of including this linkage will soon be outlined.

II. *In the Remaining Region.*—As shown in Figs. 107*a* and 107*c*, the top coil sides branch off at the angle Θ (Fig. 107*e*) to form a pair by themselves, and the lower sides branch in the opposite direction. The angle Θ varies from about 25 to 45

degrees, and the space between the pair varies from zero for small machines to perhaps $\frac{3}{8}$ inch in the largest. It might seem that it would be necessary to produce several new ϕ_{ec} curves, each drawn for a different spacing. The smaller the spacing, the less ϕ_{ec} becomes; but this vexing difficulty will be dodged by using the projected or axial length Y, rather than the actual length of the end connection; that is, if the coil sides ran straight out for a length $Y = \sin \Theta$, the length would be less but the spacing greater. The writer finds that the two tendencies do balance each other quite well; hence, the single pair may as well be thought of as extending straight out at $\Theta = 90$ degrees. This enables the one curve of ϕ_{ec} to be used. Note, however, that if the ratio a/b was, for the straight-extension region, equal to $2h/s$, where h is the height of a coil side, it is now equal to h/s; and further note that the current of the pair is $I_{cs}/2$. The axial length is called Y. The total length affecting one commutated turn is $4Y$.

Instead of stating the total flux, the flux per inch is plotted in the upper dotted curve (Fig. 107*e*). Check the shape of this dotted curve at once. Let the dimensions h and s be 0.6 and 0.3 inch, respectively, and let I_{cs} be 200. For the X-length, the ratio a/b is $2h/s = 4$. From the curve, the flux of the slot pitch per axial inch per ampere-turn is 0.7 maxwell. The total flux per inch along X is $0.7 \times 200 = 140$. For the Y-length, the ratio a/b is $h/s = 2$, for which the curve gives about 1.1 maxwells; the total flux per inch is $1.1 \times 100 = 110$. These values will give a curve like that plotted over the distance $X + Y$.

268. Flux Due to Active Slot Currents. I. *Interpole Present.*—The upper commutating zone in Fig. 105*b* shows this field. All of the active turns are sufficiently far removed from this locality to make it possible to say, practically, that within this zone they will have the same field conformation. The m.m.f. is equal to half the total m.m.f. of the active turns (in a two-pole case), provided the magnetic circuit is symmetrical; if only half the full number of interpoles is present, the circuit is asymmetrical, and the m.m.f. division is not 1-to-1. For simplicity of treatment, it will be assumed that to divide the armature m.m.f. equally between the commutating zones will give sufficient accuracy.

The author mapped enough fields for typical dimension ratios to plot the curves for ϕ_r (Fig. 106). The interpole arc was 0.9

$(s + t)$. Let N be the slot pitches in a pole pitch. Then the active slots in a pole pitch, taking out the commutated slots, is N-2. The current in each active slot is I_s. The ampere-turns acting over the path will be $I_s(N\text{-}2)/2$. The numerical values of the total linkage change of a commutated turn, using L_i as the *total* axial interpole length over the two sides of the turn, is

$$\Phi_r = L_i \left[\frac{I_s(N\text{-}2)}{2} \right] \phi_r,$$

the linkage change being negative and, therefore, delaying.

This equation will not be used as it is. The ultimate equation by which interpole length is solved for can be compressed by recognizing that this flux, so far as the commutating zone goes, is just like the reversing flux set up by the interpole itself.

II. *Interpole Absent.*—The lower zone (Fig. 105*b*) shows this field. The m.m.f. is the same as that found just above. For all usual dimensions, the flux of the slot pitch per axial inch per ampere-turn is about

$$\Phi_a = 2.$$

The axial inches are $2L_a - L_i$. The total delaying flux-linkage change for one commutated turn is

$$\Phi_a = (2L_a - L_j) \left[\frac{I_s(N\text{-}2)}{2} \right] \phi_a.$$

269. End-connection Flux Due to Active Currents.—Here is a very difficult flux field to work with, as it is three-dimensional. Refer to Fig. 108. The beginning is made by studying the two-dimensional field set up by the turn shown. Near the center plane of the machine (a plane normal to the axis), the field would be almost two-dimensional. The end view shows the equipotential surfaces as indicated by dotted lines. The equipotential surface S_2 seems to rise from the surface of one conductor, follow the air-path spaces, and drop to the surface of the other conductor. All equipotential surfaces are closed by the conductor upon which they rest. Turning now to the side view (section A-A), the surface S_2 floats out into the end space and bends down to rest on the upper surface of the end connection. It will hug the corner of the armature core end, thus raising densities in that region. One line of flux passing through the point P is shown.

In order to find the probable values of the flux per axial inch coming from the commutated slot pitch in the space near the end of the armature, the writer made various studies for various

cases. Without the expenditure of a great deal of labor (or without resorting to experimental investigation, for which there was not the opportunity), field values of high accuracy cannot be attained. One way to work up this sort of field is to resort to its

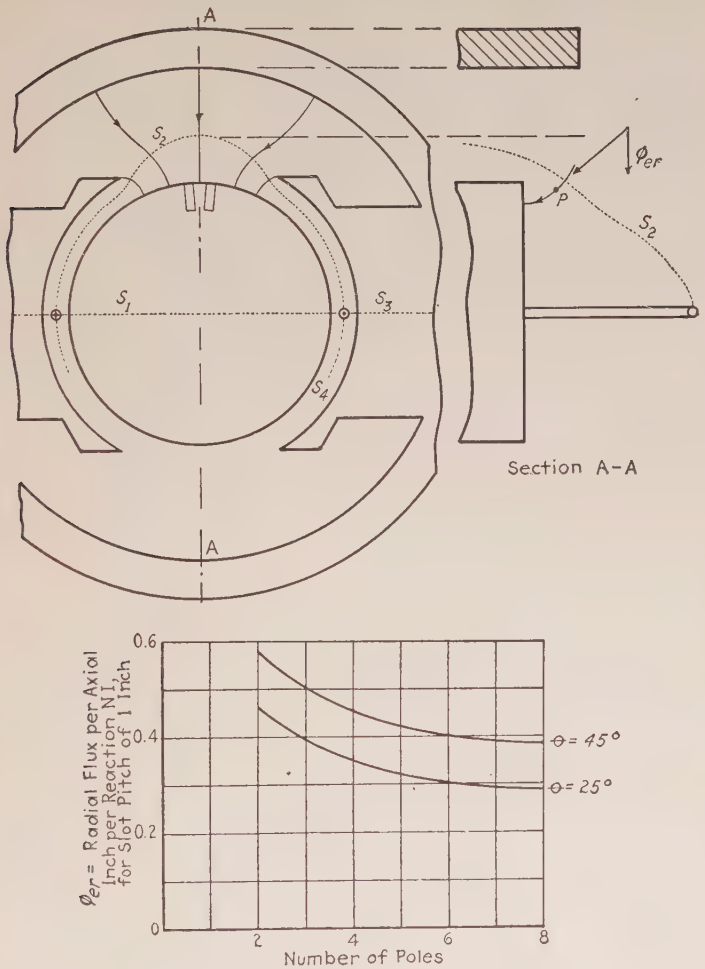


FIG. 108.—End-connection flux of active turns.

representation by the use of isometric perspective; the curves given are based on simpler methods, being tempered considerably by the writer's judgment. No claim of high accuracy is entered, but it is believed that the curves are correct to within 40 per cent, and that they are too high rather than too low.

Note that although the presence of the armature iron will cause high densities from the end of the central tooth, at the same time the flux here has a strong axial component. In using axial inches for lengths, it is the radial component which counts and which, therefore, has been given in the curves.

If the numerical values offered by these two curves cannot be defended, their shapes at least can be. The curves rise as the poles are decreased in number. This must be true, for the equipotential surface S_2 thus acquires a relatively sharper curvature, making the flux from the end of the tooth roughly approach a radial line conformation. As to the effect of the change of end-connection angle Θ, the smaller the angle, the closer the end connection will hug the end of the core, and the more sharply will the surface S_2 be pinched down around the corner to get to the end connection.

As for variation in this radial flux along the end connection of the rest of the commutated turn, every theoretical analysis, as well as test, confirms the idea that the flux falls nearly to zero in a fairly regular manner, as the outer end of the connections is approached.

The flux per axial inch per ampere-turn is then plotted in the lower dotted curve (Fig. 107*e*). The maximum value, over the distance X, is obtained by

Radial flux per axial inch $= [I_s(N\text{-}2)/2]\phi_{er}$.

The remainder of the curve, after this is plotted as shown, is obtained by drawing a straight line to zero at the end of Y.

It is the writer's belief that the formulas usually used for computing end-connection flux are untrustworthy, for the reason that the flux analyses upon which they are based are not sound. It is hoped that the analysis given here, although admittedly imperfect, will nevertheless indicate the paths for study and experimentation leading to better formulas.

It should be noted that this work has been carried out with regard to the condition that the interpole is absent. If now an interpole is placed in the interpolar space and is allowed to come clear to the end of the armature, it will cause an increase in the reaction end-connection flux in the region near the end of the core; but this increase will in all probability more than be compensated for by the fringing flux of the interpole itself.

Into the graph of Fig. 107*e* is now plotted the flux per axial inch due (*a*) to commutated end connections and (*b*) to active

end connections. These two curves are added vertically to secure the solid-line curve of totals; it is now expedient to estimate in any approximate manner the average height of this curve; the total change in linkage affecting one commutated turn is then

$$\Phi e = 4(X + Y)(\text{average flux per axial inch}).$$

Let average flux per axial inch be ϕ_e. Then

$$\Phi_e = 4(X + Y)\phi_e.$$

All other basic flux values as denoted by lower-case phi (ϕ) are "per axial inch per ampere-turn;" ϕ_e and ϕ_s are the only exceptions to the uniform practice.

If the end connections are held in by steel banding wires reacting against an iron support ring, it is probable that the flux per axial inch *in that region* should be doubled.

270. Main-pole Flux in the Interpolar Space.—The fringing flux of the main poles is extremely weak in the commutating zone where there is an interpole. If there is no interpole, and if the brushes are on neutral, this fringing flux has an effect that is usually negligible.

271. The Reversing Flux of the Interpole.—All of the fluxes heretofore analyzed for which values have been derived are delaying fluxes, in that they give a negative linkage change, which induces a positive or current-sustaining voltage in the commutated turn (the beginning current being always considered positive). The brushes may be shifted so as to move the commutating zone into a strong enough fringe flux from a proper main pole, thereby securing in many machines good commutation at all loads with a fixed brush shift; or good commutation for all reasonable loads, if brush shift is varied with load. But in practically every case, except for small machines, this expedient has gone out of date, for several reasons; the reasons are so evident or so well known that no more attention will be paid in this book to the non-interpole machine.

The interpole is introduced to give a proper reversing flux at all loads. All of the delaying fluxes are proportional to the armature current. The reversing flux must also be varied in like manner. The interpole windings are therefore placed in series with the armature. The reversing flux, over the slot pitch of the commutating zone, has the same field map as has the armature reaction flux due to active currents. The upper zone (Fig. 105*b*) shows the field conformation.

The reversing flux must be opposite to the armature reaction flux. Since, in the region immediately in the neighborhood of the interpole face there are two like but opposite fields, instead of computing the two fluxes separately, it is as well to make use of the net m.m.f. and thereby find the net flux.

The armature reaction m.m.f. here is M_r, found as indicated above. The interpole winding must have, first, a component of m.m.f. equal and opposite to M_r; and in addition thereto, a component which may be called M_{ig} which will be the net interpole gap m.m.f. setting up the net interpole gap (reversing) flux. This net flux may be called ϕ_r, the subscript signifying *resultant.* Then, in accordance with these considerations, total reversing flux linkage change of one commutated turn is positive and given by

$$\Phi_r = M_{ig} L_i \phi_r.$$

272. General Commutation Flux Equation; Solution for L_i.—Suppose the proportions and dimensions of slot, teeth, interpole face, and interpole gap are known or selected; then every basic or unit value of every commutation flux becomes fixed. Also, let M_{ig}, the net interpole gap ampere-turns, be selected in some way. It is presumed of course that I_s and I_{cs} are also known. The question then is, What should L_i amount to? The criterion, in this *average* treatment, must be that the total delaying flux-linkage change must be counteracted by the introduction of an equal amount of reversing flux-linkage change. This is stated as follows:

$$\Phi_r = \Phi_s + \Phi_{s1} + \Phi_{s2} + \Phi_a + \Phi_e.$$

If each term in the above equation is fully written out, it will be seen that the only unknown in it is L_i.

As L_i in this treatment was allowed to stand for the total length of interpole or interpoles found over the *two* sides of the commutated turn, it follows that (*a*) if L_i comes out greater than L_a, the design calls for a full number of interpoles, each half as long as L_i, or (*b*) if L_i is less than L_a, half the interpoles may be omitted.

273. Total Net Flux from Lower End of Interpole: Interpole Density.—Refer to Fig. 113*a* in the next chapter. Between the flux lines P and Q (which, note, cover two slot pitches on the armature) would be included nearly all of the flux leaving the lower end of the interpole core. The basic value of this flux will

be denoted by ϕ_i, this being the flux per axial inch per ampere-turn. $M_{ig}\phi_i$ would then give the flux per axial inch (aside from leakage) found in the core at the *P-Q* level. To make sure that the core here will not be unduly saturated for any reasonable load or overload, this flux per inch and, therefore, the density here must be ascertained or else fixed upon in advance.

The ϕ_i curves of Fig. 106 were drawn by means of the same series of field maps from which the ϕ_r curves were produced; by means of them it is easy to find the ϕ_s value, once the usual dimensional ratios are determined.

274. Partial Design of the Interpole.—Before solving the general flux equation given above, by which solution L_i would be found, it is necessary to fix certain items.

To insure that the interpole flux will be nearly proportional to the reversing fluxes, which in turn vary directly with the load, the density in the core at the *P-Q* level (Fig. 113*a*) must be kept reasonably low. A good value is 30,000. This density, and the distance *PQ* (equal ordinarily to $0.9(s + t)$) will fix the flux per axial inch here, and this value will be equal to $M_{ig}\phi_i$.

To avoid unequal interpole fluxes due to unequal gaps, the interpole gap must be reasonably high, at least equal to the average main-pole gap; and furthermore, M_{ig} should preferably be about 40 per cent or more of M_r, to give stability of interpole flux and further get away from inequality between various interpole strengths.

As M_r is known, M_{ig} can be selected proportionately; and with $M_{ig}\phi_i$ known, ϕ_i becomes known.

Use is now made of the ϕ_i curves of Fig. 106 in an indirect way. t/s is known, and so is ϕ_i; then g/t can be found by interpolation, whence g is found. If the gap length thus found is deemed sufficient for other reasons mentioned above, well and good. If not, it is arbitrarily increased and previous values corrected accordingly.

These considerations once attended to, everything is ready for the writing of the general flux equation and the solution for L_i.

In the larger machines it is usually necessary to thicken the interpole core above the *P-Q* level to keep the additional leakage flux from saturating the core.

CHAPTER XXIX

COMMUTATION: VARIATIONS FROM BASIC SYMMETRICAL MACHINE; COMMUTATION TYPES; WAVE-WINDING COMMUTATION; PULSATIONS; DESIGN OF INTERPOLE

The basic symmetrical machine, upon which the treatment of commutation has been developed, would seldom be met with in practice. Certain variations from the basic machine are now investigated, and several types of commutation are outlined.

275. Usage.—For the purposes of writing and speaking about brush arcs and interpole base arcs, it is very convenient to introduce a couple of new descriptive terms. Our standard reference machine is one in which the commutation of a coil occurs while one side of it travels through one slot pitch; this, because the brush arc is made the equivalent of one slot pitch in value. *Unit brush arc*, therefore, stands for and describes the arc of such a brush. For one coil per slot, there is one bar per slot, and the brush would cover one bar pitch to have unit brush arc; for two coils per slot, the brush covers two bar pitches to have unit arc, etc.

Unit interpole arc likewise will stand for the arc of an interpole base that is just wide enough effectively to cover one slot pitch. A word here on proper interpole arcs will be in place. The interpole base should be so designed that the reluctance of the major portion of the flux between it and the passing armature teeth shall not vary through a very wide range. A popular rule for a one-slot-pitch interpole is to make the arc less than the slot pitch by twice the length of the interpole gap. The writer investigated this proposition by drawing many maps of the flux field, being careful in the study to cover the usual extreme ranges of the ratios (s/t) and (g/t). It was found that the above rule does not work out; that the arc should be, for better constancy of gap reluctance, equal to $0.9(s + t)$. The next greater arc available, which again will avoid undue variation in interpole flux due to change of reluctance as the teeth pass underneath, is a two-unit

arc; and this should be about $1.9(s + t)$. Any other arcs than these, that is, differing appreciably from them, should not be used. All this is under the assumption that the interpole base is flat, with square corners; for any other shape, field mapping alone will tell what dimensions to use, unless experiment is resorted to.

276. Index Numbers.—It will also be very convenient to develop a system of index numbers for easily and rapidly indicating the state of currents in coils undergoing commutation. A given machine might have a path current of 40 amperes. If a coil has straight-line commutation, and if the coil currents are set down in order for the time when the coil begins commutation, when it is one-quarter through, then halfway, then three-quarters, and lastly, just finished, the series would be

$$40 \quad 20 \quad 0 \quad -20 \quad -40$$

For immediate purposes, there is no need to work with actual amperes; representative proportionate numbers—index numbers—will do better. The change is just as well indicated by

$$2 \quad 1 \quad 0 \quad -1 \quad -2$$

and if needed, the current values before and after commutation can be added, such as

$$2 \quad 2 \quad 2 \quad 2 \quad 1 \quad 0 \quad -1 \quad -2 \quad -2 \quad -2 \quad -2 \quad -2$$

Next add to the invention by the use of a bar or line to indicate the beginning and the ending of commutation, as

$$2 \quad 2 \quad 2 \quad 1 \quad 0 \quad -1 \quad -2 \quad -2 \quad -2$$

If the winding is symmetrical full pitch, one coil per slot, each number in the series can stand for the *entire current* in its slot. Under the same conditions, but with *two* coils per slot, the series becomes

$$2 \quad 2 \quad 2 \quad 1 \quad 1 \quad 0 \quad 0 \quad -1 \quad -1 \quad -2 \quad -2 \quad -2$$

where each number stands for all the current of its kind, varying as it does in its vertically divided half of the slot.

277. Relation of Commutated Slot Current to Active Slot Current.—Harking back to the basic symmetrical machine, its series is, using the simplest index numbers possible,

$$1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1$$

The commutated coil begins with full slot current I_s and ends with full slot current $-I_s$; that is, I_{cs} and I_s are in this case identical.

Now consider the series

2 2 2 0 0 −2 −2 −2

One coil begins with a slot current of 4 and ends with −2; the other one begins with 2 and ends with −4. In both cases the *total change* of *slot current* is 6; whereas, if at both ends of commutation the *full* slot current were present, the total change would have been 8. The average I_{cs} is then 0.75, or six-eighths, times I_s in this case, and all delaying fluxes set up by commutated slot currents are affected in proportion.

The *coefficient* by which to multiply I_s to obtain the average I_{cs} will be designated k_{cs}. In the present case, its value is 0.75.

278. Types of Commutation.—The various commutation factors as found in the *basic symmetrical machine* will constitute what may be called **Type One Commutation.** This machine had unit brush and interpole arcs, and 1 coil per slot. The variations from this standard involve changes in any or all of these conditioning factors. The cases will be developed in which the coils per slot are one or two; the brush arc is one-half, one, or two units; and the interpole arc is one or two units.

Underlying all of the work are the conditions that the machines are still symmetrical as to having an integral number of slot pitches per pole, that they are lap wound, and that the winding is full pitch.

Table IX includes eight types. In using this table, it should be understood that the index numbers have here been arranged vertically instead of horizontally, and that in all cases, *in moving from one group of numbers to the next,* the commutator has traveled by the amount of *one bar pitch.*

Furthermore, the various fluxes set down are not total fluxes; they are, as their symbols indicate, usually of the nature of flux per axial inch per ampere-turn. Type One occupies the first data column, and all other types are rated in terms of its basic values. If, in some other type, there is a blank space opposite ϕ_{s2}, for example, it means that in this type the flux per axial inch per ampere-turn of this kind is the same as for Type One, is found in the same way, and enters into the general equation for

finding L_i in its usual way. If a remark is found, such as "twice," it means that in effect this flux is approximately doubled. These tabulated types, and how their various factors were arrived at, will now be discussed.

TABLE IX.—TYPES OF COMMUTATION

Type		One	Two	Three	Four	Five	Six	Seven	Eight
Coils per slot		1	1	1	1	2	2	2	2
Interpole arc, units		1	2	2	1	1	2	2	1
Brush arc, units		1	1	2	½	1	1	2	½
Index numbers						2	2	2	
						2	2	2	
		1	1	1		2	2	2	
						2	2	1	
		1	1	1					
						2	2	1	
		1	1	0		0	0	0	
		−1	−1	−1		0	0	0	
						−2	−2	−1	
		−1	−1	−1					
						−2	−2	−1	
		−1	−1	−1		−2	−2	−2	
						−2	−2	−2	
						−2	−2	−2	
Commutated slot current factors	k_{cs}	1.0	1.0	1.0	1.0	0.75	0.75	0.87	1.0
	ϕ_s								
	ϕ_{s1}	...	...	twice	...	...	...	twice	half
	ϕ_{s2}	...	...	...	...	...	...	...	half
	ϕ_{ec}	...	...	twice	...	...	...	twice	half
Active slot current factors	ϕ_a	...	...	twice	...	...	...	twice	half
	ϕ_{er}	...	...	twice	...	...	...	twice	half
	M_r	...	...	less	...	...	...	less	
Interpole factors	ϕ_r	...	...	twice	less	...	...	twice	half
	ϕ_i	...	twice	twice	...	...	twice	twice	
	L_i	...	...	less	more	less	less	less	

Type One.—This type has been fully discussed in the development of the commutation treatment, which is based upon it.

Type Two.—The two-unit interpole arc does not affect to any great extent any of the basic fluxes concerned directly with com-

mutation. Therefore, for the same interpole face density, the length L_i would be the same as for Type One. The main feature is that ϕ_i is about doubled, which is undesirable, adding as it does to the yoke saturation effect and thereby unwarrantedly making greater the mutual reduction effect of main and interpole fluxes.

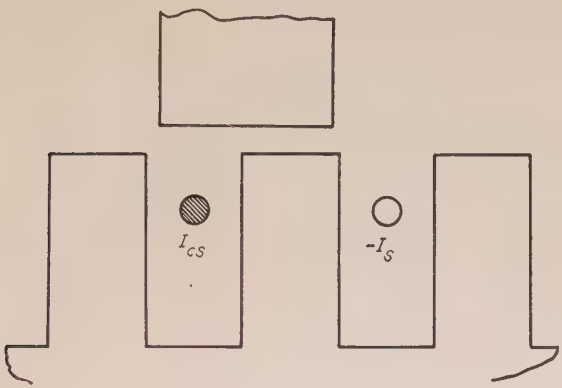
The wider interpole uses more material in pole and winding; it blocks off interpolar ventilation space; and increases leakage of all poles.

Type Three.—It is easy to see that ϕ_s is the same as in Type One. ϕ_{s1} is about doubled, for the two commutated slots are now two slot pitches apart, and the principal reluctance offered to this flux is about cut in half. As can be verified by a simple field study, ϕ_{s2} is increased but hardly enough to allow for it, this being a small flux. In studying ϕ_{ec}, it must be remembered that the pair of currents are separated by two slot pitches instead of one. The curve in Fig. 107*d* was drawn for the one-slot-pitch separation. Investigation by the writer disclosed the fact that doubling the spacing, for all practical commutation purposes, about doubled the flux. ϕ_a and ϕ_{er} are about doubled. M_r, the armature-reaction ampere-turns at the commutating zone, is reduced, for the zone is two slot pitches wide. $M_r = I_s (N - 3)/2$, instead of $I_s(N - 2)/2$. ϕ_r, still defining it as the flux per ampere-turn for *one slot pitch*, is about the same; but there are now two slot pitches passed through; so twice the regular value must be used. All basic fluxes are doubled except the slot flux; due to its remaining constant, less L_i will be required. If slot flux ϕ_s is a major quantity, the reduction in L_i might more than compensate for the disadvantages caused by a two-unit interpole arc.

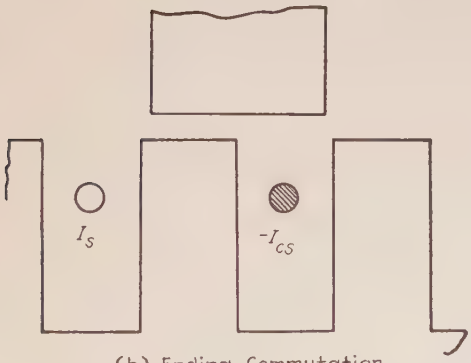
Type Four.—The flux analysis used here must be fully explained. Refer to Fig. 109. The commutated conductor is shaded. At beginning of commutation, let its slot current I_{cs} (which in this case is equal to I_s) be paired off with the current $-I_s$ of the advance slot. Then the conductor enters commutation surrounded by a full complement of every kind of flux, except reversing flux ϕ_r. At the end of commutation, pair its current off with that of the current of the succeeding slot; by this analysis, the conductor is made to end with a full complement of every flux except ϕ_r. These statements should be modified by noting that fluxes in the interpole gap, such as ϕ_{s1},

have not a symmetrical path; but their values will be approximately unaffected.

It would seem off hand that, as the travel is only half a slot pitch, the reversing flux ϕ_r would be cut in half. This is far



(a) Beginning Commutation

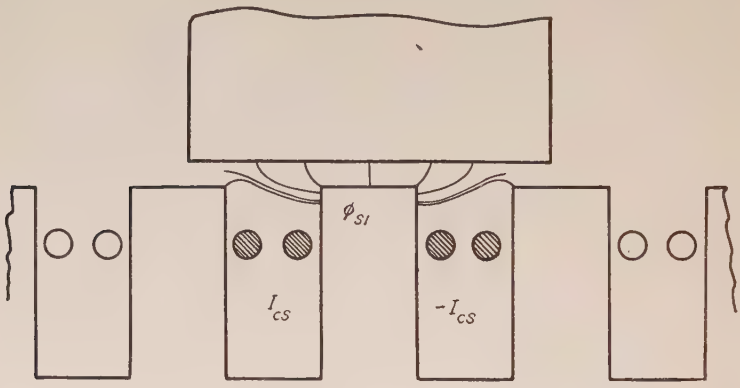


(b) Ending Commutation

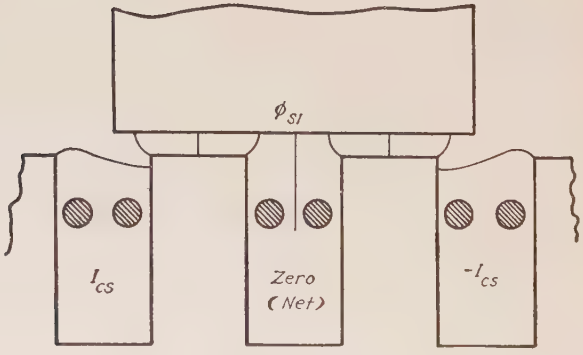
FIG. 109.—Type four commutation.

from true. As is shown later, it may still be two-thirds full value. But it is definitely reduced, whereas delaying fluxes are not. Hence, L_i will be increased. Also, the narrower the brush, the longer it must be to keep average contact density to the right figure. This lengthens the commutator.

Type Five.—The principal feature to note here is that k_{cs} is reduced by 25 per cent, thus reducing the total slot flux; whereas all other fluxes are normal. L_i is thus reduced.



(a) Type Six



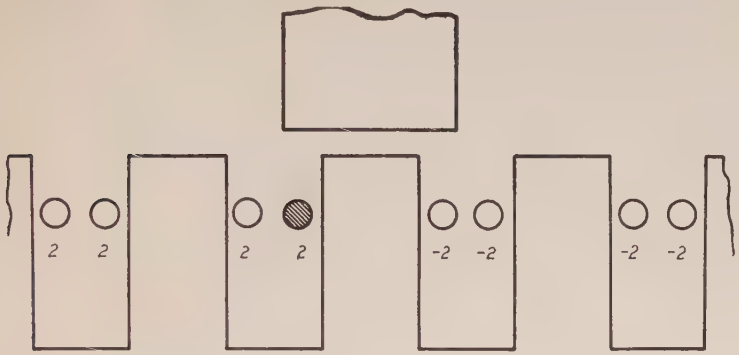
(b) Type Seven

FIG. 110.—(Showing ϕ_{SI}.)

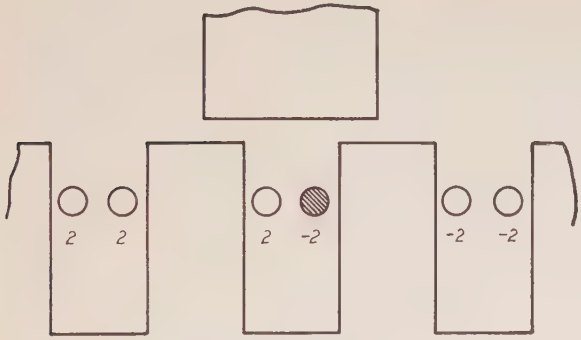
Type Six.—This duplicates Type Five, except in doubling ϕ_i, which introduces the disadvantages of the wide interpole (see Fig. 110*a*).

Type Seven.—Because the commutation periods of adjacent coils overlap, k_{cs} becomes seven-eighths, or 0.875. ϕ_s is normal, but all other fluxes are doubled. M_r is reduced as in Type Three. L_i is reduced.

It might not seem that the disadvantages of the wide interpole are sufficiently compensated for by its shortening axially. There is, however, another compensating factor. Doubling the brush arc has doubled the time, thus cutting the average delaying voltage in half. Then, if interpole design or adjustment is not exact,



(a) Beginning Commutation



(b) Ending Commutation

FIG. 111.—Type eight.

the net free voltage tending to warp reversal from the straight-line basis is only half as great; the machine has greater leeway of load capacity before sparking will begin. Refer to Fig. 110*b* for this type.

Type Eight.—Another new and different analysis is here called for (see Fig. 111), which shows the beginning and the ending

states. Using the middle of the interpole as a center, in *each case* the *four central conductors* are *grouped together*, the two on one side being paired off against the two on the other. The commutated conductor is shaded. It will be observed that commutation begins with a full complement of all fluxes, and that I_{cs} is equal to I_s. At the end, the central slot has zero net current, so that, however it is figured out, ϕ_s and all other fluxes due to these four conductors have their total linkages cut in half. It works out to the same thing, whether it is said that these fluxes are halved and k_{cs} is 1.0, or that the fluxes are normal and $k_{cs} = 0.5$. The former usage has been resorted to in forming the column of data in the table.

Setting aside four conductors as commutated slot conductors leaves the normal number to be considered as active, and so M_r is normal. There is no change in L_i.

Here, as in Type Four, the time of reversal has been cut in half, thus doubling the average delaying voltage. Imperfect interpole design or adjustment will then mean greater free voltages to circulate currents and cause local high contact density.

279. Three Coils per Slot.—Analysis shows that with three coils per slot the various commutation factors correspond quite closely to those of a two-coil-per-slot machine; that is, fixing the brush and interpole arcs, and considering the three-per-slot performance equivalent to two-per-slot performance, fixes the commutation type.

Industrial machines seldom use more than three coils per slot.

280. Brush Widths.—Brushes vary in width or arc by $\frac{1}{16}$-inch increments. It is therefore easy to find a brush of unit or two-unit width; but it is costly to make a large number of sizes of brush boxes. In practice it is often necessary to be satisfied with an arc as much as half a bar pitch away from the one desired. As a rule, no commutation difficulty should be expected, unless the arc desired is less than about a bar and a half.

281. Pulsations.—This term is meant by the writer to apply to pulsations in brush contact current density, which otherwise would ideally be constant. The subject of pulsations has never been more than scratched on the surface, and the literature on it is almost nil. Suppose the case is considered in which alternate coils all reverse alike in some definite way, the remaining coils also reversing in their own identical style. Let every other coil be marked with the numeral 1, being individually marked with

the series 1*a*, 1*b*, 1*c*, etc., and the intervening coils, by 2*a*, 2*b*, 2*c*, etc. Let the brush cover two bar pitches, and refer to Figs. 112*c* and 112*d*, which illustrate the case.

Figure 112*a* is the kind of diagram already previously used in this book. It is plotted here as before, over the brush span (or under it), and it is a *space* diagram in that when the coils arrive at

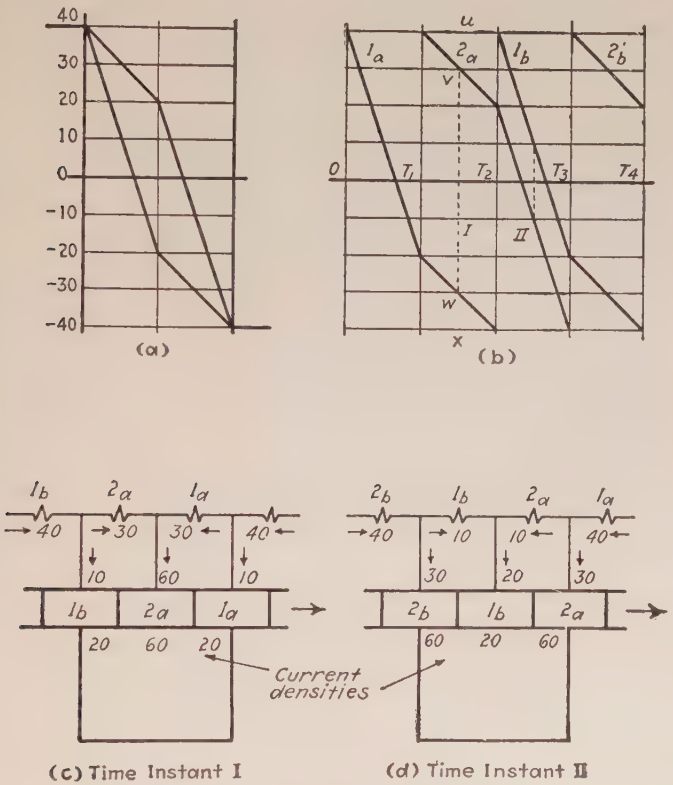


FIG. 112.—Irregular current reversal.

a certain space point, vertical reference to the diagram gives the current of the coil. It also serves as a *time* diagram, but so to consider it demands that *zero time for each coil is begun anew for each coil as it enters commutation.* Now let all coils marked 2 reverse according to the upper reversal curve, and all marked 1, according to the lower. In both curves, there is a half-time change of current at a low rate, and a half-time change at a high

rate. It is seen from the scales used that the two rates during reversal are 1:3 in ratio.

If it is desired to make a true general time diagram, the style must be changed to that of Fig. 112*b*. The beginning of commutation for coil 1*a* may be considered as fixing zero time. A bar pitch of travel later in time, coil 2*a* begins its reversal, then 1*b*, then 2*b*, and so on. This diagram is better than the other in that, by referring to any time instant such as I or II, the currents in the two coils then being commutated are found directly.

For the time instant I, Fig. 112*c* shows all current conditions, as well as brush current densities. Some very handy truths can now be established. As time progresses from instant I, current in 1*a* changes uniformly from −30 to −40 amperes; therefore current in the lead going to bar 1*a* decreases uniformly from 10 to zero. But area of bar 1*a* in contact decreases uniformly from half a bar to zero. So the current density average for bar 1*a* stays constant at 20 amperes per whole bar, or as may well be said, at 20 amperes per square inch if bar contact area is taken to be 1 inch. Likewise, bar 1*b* during the same interval will have both its current and its area increased in proportion, as it also has a density of 20. All this holds true from time T_1 to time T_2.

Furthermore, the central lead and bar carry 60 amperes during all this time, the density of this bar being constant at 60, for its whole area remains in contact throughout the interval. It is now easy to see that *uv* at once gives the current in bar 1*b*, *vw* gives that for bar 2*a*, and *wx* gives that for bar 1*a*. This is a general statement, holding for *any* kind of reversal curve.

Conditions for time instant II are shown by Fig. 112*d*. Note that while previously the outer bars had densities of 20 and the central bar of 60, during this interval the outer bars have 60 and the central one 20. As these densities, however, remain constant, the situation resolves itself into simple terms. At all times a total of one bar contact carries a density of 60, while the remainder contact, totaling one bar, has 20.

Assuming that ohms per square inch are constant everywhere, the losses will vary with the square of the current densities. The contact losses here are proportional to

$$60^2 + 20^2 = 4{,}000$$

whereas, in case of straight-line reversal, with density everywhere at 40, losses would be represented by

$$40^2 + 40^2 = 3{,}200.$$

The following conclusions may be drawn.

1. Density for a bar varies as the distance vx between reversal curves varies; queer reversal curves result in highly varying bar contact density during the bar's traverse across the brush; highly varying density means highly varying contact voltage drop.

2. These fluctuations in density (or drop) are called *pulsations;* the presence of pulsations indicates irregular reversal.

3. Contact losses are raised with increase in pulsations.

282. Wave of Reversing Voltage Induced by Interpole Flux.— The subject of pulsations leads directly to the question, What will be the shape of the wave of voltage induced by the reversing flux of the interpole? This constitutes another phase of commutation about which little or nothing is published by which actual performance data can be predicted, and which is taken care of nicely by the science of field mapping.

In Fig. 113 are shown three positions of the armature when passing under the interpole. The complete travel studied is to be one slot pitch. The positions in order are for beginning, one-quarter advanced, and one-half way through. The remaining positions would give field maps which are reversed duplicates of these.

This interpole arc is $0.9(s + t)$; note that in each position, counting up the flux tubes emerging from the interpole through the surface reaching from P to Q, a count of 18 is obtained. Thus it is shown in this case that the reluctance offered ϕ_i is about constant; this P-Q flux, be it noted, covers two slot pitches, which defines ϕ_i.

The induced-voltage wave is arrived at by first plotting the flux which the conductor has passed through in coming to various positions. Counting only from the flux line out of P, it is seen that when the conductor has arrived at its first position shown, it has passed through about 2.3 tubes. At the next position it has passed through 4.6 tubes; and for the central position, 9 tubes. These values and those for the next half of the travel are plotted in the upper curve of the graph.

The induced voltage at any point is proportional to the slope of the flux curve; the lower curve, of voltage, was so obtained. This wave is practically flat topped. On large machines the edges of the interpole are often beveled off to various degrees. This would tend to make the wave sharper, with a narrower crest.

283. Pulsations Due to Reversing Voltage Wave Shape.—Suppose that, by assuming straight-line reversal for all coils in a machine, the delaying voltage could be predicted for each instant of commutation for each coil. If there are two or more coils per

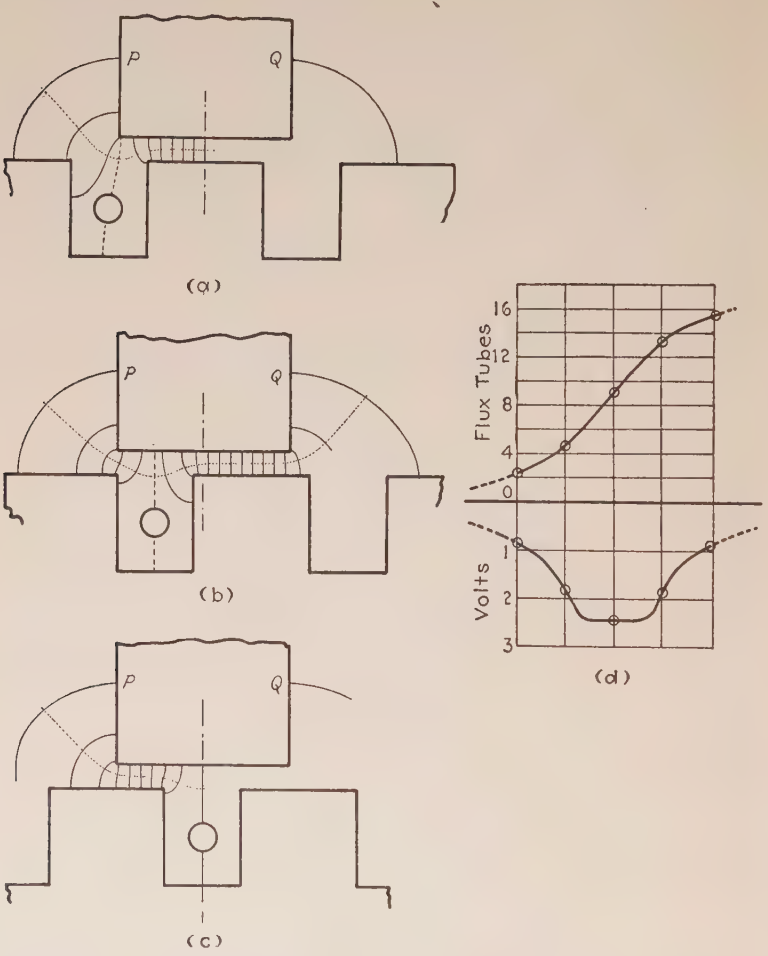


FIG. 113.—Interpole flux and reversing voltage.

slot, the overlapping of commutation periods for coils in the same slot would give rise to a mutual inductance voltage in a particular coil for part of its period, and only a self-inductive voltage over the remainder of the period. It would be found that

the several coils in a slot would have different delaying-voltage wave shapes. But the interpole is capable of inducing but one wave shape of reversing voltage. It is clearly impossible to avoid, in any one coil, instantaneous differences between the one kind of voltage and the other. Therefore, the reversals will be anything but straight-line reversals.

Consider the case of two coils per slot, with unit brush arc. The first coil up, in a given slot, is halfway through its commutation period before the other coil begins. During the first half of the period of the first coil's, it commutates alone in the slot; during the second half, both are commutating; and during the second half of the second coil, it commutates alone—all so far as flux changes *in that slot* are concerned. Now, if the reversing voltage were *constant*, the tendency would be for the two coils to reverse like those of Fig. 112. The ordinary type of interpole, however, as studied above (Fig. 113) yields something like the type of reversing wave this case needs, for, as concerns commutated slot flux, straight-line reversal would induce twice as much delaying voltage in a commutated coil during its overlap with the other coil as is induced when it is commutating alone. The brushes would normally be set so that the two coils would commutate in equally favored positions; hence the overlap part of the period would find the coil conductors under the central part of the interpole, which is where the peak reversing voltage occurs.

284. Pulsations Due to Active Slot Current Shift.—As shown in Chap. XXVII, during a travel of a slot pitch, all the active slot currents cause a very large reversing flux linkage to occur, which, however, is equalized *on the average* by the opposite change in linkage of the commutated coil. The flux path concerned, it will be recalled, is mainly by way of the main-pole circuit. The active current linkage will occur at a varying rate during the slot pitch of travel; it could be made constant only by specialized design toward that end, such as by using closed slots. In general, with ordinary design and for a symmetrical machine, the linkage mentioned would vary roughly as one cycle of a sine wave; and the commutated coils, acting as damping circuits enclosing this changing flux, would respond by circulating a total net current of such a value as would give the equal but opposite linkage change at all times. It is by no means commonly recognized that here is an ever-present factor which may

easily—and perhaps usually does—entirely ruin all chance of straight-line commutation taking place.

285. Pulsations Due to Main Flux Reluctance Variations.—If a small number of slots per pole is used, if the main air gaps are small, and if careful design has not been carried out, the passing of teeth and slots under the main-pole arcs may cause a variation of 5 to 8 per cent in main flux-path reluctance. As the field m.m.f. is constant, it might seem that with 8 per cent reluctance variation, an approximate 8 per cent variation in flux would occur. If it did, it would certainly have a radical effect in inducing pulsations, linking as it does entirely with the commutated coils.

There are two main elements acting to prevent any such flux variation. The main field windings, working in connection with whatever they are connected to, constitute a damping winding of many turns, and calculation shows that their effect alone would usually keep the flux variation to less than one-half of 1 per cent. The other element is the solid yoke, which acts by virtue of eddy currents set up in it, also to damp out variations in its flux.

Tests carried out under the writer's direction substantiate the results of calculations, that such variation is usually negligible.

The short-circuited commutated turns themselves, however, offer a third element tending to help the other two in preventing main flux variation, and it is certain that, in part at least, they will carry circulating current components caused thereby. To what degree pulsations will thus be introduced, it is very difficult to say, but until it is definitely established that main flux reluctance variation has a negligible effect on commutation, if indeed it can be established, it would be better to design for reasonably low variation here, by using what is effectively a full number of slot pitches in the pole arc.

286. Pulsations as Investigated by Test.—It has long been customary to check the commutation of important machines by taking "brush drop" curves. A direct-current voltmeter (low reading) is connected on the one side to the brush or brush box; the other terminal is connected to a sharp-pointed carbon rod (or to a spring brass strip, etc.) which is held against the commutator at various points opposite the brush. Suppose readings are taken for three, four, or five equally spaced points along the brush arc. If the readings are all equal, it has been supposed

that the brush contact current density is constant; if they plot into a curve which rises to one side or the other, a corresponding curve of current density is indicated. In practice, the curve is taken, and adjustment of interpole strength is made immediately.

If at some point the drop reading were zero, it would indicate that the *average* density thereat is zero; but the direct-current voltmeter can give no good account of pulsations. The oscillograph can.

Figure 114 is an oscillogram taken at one point on the brush arc of a three-coil-per-slot, wave-wound, four-pole generator. Whereas the average drop here was in the neighborhood of 0.3 volt, as read by a voltmeter, the oscillogram shows an extreme variation of from less than zero to three times the average of 0.3. This wave is typical of all machines tested, that is, as to variations noted. The oscillograph investigations, as carried out by Whipple and Robertson under the writer's direction, revealed the fact

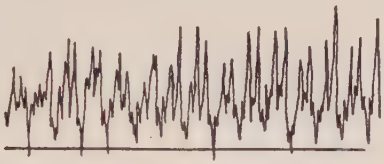


FIG. 114.—Oscillogram of brush drop.

that there is yet a good deal to learn about the cause and the extent of pulsations and of their ultimate effects.

287. Short-pitch Windings; Wave Windings.—Lap coils are nearly always either full pitch, or else are within half a slot pitch or less of being full pitch. In all such cases, so far as the purposes of this book are concerned, the cases will be considered to perform like full-pitch windings.

If there is any type of commutation that will always defy attempts at close analysis, it is the wave winding, especially when a dead coil is present. When the dead coil is eliminated, as by the use of three coils per slot, the reversal curves for all coils in a particular part of the slots are duplicates; and contact pulsations are cyclic, with a travel of one slot pitch as the length of a cycle. With the dead coil present, as in the case of two coils per slot, one cycle of pulsations is nearly like the next, except when the coils near the dead coil come up for commutation, when there is a

distinct break to be expected in the chain of events. Analysis shows that, within the necessary limits of accuracy, the wave winding can be considered to be equivalent to the nearest symmetrical lap winding that could replace it.

A difference in what constitutes unit brush arc for the wave winding should be noted. Consider that the commutation of a coil is effected by the two like brushes with which its terminal bars are in contact. Assume the machine to be four-pole, and the brush to be one bar wide. Then, for either progressive or retrogressive style, the commutation period would be a *bar and a half* long. The reader can easily verify this feature for himself. For six poles and similar brushes, the period would be a bar and a third long, and so on. Thus, if the *full* number of brush sets is used, unit brush arc would be the equivalent of a slot pitch, minus a fraction of a bar pitch, the fraction being 1 divided by the number of pairs of poles.

288. Interpole Design Notes.—The greater the main pole arc, the more the flux per pole becomes for a given investment in field m.m.f. But beyond a certain limit, reduction in ventilation space and increase in leakage enter to cause a reduction in ultimate economy of design. Thus, main-pole arc usually lies between 65 and 72 per cent of the pole pitch.

Small machines have fewer slots per pole than large ones. With the smaller four-pole units, it is usually distinctly uneconomical to use an interpole arc of two slot pitches—a two-unit arc. Multipolar machines may likewise demand a one-unit interpole arc, but many of the larger ones, having a high number of slots per pole, can easily accommodate the two-unit interpole arc.

Small interpoles are usually simple rectangular blocks of forged steel or are cut from thick rolled stock. Intermediate sizes often are given a long bevel, which does not materially affect the interpole flux distribution but which swells the core body to enable saturation therein due to leakage to be avoided. The largest machines have laminated interpoles. Pole-face losses are effectively reduced, and the laminations also pretty well eliminate damping eddy currents which flow in the solid pole when there is a sudden change of load; the damping currents would oppose the desired proportionate change of interpole flux. Laminating the interpole also gives easy opportunity for spreading out the pole core where it joins the yoke, in case the yoke is cast iron; this avoids saturation in the yoke where the flux enters.

The selection of the brush and interpole arcs, together with the known number of coils per slot, fixes the type of commutation as one of the eight types tabulated; or else, if necessary, a new type is created and listed. The next factors of immediate concern are the interpole gap length and the interpole core density.

Up to full load the core density, after taking leakage into account, should never be high enough to require any but a very low m.m.f. as compared with the gap. On the other hand, the use of an extremely low density may bring out L_i so great that there is not enough axial length in the whole machine to accommodate the interpoles; or in any case, an unnecessary length would result. As referred to the cast-steel saturation curve given in this book, pole core density may be satisfactorily placed at around 50,000. Then a 100 per cent overload can easily be carried for a while.

The solution for L_i has been given in the preceding chapters.

289. Reactance Voltage; Total Delaying Voltage Allowable.—As mentioned before, it has long been customary to calculate the self- and mutual-inductive linkage changes of commutated coils only and, after calculating the *average* voltage induced thereby, to call this the *reactance voltage*. The following statements should be made about such a practice: (*a*) the "reactance" as found in this way does not include the major reactance component due to flux through the main-pole circuits, which is a fact not commonly recognized or called attention to; (*b*) armature-reaction flux components are omitted by the practice and, as Lamme points out, such flux components should be brought into consideration.

Instead of making use of the term *reactance voltage*, the writer prefers to give all commutation fluxes full credit for their effects, and to call the delaying voltage so induced *the average delaying voltage*, or simply the *delaying voltage*.

As has been demonstrated, it is usually impossible to design interpoles that will give each coil at all instants a reversing voltage exactly equal to the delaying instantaneous voltage. The inevitable instantaneous differences cause pulsations. The less the delaying voltage average, the less the differences become. It is therefore likely that the average delaying voltage in a coil in interpole machines should be kept to less than 10 or 12 volts, unless it is possible by careful design to keep pulsation causes to a minimum.

CHAPTER XXX

DIRECT-CURRENT MACHINE DESIGN: NOTES ON DESIGN CONSTANTS; PROPORTIONS, RATIOS, VALUES, ETC.

Basic studies of individual problems found in the study of the direct-current machine have been made in chapters up to this point. Field mapping, thermal studies, armature windings, equalizers, commutation, leakage, fringing, magnetic circuits, armature reaction, and gap-and-tooth relations have been taken up.

This work has been handled on its own merits and without, so far, being treated in the sense that it had to be gone into because the design of a machine waited around the corner to be done. This work is basic to the full understanding of electrical machinery, whether one goes on to designing machinery or not; it was therefore intentionally arranged so that it could be studied without direct reference to a full design problem. The writer believes that possibly 75 per cent of the time of an undergraduate course should be given over to this basic work and that perhaps only the remaining and last 25 per cent—if that much—should be devoted to a full machine-design problem.

The next job is that of taking up that complete problem—the design of a direct-current machine. In this chapter, notes on the more important and useful factors, coefficients, ratios, constants, and proportions are made available.[1] Even this material has been arranged so that it is useful on its own merits; it is not limited by being tied in with any specific design problem.

290. Least First Cost; Maximum Efficiency; Maximum Economy.—It is decided to build a direct-current motor for a given voltage, speed, and continuous output; or an order is placed in which these items are dictated. The first question

[1] Slichter, W. I., "Design of Electrical Machinery," John Wiley & Sons, Inc.; Gray, Alexander, "Electrical Machine Design," McGraw-Hill Book Company, Inc.; Still, Alfred, "Elements of Electrical Design," McGraw-Hill Book Company, Inc.

arising in the design of the motor is, How big is it going to be? Size has several aspects, length, diameter, weight, volume. Unless special considerations fixing one of the external dimensions are encountered, and if the factory has freedom in other respects, then the design might be carried out in accordance with one of three differing aspects:

1. To design for least first cost.
2. To design for maximum efficiency.
3. To design for maximum economy, consideration being given to first cost, efficiency, operating-load cycle, cost of power, expected years of life, depreciation, upkeep, etc.

Most motors are built with as much care, out of as good materials, and with as high manufacturing accuracy as present trade conditions permit; these statements interpreted freely mean that, at present, reliable manufacturers turn out very reliable machines. Beyond these criteria, the great majority of machines are built to operate with a temperature rise of from 40 to 50°C. This fact amounts to an admission that, granting reliability, we are now mainly building for least first cost, irrespective of the long-time economics of the question.

The work to follow will be directed along the usual lines, the discussions and the design being pointed toward the making of machines operating with a 40°C. rise.

291. Dependency of Output on D^2L.—Suppose that in one factory several equally good designers would separately carry out designs for a given motor rating. It is likely that most of them would independently fix upon the same armature diameter, for factory standards are fixed at any one time, and only so many diameters are made available to the designer. But assuming that the designers had freedom of choice as to diameters, then six men might very possibly turn out six different designs, one about as good as another, and among which the ratio of armature diameter to length would vary by as much as 25 or 30 per cent. Yet the D^2L values of the six armatures would vary but little.

To see why this is true, consider some definite armature design. Let average gap flux density and slot copper density be kept constant. When a change is made below, let the armature winding or the applied voltage be changed, so that *speed* remains *constant.*

First, let the armature core length L be increased by a small amount. It is evident that the torque produced, the armature

losses, and approximately at least, the armature cooling surfaces would all increase in proportion. The temperature rise would be about the same as before, and output has changed in proportion to length.

Next, bring the armature back to normal, then increase the diameter. The easiest way to follow this argument is to assume that the same tooth-and-slot design is kept as nearly as is possible, and that slots are increased in number in proportion to diameter increase. The force produced per slot remains the same, but torque lever arm increases with the diameter; hence, torque varies with the square of the diameter. This finding, however, is subject to the condition of preserving the previous temperature rise. Assume for the moment that as slot-and-tooth units have increased in proportion to the diameter, the losses have increased the same way. The outer cylindrical armature surface cooling area has likewise increased in proportion, and radial duct surfaces have also, at least approximately. In short, cooling area per unit package of tooth-and-slot material remains constant, losses per package remain constant, and temperature rise would seem to remain constant.

Two factors of importance must now be remembered and applied. First, the higher peripheral velocity gives a higher heat dissipation coefficient; second, the greater pole arc covers more armature current sheet, which causes greater gap flux distortion, higher tooth maximum densities and higher losses per tooth than were considered above. These two factors tend to balance off, so that approximately, for the same temperature rise, the output varies with the square of the diameter.

If, under the restrictions laid down, the output is found to vary with the length and the square of the diameter, it varies as D^2L; as the six designers are working for the same output, they will have armatures with nearly the same D^2L values.

The above arguments, like most of those encountered in design, are good only within narrow limits. If pushed to extremes, it would seem that for a given speed and temperature rise, all motors, large and small, would have a constant ratio between output and D^2L. This is very far from being true, as is later made evident.

292. Variation of Output with Speed.—Within reasonable limits, the output of a given size of machine is proportional to its speed. This statement depends, of course, on good design.

It is best illustrated by the fact that manufacturers are able to make, for instance, a 600-, a 1,200-, and an 1,800-r.p.m. motor, rated respectively at 5, 10 and 15 horsepower, all three being identical as to dimensions of magnetic parts. This proportionality, though simple enough to state, is by no means simple as to explanation, and like the previous D^2L relation, breaks down if pushed too far. An illustration of how it works within proper limits will be useful.

Consider a 5-horsepower motor at 600 r.p.m. It is proposed to build a 1,200- and an 1,800-r.p.m. motor from the same steel punchings and castings. Knowing that manufacturers are able ordinarily, by good redesign, to get 10 and 15 horsepower out of the two higher-speed motors, the same result will be attempted here.

As there is the same magnetic circuit, it might be decided to operate all three motors with the same flux. As rating is to be proportional to speed, torque is to remain constant, and using the same flux demands that copper densities in armature windings shall be kept the same. This means that for the higher speeds, copper losses will be the same, whereas, with increased frequency, iron losses will be something like double and triple for the 10- and 15-horsepower motors. If fixed and variable losses are roughly equal in the 600-r.p.m. motor, fixed losses will be much greater than variable in the other two. This fixed-flux idea turns out to be a poor one.

The best results are obtained by keeping something like the same division of losses in each machine, allowing neither iron nor copper losses to become predominant. The following table gives approximate figures for proper redesigns:

VARIATION OF DESIGN WITH CHANGE OF SPEED

Revolutions per minute	600	1,200	1,800
Horsepower rating	5	10	15
Flux per pole, per cent	100	80	70
Circular mils per ampere, armature copper, per cent	100	80	70
Iron and copper loss of armature	100	150	200
Watts dissipated by armature at same temperature rise	100	150	200

As can be seen, if torque is to remain constant, the inverse of copper current density, which is expressed by circular mils per

ampere, must be proportional to flux per pole. For doubled speed, flux has been reduced 20 per cent; for tripled speed, 30 per cent; each with corresponding increases in copper current density.

Such changes will assure that the ratio of copper to iron losses will remain roughly constant. Note that for doubled speed, losses increased only 50 per cent, and for tripled speed, the losses have only doubled. Also, that heat dissipation has not increased in proportion to speed, but only as fast as losses have increased.

An important factor to take note of is that, as losses do not rise in proportion to speed, the higher-speed motors are the more efficient.

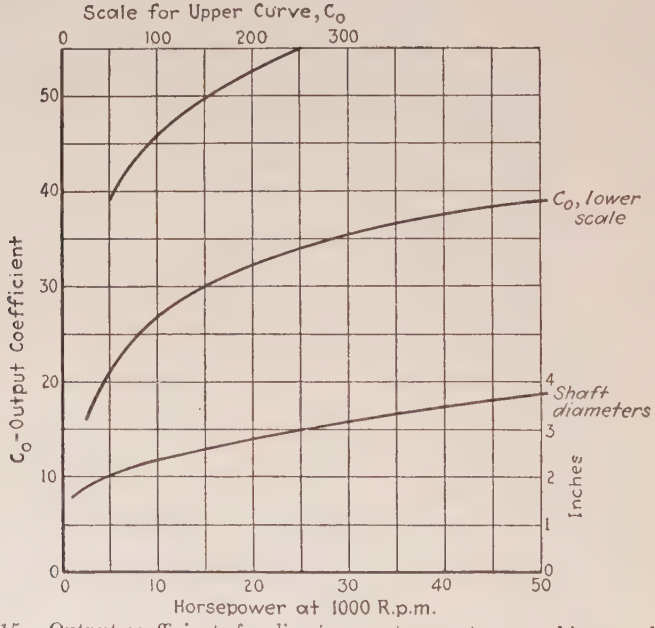


FIG. 115.—Output coefficients for direct-current, open-type machines; and shaft diameters at core part of shaft.

293. Output Coefficient for Direct-current Machines.—A curve of output coefficients is given in Fig. 115. This curve is used for finding the approximate preliminary value of D^2L to use for given values of speed and output. The equation is

$$D^2L = \frac{\text{Horsepower} \times 10^6}{\text{Revolutions per minute} \times C_o}$$

where C_o is the output coefficient corresponding to what the horsepower would be at 1,000 r.p.m. Suppose it is desired to design a 70-horsepower, 700-r.p.m. motor; its armature should have the same D^2L value as that of a 100-horsepower, 1,000-r.p.m. motor. Looking up the C_o for a 100-horsepower motor, we get a value of 46. Then

$$D^2L = \frac{100 \times 10^6}{1{,}000 \times 46} = \frac{70 \times 10^6}{700 \times 46} = 2{,}175.$$

The curve holds for modern, open-type, self-ventilated machines, designed for a 40° temperature rise, and in general, not equipped with fans mounted on the end of the armature to act as blowers.

In the above equation, if C_o were constant for all ratings, armature volume would be proportional to output. But C_o increases with rating; hence the pounds of armature per horsepower decreases; this figure is twice as great for a 5-horsepower as for a 50-horsepower motor, granted that their speeds are the same.

294. Peripheral Speed of Armature.—Up to 100 horsepower the armature peripheral speed is normally not more than 4,500 feet per minute. Larger sizes may run up to as much as 7,000 or 7,500.

295. Main Proportions for Economical Design.—The rectangular volume in which a wound field pole unit might be enclosed should be roughly cubical. Extreme departures from this criterion lead at once to disadvantages of copper or iron waste, and excess copper losses. If axial armature length is excessive compared with peripheral dimension of pole face, the rectangular pole winding becomes excessively long in the axial direction, requiring excess field copper and causing excess field loss. On the other hand, if axial armature length is excessively low compared with peripheral dimension of pole face, armature end-connection weight and copper loss become excessive. An excessive pole pitch also may mean that the field winding is stretched out peripherally from the most economical square shape, and far enough so as to run up the field weight and loss too much.

It is usually found best to make the pole face arc somewhat greater, and the pole core width somewhat less than the length of the armature.

Discussing now the radial pole dimension, if this is excessively high, pole core and yoke magnetic path lengths are great, and the

excessive NI drops therein require unnecessarily high field ampere-turns. The magnetic parts also weigh too much. If this dimension is made too low, the height of field and interpole coils may be reduced enough to cause so much thickening of these windings that they choke off each other's ventilating air, or they may actually interfere.

296. Number of Poles.—The fewer the poles, the greater the total amperes in the current sheet under the pole arc. This value fixes the reaction ampere-turns at the pole tip. With too few poles, reaction is excessive, causing high flux distortion under normal load, excessive maximum tooth densities, and excessive tooth losses. To make the design practicable the air gap may be increased, but this at once demands excessive field m.m.f., field copper, and field loss. Also, high reaction calls for large interpole windings, leading into heating difficulties and high leakage coefficients. Furthermore, high flux per pole, from using too small a number of poles, means a large yoke section and excessive machine weight.

That there may be such a thing as too many poles is seen when it is remembered that there is an underlying cost to producing every pole unit and every brush unit, irrespective of size. The size of the commutator segments can, in some cases, offer a limitation; for as average volts between adjacent bars have a limit of 15, fixing the machine voltage fixes the minimum bars per pole pitch, and too many poles might give too small a bar pitch for building a practicable commutator.

Redesigning for an increased number of poles, if the pole unit is to be kept roughly cubical, means shortening the armature and increasing its diameter; and this process is limited in some cases when economical peripheral speeds are exceeded. An excessive peripheral speed, from the manufacturing standpoint, is one for which unusual construction must be resorted to, to take care of centrifugal forces.

297. Stability; Gap-flux Distortion; Armature Reaction.—If main field ampere-turns for gap and tooth, M_{gt}, are equaled at full load by armature reaction ampere-turns at the pole tip, M_t, then the gap density there is zero. This condition means a rather high distortion of gap flux by reaction m.m.f. Such a distortion means that in a plain shunt motor the full-load flux may be from 2 to 10 per cent less than the no-load flux. The per cent. decrease in flux may exceed the per cent. decrease in

generated voltage; if so, the full-load speed would be greater than no-load speed. In itself, this fact has little importance. But note that, while IR drop increases uniformly with the load, change in flux due to reaction takes place faster for greater loads. In a given case, the flux might be reduced 2 per cent from no load to half load, and 6 per cent from half load to full load. Thus the speed may drop for light loads, begin to rise for more load, and at full load be rising so fast that it may get the motor into trouble due to speed instability. It is a machine that is too "ambitious;" that is, the more it is loaded, the faster it runs, and with unusual loads, the faster it runs, the more the load becomes, and so on. Not only may the motor, on certain loads,

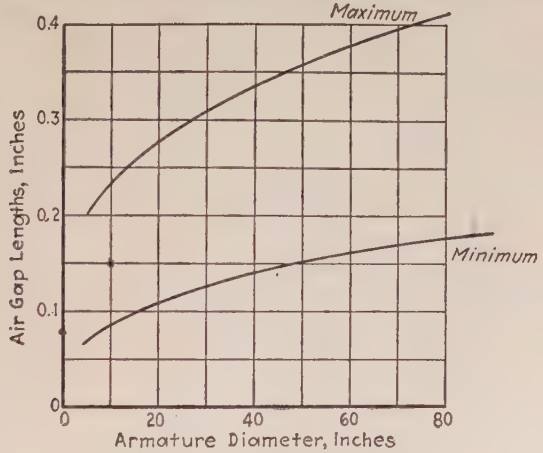


FIG. 116.—Range of gap lengths.

give evidence of difficulty in finding a definite speed at which to run, but with certain inherently fluctuating loads, a swinging cycle of speed and current changes may occur. The armature current may swing rapidly up to overload values, fall rapidly perhaps to zero or reverse, and then back up again, the speed in the meantime performing some excessive variations.

One of the older ways for meeting this problem is to use large air-gap lengths, which increase M_{gt} and reduce distortion and flux changes. Another device used in very special machines is the pole face or compensating winding. A more recent method is to retain the short air gap which means field copper economy, and to stabilize the speed by using a stabilizing winding. This winding is in fact nothing but a weak series field winding whose

only purpose is to prevent flux reduction due to armature reaction. Its use, of course, does not prevent distortion taking place, as does the pole face winding. Short-gap machines are successfully built and operated in which reaction is so heavy that the flux density at the weak pole tip is actually reversed somewhat at full load.

High distortion causes greater tooth losses, as the tooth loss is determined by the maximum density experienced by the tooth in passing through the pole's flux.

High distortion may also cause a commutator limitation to enter. The maximum voltage between adjacent bars has a safe ordinary limit of 25, to prevent flashover possibilities. If a curve of voltage between adjacent bars is plotted over a pole pitch, the curve will be practically like that of gap-flux distribution, because the generated voltage of a coil depends directly on the flux density through which its sides are passing.

The ratio of maximum gap density to the average density taken *over a pole pitch* then gives the ratio of maximum voltage between adjacent bars to average between bars. The greater the distortion, the greater becomes this maximum voltage.

As pointed out above, high reaction may also call for such a heavy interpole winding that it becomes undesirable or out of the question because of excessive winding losses, overheating, or excessive leakage.

Motors ordinarily can and should be designed so that the flux density at the weakened pole tip is at least not reversed, and preferably so that the full-load gap density here is perhaps 25 per cent of the average pole arc density.

298. Ratio of Pole Arc to Pole Pitch.—To avoid excessive leakage, in interpole machines the pole arc can usually be made to cover not more than 65 to 70 per cent of the pole pitch. The exact amount depends on the individual machine. Here enters the question of choice of number of slots and teeth. It will be remembered, from the work on commutation, that the least arc for an interpole face is nearly a slot pitch. The fewer the slots per pole, the larger the slot pitch and the interpole width become, and, remembering dangers of high leakage, widening the interpole means pushing the main pole arc back just so much further. This effect argues for the use of many slots per pole, because the capacity of the machine to produce torque, and therefore output, depends on the amount of pole arc available.

299. Tooth-and-slot Proportions.—Due to the trapezoidal shape of teeth of all machines of any size (caused by the use of form-wound coils which demand parallel-sided slots) tooth-root densities present an important limiting factor. If this density becomes excessive, tooth losses and tooth NI drop get out of bounds. Maximum tooth-root density, B_{rm}, ordinarily runs from 100,000 to 160,000 maxwells per square inch. This value is figured for when the tooth arrives at the position of greatest gap density. The higher tooth densities, as indicated in the study made above of variations occurring with change of speed (Art. 292), are used with the lower speeds, or frequencies.

A highly useful and important relation between tooth and slot is found in the fact that, roughly, tooth volume and slot volume are equal. This property grows out of several factors and of course represents a compromise adopted to get the best results from a given amount of space and material. If the slot is too wide, cramping the tooth space, the output goes down; and in turn, if slot space is encroached upon by wide teeth, output again suffers.

Suppose, for getting up a simple argument to carry this point, that in a given armature, slot current will be kept proportional to slot width, and tooth flux proportional to tooth width. This amounts approximately to saying that the copper current density and the tooth flux density shall be kept constant—not unreasonable assumptions to make. Also, let the slot pitch be fixed, but vary the width of slot. The torque produced depends on the product of slot amperes by tooth flux, which product is proportional to that of slot width by tooth width. If slot plus tooth width is to remain fixed, their product is greatest when they are made equal.[1]

Slot depth is from three to four and a half times the slot width. This relation can be seen evidenced in any number of good designs, or it can be arrived at by trying out different proportions in several distinctive designs. Shallow slots or very deep slots will inevitably cut the output. The very deep, narrow slot is also likely to lead to commutation troubles because of high slot flux values reached; and it tends to run up the eddy-current loss on heavy armature conductors.

300. Type of Armature Winding, Conductors per Slot, Etc.—The chapter on Armature Windings is referred to here.

[1] This argument is due to Dr. Benjamin F. Bailey, University of Michigan.

301. Ampere Conductors per Inch of Periphery.—This term is misleading, but is so well implanted in the general usage that it is likely to stay. It means, conductors per inch of periphery times the amperes in each armature conductor. This product really gives amperes per inch of periphery. If the slot pitch is 1.2 inches and the current per slot is 600, then ampere conductors per inch of periphery, or q, is $600 \div 1.2 = 500$.

The value of q is one of the important factors in design. q times one-half the pole arc (in inches) gives the ampere-turns of

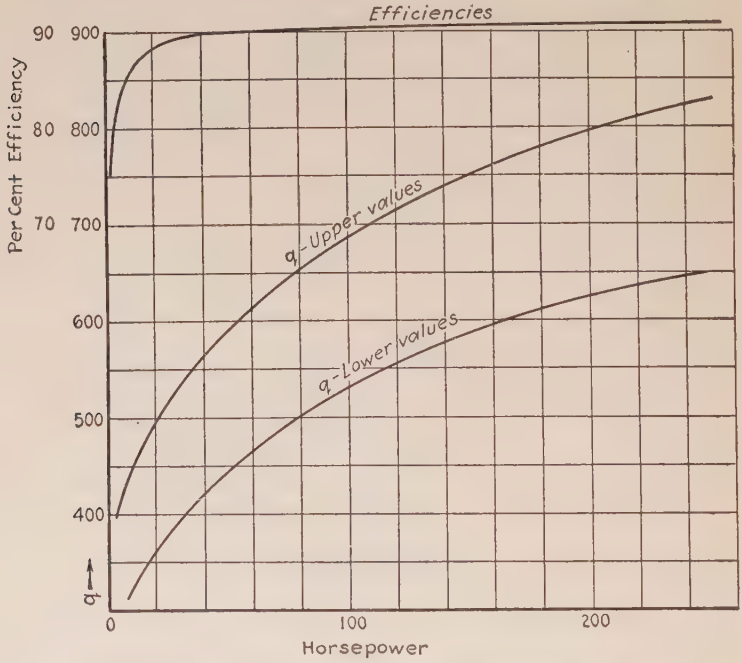


FIG. 117.—Direct-current motors, open-type. Efficiencies, and ampere-conductors per inch.

armature reaction at the pole tip M_t; and half the pole pitch times q gives (approximately) the reaction at the interpole. q is in a class with B_{rm} as being a limiting factor. Figure 117 gives usual values.

302. Gap Density; Tooth-root Maximum Density.—Figure 118 gives the usual range of average densities used.

303. Number of Slots per Pole.—The number of slots in a pole pitch or of slots per pole varies from about 8.5 to 15. Small machines must have fewer slots, else the tooth root becomes

unduly pinched; t_r, the tooth-root width, should usually be at least 0.2 inch. Also the small machine is bound to have pretty narrow slots anyway, and narrow slots lead immediately to poor space factor. The larger the unit, the more slots it can have per pole.

304. The Commutator.—The commutator diameter is very often made from 60 to 75 per cent of the armature diameter. Peripheral speeds should not exceed 3,000 or 4,000 feet per minute, unless special construction is used.

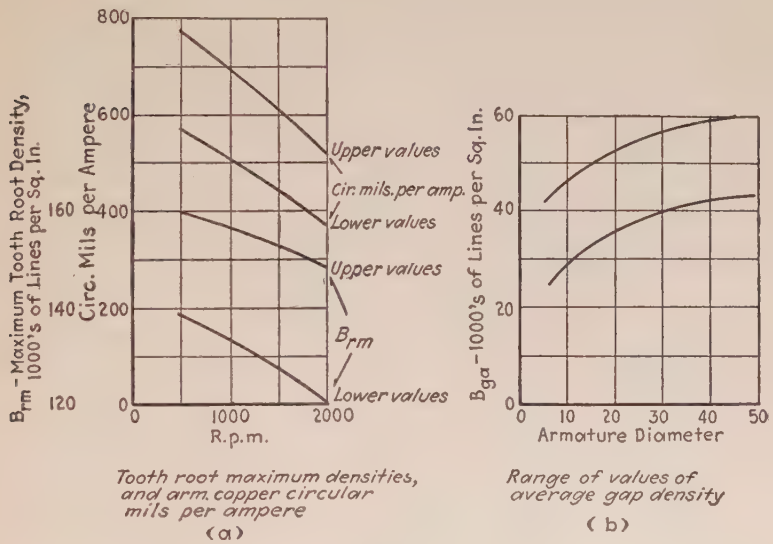


Fig. 118.

The bar pitch should be not less that 0.2 inch; if less, the assembly becomes unreliable mechanically; bars will move out of place.

Although commutator design can usually be made to wait for the completion of armature and field design, yet it must be held in mind when fixing upon the armature-winding details. The requirement of average volts per bar of 15 or less helps to fix the number of coils per pole, and it often influences the choice of exact number of slots.

305. Armature Copper Current Density.—The copper density, or rather, circular mils per ampere, has a usual range as given by Fig. 118*a*.

306. Resistivity.—A machine designed for a 40°C. rise may operate in a room with the air at from 20 to 25°C., which means that the machine surface temperatures will be at from 60 to 65°C. In ordinary designs, armature and field coil conductivities for heat are so fixed by good design that the highest copper or iron temperature is not more than 10 or 15° higher than the surface measured temperature. Probably 75°C. is a usual safe copper temperature average figure, at which temperature, 1.05 ohms per circular mil-inch is the proper resistivity to use.

307. Flexibility of Suggested Values.—It should by no means be taken for granted that the curves given in this chapter for such quantities as q, circular mils per ampere, flux densities, C_o, etc., are rigidly correct. Any individual design factor is not important enough to have attached to it the "shall" or "must" idea. All design is a matter of compromise, and the best design is in any case the best compromise that can be made among conflicting requirements. An individual design should be judged more on its own merits than according to the questionable criterion of how nearly all of its quantities hit to certain average marks attained in a great number of machines already built.

CHAPTER XXXI

DESIGN OF THE DIRECT-CURRENT MACHINE: LOSSES, TEMPERATURE RISE, EFFICIENCY

In this chapter will be found methods for calculating the various losses of the machine, methods for predicting the temperature rise of its various parts, and a note on the efficiency calculation.

The methods given for predicting the temperature rise of the armature, end connections, and commutator are unique within the field of books devoted to design problems, in that they are based as directly as possible on the modern research results of G. E. Luke. Our information on the air flow obtained in self-ventilating apparatus is very incomplete, as is also information on dissipation coefficients for various surfaces of irregular passages. But there is every reason for breaking away from the traditional empirical methods of temperature prediction heretofore in general use.

The methods and values given here should be viewed as suggestions rather than as well-tried and proved practices. When full research shall have been completed, as it ought to be within a few years, fairly complete methods such as are given here must in some way form the basis of whatever temperature predictions are then made.

308. Iron Loss. I. *Tooth Loss.*—The maximum gap flux density through which the teeth pass determines the maximum densities the teeth experience in passing through a pole pitch. Tooth loss depends, not absolutely, but pretty largely, on the maximum density, as, with proper laminating, the loss is mainly hysteresis. When B_{rm}, the maximum root density, is known or decided upon, density in all parts of the tooth can be calculated, the tooth can be divided into sections, and the loss for each section found by using the iron-loss curve of Fig. 21, or by use of the curves of Fig. 119, tooth loss can be found much more easily. These curves were made up for a basic tooth of 1-inch depth, and 1-inch root width, tooth constant $k(= t/t_r)$

being used as the parameter. If W is the value taken from the curves, tooth loss becomes

$$\text{Tooth loss watts} = Wdt_r L_g Sf,$$

where d = tooth depth, t_r is root width, L_g is gross iron length of armature, S is number of slots or teeth, and f is frequency in cycles per second.

II. *Core Loss.*—By finding the product of average core density, core weight, and loss per pound as found from Fig. 21, a preliminary value of core loss is obtained. This value, however, is based on the assumption that core density is uniform over the

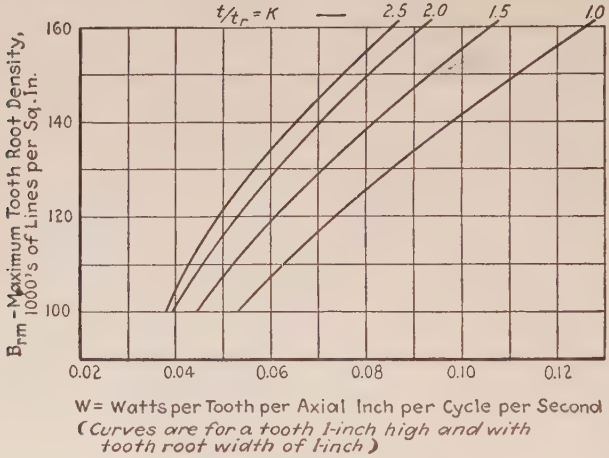


FIG. 119.—Tooth loss curves.

path section, and such is rather far from the truth. A closer value of core loss is now obtained by adding 25 per cent to the watts already obtained.

309. Armature Copper Loss. I. *Slot Copper Loss.*—It is likely that by the time slot copper loss is computed, a value for armature resistance as a whole has already been set down. But there are plenty of chances for error in making this computation, and an opportunity for making a check should be welcomed. The number of slot wires is known, also their circular mils section, length, and current. A direct computation of I^2R losses based on these figures, rather than to figure it as a certain proportion of total armature copper loss, had better be made.

The watts loss thus obtained should in general be increased, because of the fact that main-pole flux entering the slot does not induce equal voltages in the upper and lower sides of a relatively deep conductor; this inequality sets up a circulating current in the slot part of the conductor which inevitably raises the copper loss. The slot loss as regularly computed should be increased by from 5 to 15 or more per cent, depending on the conductor depth and section. No trustworthy figures to guide one's allowance seem to be available; so the coefficient f_e must be selected according to judgment.

II. *End-connection Loss.*—This can be figured by the method suggested for slot copper losses; no excess allowance for eddy currents need ordinarily be made.

310. Armature Temperature Rise Prediction. I. *Method.*—The surface temperature rise of the mass of material made up of armature iron and slot copper, which is exclusive of end connections, is usually figured all at once; that is, the fact that the surface temperature over the various surfaces of this mass will vary a few degrees is usually disregarded. Certain highly empirical practices are in wide use for the prediction of this temperature rise, most of them having little merit except when based upon a given line of machines of the same kind of construction, and likewise applied only to that type of machine.

The method used here is based as directly as can be on Luke's investigational findings of heat dissipation, and is padded with the writer's judgment only where necessary. Three surfaces are involved: the outer armature surface, the inner armature surface under the core, and the radial duct surface. The ends of the armature are included in the duct surface. The total watts these three areas (if all are present) are able to dissipate, at the desired temperature rise, is computed. The loss to be gotten rid of by these areas is the sum of iron and slot copper losses. The object in design, at this point, is to make the loss watts equal to the dissipated watts, else the rise will be too high or too low.

II. *Dissipation by Outer Armature Surface.*—This surface, also called the outer core area, is πDL_0. Various authorities have given dissipation coefficients for this surface, as a result of experience and judgment. Walker's suggestion is embodied in the curve marked with his name in Fig. 120. Luke's curve for the open, rotating smooth rotor body is given below. It is

quite evident, from Luke's various findings, that any such straight curve as Walker's does not have the right trend; it should bend over to the right. The writer's suggestion is given by the intermediate curve, and this curve is used hereafter.

The watts dissipation is computed by taking the product of the outer core area, the dissipation coefficient, and the rise of this surface above the air passing the surface.

The air passing this surface will be a mixture of air coming from over the ends of the armature, and that coming out through

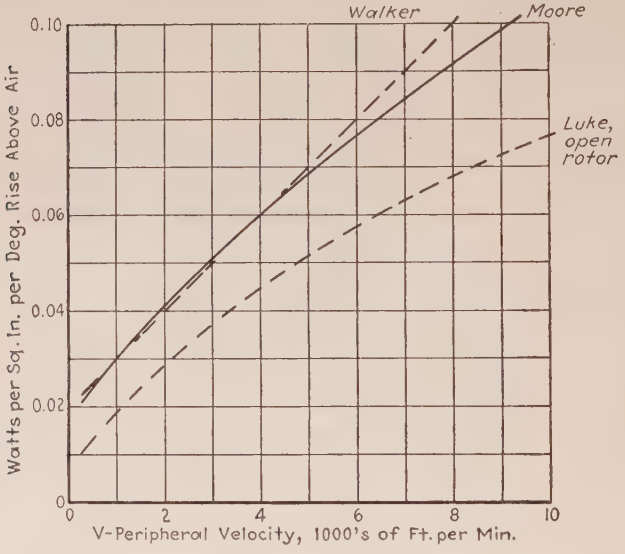


FIG. 120.—Dissipation coefficients for outer armature surface.

radial ducts. Its average temperature will be higher than that of the room air, and this fact should be allowed for. Apparently, no investigations whatever have been made of this rise in air temperature. The longer the armature, the more ducts it will have. Seemingly, then, the more ducts there are, the greater will be the average air temperature. The writer suggests the values in the following table:

Number of radial ducts.....	0	1	2	3	4	5	6
Air passing outer armature surface is this number of degrees above room air (Θ_1).....	4	5	6	7	8	9	10

As an example, suppose the desired or assigned temperature rise of the armature is 40°C. and that the armature has three ducts. By the table, Θ_1 is 7°. Thus the actual rise of surface above passing air is $40 - 7 = 33°$ rise. This actual rise, times area, times coefficient, gives watts dissipated.

III. *Dissipation by Duct and End Surfaces.*—Here, a quite complicated problem is presented. Different writers suggest that the duct air velocity be taken at from one-third to one-fifth of the armature peripheral velocity. Actually, no one knows what the duct velocities may be. The writer suggests $V/4$, or one-fourth peripheral velocity, for average duct air speed, not because it represents an average of usual guesses, but because it seems, when used in connection with Luke's coefficients for short ducts, to give reasonable results.

The total area used included duct side walls, the two armature ends, and areas of ventilating or spacer fingers. The dissipation coefficient used is taken from the curves for short ducts (Fig. 39); the length of duct being taken as outer armature radius minus inner core radius. The radial areas are figured on this same radial dimension. As the air, in passing through the duct, is warmed, air temperature should be corrected. It is suggested that the following corrections be applied:

Radial dimension of duct, inches	3	6	9
Number of degrees by which average air temperature is higher than room temperature (Θ_2)	6	8	10

As an illustration, let the armature radius be 16 inches and inside core radius 8 inches. Then duct length is 8 inches, for which the air has an average of 8° rise, by the above table. If 40°C. rise is the specified rise for the armature, $40 - 8 = 32°$ rise is used to find the dissipation, which is this value times area, times coefficient.

IV. *Dissipation by Inside Armature Surface.*—A part of the inside core area will be covered by the supporting hub or spider; but heat will be conducted across the contact areas, so that the hub or spider will help to dissipate heat. The inner core area therefore is figured as being entirely active. The area is then $\pi D_2 L_g$, where D_2 is inside core diameter.

The air here will be practically at room air temperature, and no correction for air temperature rise need be made.

The dissipation coefficient is perhaps best taken from the curve for the 1.75-inch round duct (Fig. 35), air velocity being assumed to be $V/4$.

V. *Temperature Rise.*—Let the sum of tooth, core, and slot loss be W_l; the sum of watts dissipated from the three types of surfaces, at the *desired rise*, be W_d; and the desired rise be Θ. The rise to be expected will then be, approximately,

$$\text{Armature temperature rise} = \Theta(W_l/W_d).$$

VI. *Modifications of Temperature-prediction Method.*—If the ventilation of the armature is changed due to the use of a different construction, the above method of temperature prediction must be modified to suit. Large units have plenty of room for air access inside the core, the air flow here being that which feeds the radial ducts.

Smaller machines begin to make axial flow inside the core difficult. The required core magnetic section begins to make the core approach too closely to the armature hub, which by this time has degenerated into a cast sleeve pressed over the shaft, with very short arms for supporting the core. As long as the actual cross-section of air-flow path axially inside the core equals in area the average area of radial flow—counting as radial-flow area that of the ducts used plus the two end finger-plate ducts each counted as a duct—as long as axial feed area equals or exceeds radial-flow area, the above method may be used intact.

In still smaller machines, cooling effect of radial surfaces had better be cut in proportion to the cut suffered in axial feed-section area. Anyway, such small machines will have such low duct velocities that ducts begin to become poor heat dissipators, and would likely be left out for that reason alone.

In armatures with no space for axial flow and no ducts, only two surfaces are directly to be included in heat dissipation: the external armature surface, and the end of the armature (including spacers at end, if any) away from the commutator. There is always likely, however, to be a worth-while flow of heat in these small, short armatures along armature conductors to the end connections, which usually run so cool as to be able to help out the main armature mass.

Figure 115 gives reasonable shaft diameters for smaller machines.

311. Temperature-rise Prediction for End Connections.—The curves of Fig. 45 show Luke's results obtained in experiment-

ing with a set of end connections. Based on these, the writer has drawn the curves shown in Fig. 121.

The entire surface area of the coil ends is to be used, even though in many cases by no means all of the surface is exposed. The calculation for temperature rise of the surface of the *insulation, not the copper rise,* may as well be figured for an inch length of coil instead of the whole end-connection surface. For an inch length of coil end, the surface is equal to the periphery of the coil total or insulated section, and this is approximately $2(s + d/2)$. The watts loss per inch is easily figured, the number

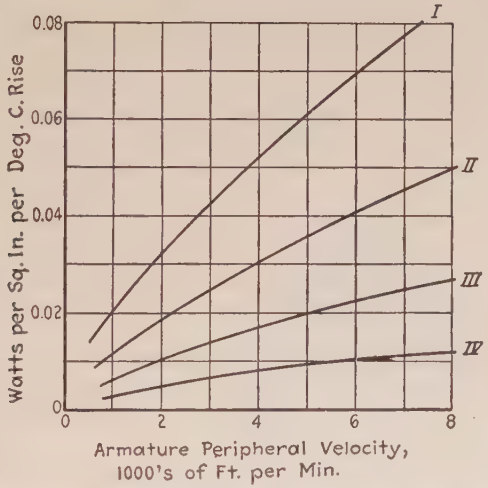


FIG. 121.—Dissipation coefficients for end-connections. (See text for explanation.)

of wires, their section, and their current being known. The coefficient is taken from one of the curves (Fig. 121).

The conditions for which these curves hold follow:

Curve I.—Adjacent coil sides in the lower layer kept at least ⅛ inch apart, to permit air passage up through the two layers; *curls* or *drops*—the extreme coil ends—made in a radial plane so as to be entirely open; perfect air access under coils.

Curve II.—Coil ends laid over to make shortest end connection —no space between adjacent coils in lower layer and no air passing up through; curls open and accessible to air passage; inner surface ventilated.

Curve III.—No air space between adjacent coils, and ends of coils returned in shortest way possible, which eliminates curls

and good ventilation thereat; fair air access to inner coil surfaces.

Curve IV.—Same as preceding case, but with air access to inner surface cut off, as at the commutator end of a small motor.

312. Commutator Losses. I. *Friction Loss.*—Pressure necessary ordinarily is 1.5 pounds per square inch, with a friction coefficient of 0.25. The friction watts loss is easily figured.

II. *Commutator Electrical Loss.*—The measurable voltage drop for a pair of brushes is usually less than 2 volts, but in view of the fact that uneven contact densities are bound to increase the contact loss, a drop of 2 volts had better be used. This, times the armature current, gives electrical watts loss.

313. Prediction of Commutator Temperature Rise.—The commutator is fireproof, but this does not guarantee the right to make it work at extreme temperatures. Excess heating immediately shows up in bad commutation, which itself is merely an excess-heating condition; and high temperatures may cause enough expansion and later contraction of commutator parts to cause high-bar troubles or loose bars, to develop. The rise should ordinarily be kept to 55° or less.

The dissipation coefficient is taken from the curve given by the writer (Fig. 120) for the outer armature surface. The surface is the cylindrical surface of the commutator, plus an allowance for cooling effect of risers, if any. For every inch long the riser may be, an inch can be considered as added to the commutator length, until a 2-inch length is reached. After this, little if any addition is made. Large commutators with axial ventilation through the commutator hub will run somewhat cooler than the above prediction method would indicate. Perhaps, in the best case, as much as 20 per cent of commutator heat is dissipated by hub and brush boxes combined.

314. Shunt Field Loss.—Nothing need be added here about making this computation, except that a resistivity of 1.05 should be used.

315. Prediction of Shunt Field Temperature Rise.—Figure 122 gives dissipation coefficients, the upper curve being for thin coils having plenty of ventilating space about them, and the lower curve for coils closely jammed in among interpole windings.

The area considered is the entire superficial coil area, which is equal to coil-section perimeter times mean turn. In the usual case probably half the heat of the coil is dissipated by the exposed

surfaces to passing air, the other half going by conduction to the pole piece and from it to pole-face surface and yoke surfaces.

316. Interpole Winding Loss.—This is easily calculated.

317. Prediction of Interpole Coil Temperature Rise.—Carried out as for the shunt field coil, and by use of the same coefficient for dissipation. An exception would be made in the case of the strap coil with turns separated for ventilation between them. Figure the area as usual, and then increase it by 50 per cent to make allowance for spaced-turn effect.

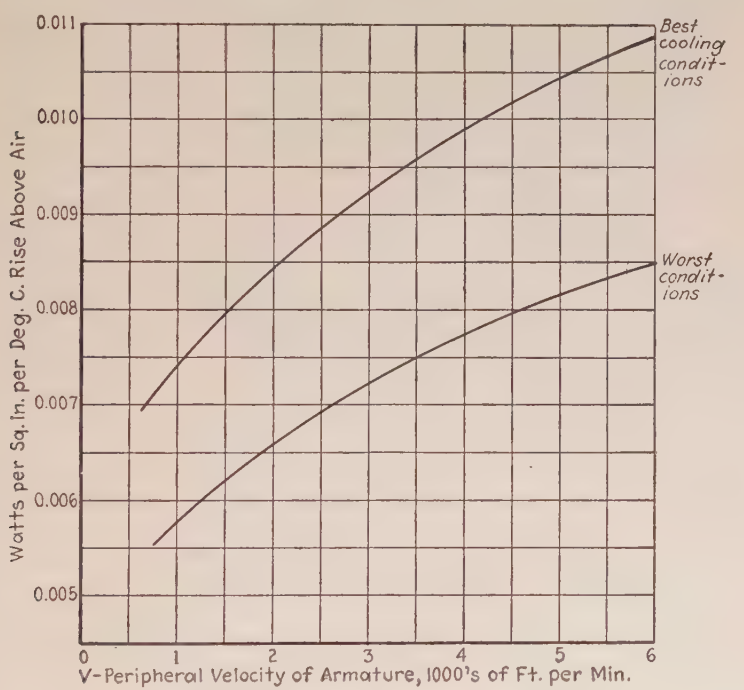


FIG. 122.—Dissipation cœfficients, field coils.

318. Series Field Loss.—Assuming the series field to be placed around the outer end of the shunt coil and separated from it by an air space, the series turn section can be most easily found by making it the same as the interpole conductor section; it may be made heavier if greater stiffness is needed, and in most cases it might well be heavier, there being plenty of room for it. Temperature rise need not be figured, if its conductor section is at least as great as for the interpole conductor.

319. Bearing Friction and Windage.—Bearing friction can be calculated, but it is much smaller than windage, which cannot be calculated except empirically from data taken on a given line of machines. The sum of the two losses runs about as follows:

Rating, kilowatt	5	10	20	50	100	200
Bearing and windage loss, per cent	4	3.5	3	2.5	2	1.8

320. Efficiency Calculation.—There are no particular difficulties encountered in computing the full-load overall efficiency of the machine. Its maximum efficiency, if that becomes of interest, occurs very close to the point when fixed and variable losses are equal.

CHAPTER XXXII

DESIGN OF A DIRECT-CURRENT MOTOR

A practicing designer, with extended knowledge of the standards, practices, and facilities of his organization, and having plenty of experience in the field of design, designs an electrical machine; that is one thing. A student in a design course, having only a little knowledge of various standards and practices, and no experience even in making the kinds of calculations and allowances called for in design work, attacks a design problem; that is quite another thing. The two situations are widely different in several respects.

The designer is hired to produce designs commercially and to lose no time about it. The student, who may never become a designer, is studying electrical engineering. The designer may take up his assigned job, reach for a specification sheet, consult various curves, refresh his memory by referring to a standards book, perhaps look up some performance data on similar machines already built, run a slide rule for a while, and complete the design, as far as he is concerned, in half an hour. Put the student up against the same problem: he has, usually, one book to consult; he wonders vastly about what to do; he makes a beginning; every step in twenty, or oftener, he makes a mistake in application of theory or in calculation, and the instructor usually has to find that there is a mistake and to indicate where it may be found in the work; finally, after many hours of struggle, the student has worried through to the attainment of a complete set of figures which, if translated into actual material, usually might work reasonably well as a machine. The designer has tucked all of his important results onto one sheet of paper. The student, unless furnished with a proper form, has a mass of figures and notes covering many sheets, out of which no one could make any sense but himself, and often he cannot.

321. Value of Completing a Design Problem.—As stated in a previous chapter, the writer believes that most of the time in an undergraduate required design course should be given over to the basic studies underlying design work. But there is much to be

said in favor of having the student go through at least one complete design job.

1. In preliminary courses, the student has been exposed to theory; the poor record he makes in applying theory in a design problem is itself proof that he needs to make that application. And applying the theory is the one sure way of getting hold of it.

2. Accuracy becomes paramount. In early courses, the problems are short; a mistake in a problem ruins only that problem. In a lengthy design problem, the student who carelessly makes a mistake early in his work reaps a rich harvest of trouble if it is not discovered fairly early. No one will deny that engineering students need plenty of opportunity for learning how to be accurate.

3. The discipline undergone in making accurate applications of theory, in making a record of many calculations in such a way that each figure later may be referred to and understood, and in revising preliminary values so as to attain a better result—the discipline involved in these processes is a much needed factor.

4. The relative importance of one item as compared with another is best brought out in a design problem. Until one has worked with various factors and has seen their relative effects, one can have no proper judgment of them. Courses in theory cannot develop judgment, for theory problems must be stripped of unimportant factors in order that the desired theory may be emphasized and developed. The true design problem presents itself stripped of nothing; it fairly bristles with questions to be answered; it involves decisions as to all the factors, big and little, important and unimportant, that the given apparatus itself represents. If a student has any possibility for developing that valuable quality known as engineering judgment, design work will give him the chance to bring it out.

322. The Design Form.—When creating a form for holding the figures of the design of a direct-current machine, two extremes present themselves. On the one hand, the form may be made as short, concise, lacking in detail, and cryptic as that which the professional designer uses, or on the other, the form may be made so complete as to include space for every term of every calculation made. For undergraduate use, the form should lean towards the latter kind.

The direct-current machine form given here has been revised time and again by the writer, who is still far from being satisfied

with it. It will be noted that it contains the appalling number of 378 items. But a large number of the items simply provide entry space for figures incidental to calculations and not important in themselves. The reasons for so providing space are two: to prevent the possibility of a term in a calculation being completely overlooked, and to make the early records so complete that they can later be referred to easily and accurately.

Quite a few of the items are repeats, as indicated by the use of the bracket following the descriptive phrase of the item. When a once-used item is needed again, it is often best to repeat it in the record, for then the instructor can easily check the immediate calculation; also, by gathering together the figures that enter a given computation, direct slide-rule operations can then be made in many cases; otherwise, the figures would be collected on scrap paper, and later thrown away.

A good share of the extensive detailing of the form is due to the inclusion of features often neglected or allowed for by judgment, if a practical designer were doing the job. It must be remembered here that the student does not have this judgment, and that the only way in which he can acquire it is to take all factors into account in order to evaluate the importance of each. He must *do too much work* in order to find out *how little* and *what kind* of work he really would have needed to do to accomplish an equally good result, had he been equipped with much experience.

323. The Order of the Form.—In general, the items as numbered are not and cannot be always sequential. To attempt to outline a table of values occurring in true sequence would be to dictate rigidly the procedure to be followed. This is undesirable in the extreme. Sequence must vary as conditions vary. The student will nearly always find that the sequence used in his preliminary calculations may be changed with advantage in later calculations. Proper sequence is, of course, adhered to as far as it is of advantage to do so. In general, the items which naturally group together are collected under a proper head, which means that preliminary work must involve a good deal of skipping around among various items before much sequential work appears.

324. Symbols Used; Cross-references; Form of Discussion.—The various quantities appearing in the design form are designated first by an item number, second by a descriptive phrase, and third by a symbol. It will be found that the symbols

correspond to those used for the same quantities where they have been treated in the earlier chapters.

Symbols starred with an asterisk (*) are of temporary significance and are used, in some cases, several times in the form. The asterisk indicates that the symbol has a temporary significance, that it stands for a quantity needed in the group of quantities where it is found, and that once so used, its meaning is usually dropped. This allows the symbol to be used again. This device was invented here in order to avoid using different symbols for every insignificant quantity. Symbols are the bane of design work, and yet must be used in reasonable number. It will be noted that very few Greek letters have been used. The writer's typewriter is not equipped with them, and it must be confessed, his memory suffers in the same respect. It is perhaps true that most engineers and engineering students are likewise handicapped.

When such items as fringing coefficient, flux leakages, slot-insulation allowances, tooth iron loss, etc. appear, they are accompanied by a notation in parenthesis, as in Fig. 156 or Table IV. Thus, if one has forgotten the meaning of a symbol or the treatment leading to some quantity, the reference shows the way to that part of the text where the quantity is treated in detail.

This reference system more or less obviates the necessity for any extended detailing of the discussion which follows. The succeeding articles take up in a very brief way the general method of designing a machine, but discussion of the specific design problem undertaken is reduced to the minimum. This is as it should be. Granted that the reader has a sufficiently clear conception of the work of the foregoing chapters, or that, in detailed spots, he has a ready-reference ability with respect to it, then, if the design items are sufficiently and clearly labeled and detailed, the figures as they appear in the form stand for themselves.

325. The Specific Design Problem.—The specific problem selected is the design of a 230-volt, 450-r.p.m., 250-horsepower shunt motor, self-ventilated. A design of it will be carried in such a way as to leave room for improvement, allowing the data to indicate how things could be bettered.

326. Preliminary Estimates.—Items 1 to 4 simply give the best guesses to be made on efficiency and currents. They are

set down here and used until they can be bettered by substituting calculated results for guesses.

327. Armature D^2L, Diameter, Length, Poles, Etc.—The preliminary determination of these items at once furnishes an illustration of how a rigidly dictated sequence is not desirable. What really is done here is this: The various sources of information given in earlier chapters are looked up, some figuring is done on the scratch pad, and then several items are decided upon and carried over bodily into the design form.

By use of the output coefficient curve (which has to be extrapolated in this case) D^2L_a is found very approximately. The number of poles is tried out at six, and D^2L_a is split up into proper D and L_a values to given good proportions to the pole, pole coils, and armature coils. The number of slots may next be fixed, to give a reasonable number of slots per pole. Pole-face area may be figured, pole arc being selected to cover as much ground as possible and yet leave proper interpole space. Without doing anything more about fringing now than to make an allowance for it, a proper gap average density is selected, and flux per pole found.

328. IR Drops; Generated Voltage.—Before or after the above steps are made—it makes no difference—preliminary allowances for the various voltage drops in the armature circuit must be made. These simply are the best guesses one can make, and of course are all, except brush drop, open to later correction when actual results begin to come in.

329. Number of Conductors; Reaction; Armature Winding.—With approximate figures for flux and generated voltage at hand, the number of armature conductors per path is calculated. The type of winding—lap or wave—is decided, so that total armature conductors may be figured. The first demand for change is now met, for, computing conductors per slot, the chances are that the number will be a whole number plus a fraction; whereas, the double-layer winding requires certain definite things of this value. The number of slots can be shifted to another value, or the number of conductors changed, or the gap-flux density changed, or all three, until a satisfactory number of conductors per slot is found.

At this point it is wise to compute q, the ampere-conductors per inch of periphery. In our present case, q has a rather high value, and trouble may be expected from it. If q seems to be

satisfactory, the earlier figures may be retained; if not, they can be modified. In fact, to work a revision, q might now be selected and other values be warped to fit.

330. Slot and Tooth Dimensions; Circular Mils per Ampere; Volts per Bar.—A check on the winding figures can be made to see if the group coils can, if necessary, be split up into single coils to give enough commutator bars; this, to keep volts per bar well below 15.

If that feature is satisfied, a value of circular mils per ampere can be selected, which, with the estimated armature current and known number of paths, will fix the section of the armature conductors. Then follows some juggling to fit the conductors in properly proportioned slots and yet leave teeth which will roughly equal the slots in volume and have reasonable tooth-root densities.

331. A Different Sequence for Preliminary Armature Design. As stated above, there are many ways in which the preliminary armature design may be approached. A sequence different from the above general attack will now be given, one in which the steps appear to be taken in a more definite order.

Instead of estimating a D^2L_a value, let the armature diameter D be selected and also the number of poles and the pole arc. Let q be selected, and B_{ga}. Let us now speak of an inch of periphery under the pole arc. q gives the amperes found in this much of the arc. If, furthermore, we speak of an axial inch, then a square inch is defined under the pole arc. The flux through this unit area is equal to the B_{ga} value selected. It should be evident that, with current and flux fixed, the peripheral (tangential) force produced by the interaction between the current and the flux is open to computation. No formula is given for this value; the student can easily derive it.

The force of this unit area times armature radius gives an increment of torque. This, divided into torque required, gives total square inches of pole-face area needed; and this total area, modified by a properly estimated fringing coefficient, as pole arc and number of poles are known items, will give armature length. If desired, the D^2L_a value can at once be found and checked against the value given separately by C_o. No great alarm need be felt if the two values so figured differ by, say, 25 per cent.

One way of arriving at the number of slots is simply to select a reasonable number per pole pitch, which will then fix the slot

pitch. Slot current is thereby fixed, as q is known. The number of paths being known from number of poles and type of winding, the current of the armature conductors is figured; whereupon, the number of conductors per slot is fixed. With this sequence, the conductors per slot will only by good luck be a proper number for use with a double-layer winding; at this point, number of slots and the value of q can be shifted until conductors per slot is an acceptable number. The total number of conductors Z, thus arrived at, must of course check approximately that which would be obtained by using the flux per pole, which can be found, and the speed, and the estimated generated voltage.

From here on, it is a matter of fitting conductors into slots so that current density and tooth-flux-density limitations are reasonably satisfied.

It should be apparent by this time that the attack on the problem can be greatly varied.

332. Preliminary Estimate of Temperature Rise.—If all of the above values have been decided upon, and if they all, or nearly all, seem to be reasonable values, one is then warranted in making a preliminary estimate of armature temperature rise. Inasmuch as this may come so far off the mark that much revision will be necessary, it would have been distinctly unwise to have gone into any detailed calculations up to this point.

Slot loss is now approximately calculated. Tooth loss is found approximately, after making an estimate of maximum gap density sufficiently above average gap density to allow for expected gap flux distortion due to reaction. Core depth is figured by allowing for a reasonable core density, following which core loss is approximately computed.

A reference to the curve (Fig. 115) giving axle sizes, and a scaled sketch of proposed armature-hub design will show how much axial flow of air inside the core can be expected. In a large machine, plenty of flow can be expected with confidence, and the point need hardly be raised; but the smaller machines vary from giving little axial flow space to none at all. This study indicates whether to include radial ducts and how useful they will be.

The probable heat dissipation of the armature body is now obtained in an approximate but trustworthy manner, and the armature temperature rise predicted. If this is within 5 or 10° of 40° rise, well and good; it is probable that a little increase in diameter, or length, or both, or the addition of a duct if plenty

are not already figured on, can fix the machine all right; in some cases, a more or less complete revision of values is necessitated. If the temperature rise is 15 or 20° off, and if all densities, constants, etc., are of normal value, it is likely that one or more errors are lurking in the work somewhere. In any case, armature rise must be acceptable before going ahead.

333. Gap Study.—Once the preceding figures are in fairly good shape, and it seems that an armature has been designed that will permit the rest of the design to be carried through successfully, closer attention can be paid to detail. But it is a great waste of effort to go on from here unless the figures so far attained are made quite sure of.

Before making the complete and final gap study, a preliminary computation should be made to see what is going to happen at the strong and weak pole tips at full load. The average gap density and the maximum gap density from which tooth loss was figured are known. So are the armature-reaction figures. A preliminary, even though quite rough calculation should be made to see what the gap design must be like. This scratch-pad study may indicate desirable changes in the armature design. If not, then the gap study can be carried out in full.

In the design carried out here, q is undesirably high. Referring to the curves given for typical air gaps (Fig. 116) the minimum gap g_1 is selected at 0.13 inch, the other two gaps being chosen at 0.23 and 0.33. The average gap density to be attained is 49,000; realizing that q is high, a rather high allowance for maximum gap density was made, this being placed at 70,000. By methods outlined elsewhere, the gap design is arrived at (Fig. 123). Curve ABC shows the full-load distribution; $A'BC'$, the no-load distribution obtained with the same M_{gt}.

The average and maximum gap densities are attained, but the density at the weak tip is actually reversed. Tentative earlier figures on gap details would have indicated as much, and in the usual case, such a high distortion would have been avoided. We shall go on, however, with the present figures, as the difficulties encountered in this high-reaction machine will be highly instructive.

Item 140, the gap-and-tooth NI, is 4,400. The next step is to investigate the stability, since it is found that if no stabilizing winding is used, the flux reduction due to reaction will be excessive. The game then is to divide the 4,400 ampere-turns into

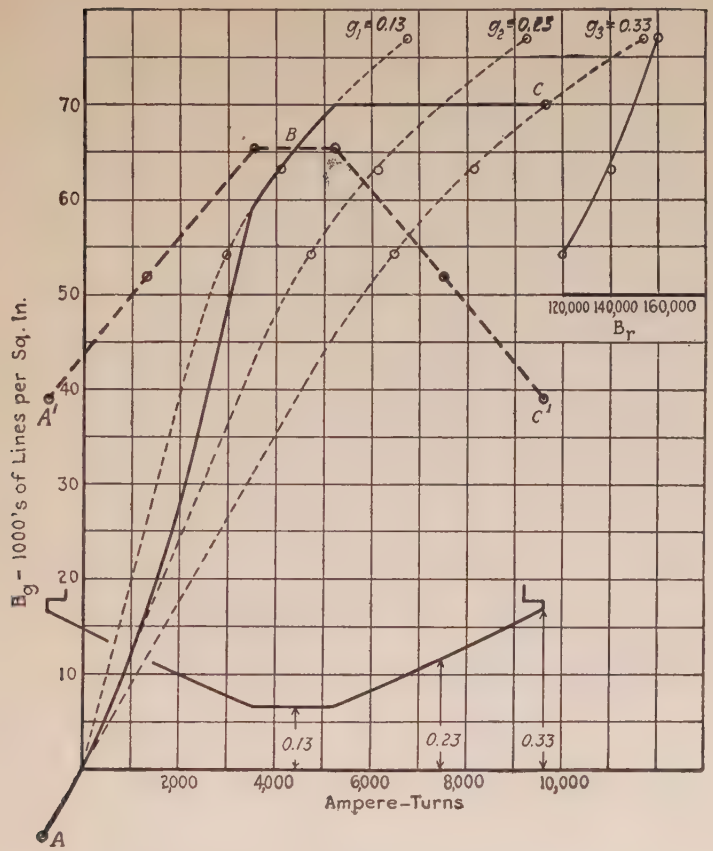


FIG. 123.—Flux distribution and gap design.

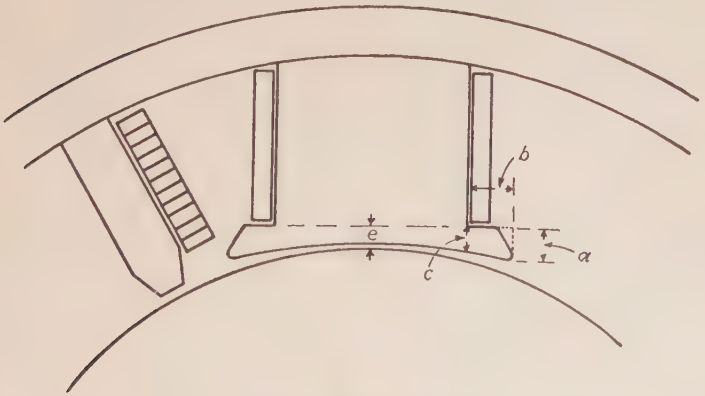


FIG. 124.—Scale drawing.

two parts, one furnished by the shunt, and the other part coming from the stabilizing winding. In this case, a series winding of a half-turn per pole will give more than enough series or stabilizing m.m.f. The remainder is left up to the shunt coil. If there seemed to be any question as to whether the no-load speed might not be sufficiently above the full-load to give stability, the no-load flux obtained by this design should be figured out; but here, there is evidently no cause for alarm.

334. Pole and Yoke Design.—The design form is straightforward enough here to need no additional discussion.

335. Shunt Field Coil Design.—The preliminary figures indicate a wire section that is so far from either of the two nearest sizes that it is decided to build a coil of those two sizes placed in series. Before doing this, it is necessary to find just about what the final coil dimensions will be; so the preliminary design is carried out in terms of the desired wire size, insulation allowance being made equal to that of the nearest sizes. The temperature rise comes out too low, 28°.

In starting the two-wire design, the coil thickness is reduced to as nearly what it should be as estimate can predict, in order that the desired temperature rise shall be obtained. The final design is straightforward enough to be clear in itself. Note that whereas it is desired to have 4,705 ampere-turns per coil, the actual NI amounts to 4,920, with a temperature rise of only about 34°.

336. Commutation.—The commutation items follow in sequence as nearly as possible, granted a certain attack. Previous statements on interpole design will be made much clearer now by their application to this specific design; thus a discussion will be in order.

After determining the commutation type (and having long before this decided to use a unit interpole arc, when pole arc was fixed), the actual interpole arc is figured out, and "face density" is selected; this is really the density at the P-Q level. Flux per axial inch at this level follows. A unit leakage value is looked up. Tentative interpole total m.m.f. is obtained, and leakage per axial inch figured, which here is at an excessively high value (immediate evidence of the undesirably high armature reaction). Leakage is added to the above resultant flux per inch to get a total, and selecting as high a pole-core density as judgment permits, the core thickness is obtained.

A tentative k_i is looked up, and this, divided into the tentative excess interpole NI, M_e, gives resultant gap NI. The resultant flux per inch at the P-Q level, which is ϕ_i, divided by gap NI, gives the unit ϕ_i value, or flux per axial inch per ampere-turn. This, with the known ratio t/s, and the use of Fig. 106, enables the ratio g/t to be found, when the gap g becomes known. This gap is satisfactorily long as compared with the minimum main-pole gap, in this case; it does not always come out so; if not, the gap is arbitrarily increased, and previous figures rectified.

From here on, all other basic commutation flux values are found in the usual way, leading to the solution of the equation which equates delaying to reversing flux, and in which L_i is the only unknown. Values which had to await the solution are distinguished by being placed in the second column.

As is obvious from the various commutation flux values, some are very much higher than others. If one wishes to make the commutation analysis much simpler, this means that there is ample opportunity to do so.

337. Interpole Coil Design.—This is quite straightforward. In our specific design, even though the interpole core is very thick, and in spite of using a coil thickness of 1.25 inches, the temperature rise is 57.5°, which is too much. Any greater coil thickness is prevented by the nearness of the main pole. About 8 turns might be put in the inside layer, instead of all 11 turns, and 3 in a second layer around the outer end. But in this case, this means simply resorting to desperate measures to patch up a bad design.

338. Commutator and Brush Design.—Nothing need be added here, except to say that the computed rise, 59°, is likely high by at least 10°, for no allowance was made for cooling effect of the cored commutator hub.

339. Summary of Losses; Corrected Currents; Efficiency.—The losses are summarized at first, so that when armature circuit losses are added to the output, armature current may be calculated. The new value of I_a is now used to correct the variable losses taken as a lump sum, as it is assumed that the total variable loss varies as the square of the armature current. This is not exactly true, for commutator electrical loss does not vary in this manner. The error is inconsequential. Also, the corrected current is not exactly right, for by this method it is only the first

approximation in a series of approximation steps; but again, the error is slight.

The field loss is now added in, line current is found, and the efficiency figured out.

If the preliminary assumptions of efficiency and currents had been very bad, the corrected values might now have to be used in correcting some of the design figures now on record. For instance, a corrected I_a, to be rigorous, calls for the correction of q, and of reaction effects, including tooth loss. Usually, with any kind of a decent start, one need not go back over the design and bring it to such a state of close accuracy.

340. Discussion of the Design.—The armature temperature rise is satisfactory, indicating that armature volume is also. Reaction is excessive, as is evidenced by high distortion, large flux reduction calling for stabilizing turns, and an interpole winding that is nearly impossible from the heating standpoint. The interpole leakage flux is out of reason as compared with its active flux. The armature is strong and the field weak. It is a "strong-armature, weak-field" machine. The whole machine needs a redesign on an eight-pole basis, which furnishes a very good job for the student to tackle.

Problems in Redesign of Armature

50. Revamp the preliminary design of the armature, keeping to the limitations given below:

8 poles
11 to 15 slots per pole
$q = 600 \pm 50$
$B_{ga} = 45{,}000 \pm 5{,}000$
$B_{gm} = B_{ga} + 12{,}500 \pm 2{,}500$
$B_{rm} = 145{,}000 \pm 5{,}000$
$M_{gt} = 1.3M_t$
Circular mils per ampere $= 700 \pm 100$
$T/P_p = 0.7$
$d/s = 4 \pm 0.5$
$s/t_{ave} = 1 \pm 0.25$
$g_{min} = 0.13$

Note that with such limitations imposed, D^2L_a cannot be independently fixed. It will be found to be in the neighborhood of 12,000. This armature will likely run well below a 40°C. rise.

51. Figure the armature losses and armature temperature rise for the redesign.

52. Make the air-gap design.

53. Complete commutation figures and design the interpole body.

54. Design the commutator.

55. Design the field system, complete.

56. Complete the design in all respects.

Direct-current Motor Design Form

Kind: Interpole shunt	Type: Open, self-ventilated.
Volts: 230.	Horsepower: 250.
R.p.m.: 450.	Temperature rise: 40°C.

	Symbols		
Preliminary estimates:			
1. Motor efficiency, per cent (Fig. 117)	%	92.5	89
2. Motor line amperes	I_l	876	91.1
3. Field amperes	I_f	16	3
4. Armature amperes	I_a	870	88
Armature data:			
5. Armature diameter	D	32	14
6. Armature perimeter	....	100.5	44
7. Peripheral velocity, feet per minute	V	3,765	
8. Number of poles	P	6	4
9. Pole pitch, inches	P_p	16.75	11
10. Core length including ducts	L_a	8.375	5.7
11.	D^2L_a	8.575	1110
12. Number of ducts	$*n$	2	1
13. Duct width	$*w$	0.375	.375
14. Gross core iron length: $L_a - nw$	L_g	7.625	5.325
15. Number of slots	S	75	11
16. Slot pitch $s + t$	....	1.34	1.00
17. Slot pitches in pole arc	$*n$	9	7
18. Pole arc, inches: $n(s + t)$	T	12.07	7
19. Pole arc/pole pitch: T/P_p	....	0.721	.636
Gap values:			
20. Average gap clearance (Fig. 116)	g_a	0.22	0.2
21. Average gap/duct width: g_a/w	....	0.587	.534
22. Duct fringing coefficient (Fig. 88)	k_{fd}	0.75	
23. Addition to L_g for duct fringing: $nk_{fd}w$	$*l$	0.563	
24. Fringing coefficient: $(L_g + 1)/L_g$	k_f	1.073	1.05
25. Arbitrary pole face area: L_gTk_f	$*A$	98.8	39.1
26. Flux per pole $\times (10^{-6})$	Φ	4.84	~~1.58 × 10~~
27. Average gap density: Φ/A	B_{ga}	49,000	~~40,400~~
Winding data:			
28. Winding, wave or lap	....	lap	lap
29. Number live conductors	Z	600	704
30. Number live + dead conductors (if any)			704
31. Conductors per slot	....	8	16
32. Coils per slot	.....	4	2
33. Number of live coils	....	300	
34. Number live + dead coils (if any)			
35. Average volts per bar	....	4.6	
36. Amperes per path	I_c	145	
37. Ampere-conductors per inch of periphery	q	865	
38. Reaction NI at pole tip: $qT/2$	M_t	,220	2410

Note: Install half the full possible number of equalizer connections.

Mean turn, etc.:			
39. Extend straight beyond slot	X	0.625	.75
40. Coil side length: $L_a + 2X$	$*a$	9.625	
41. Length of a curl: $\pi d/2$, approximately	$*b$	2.5	
42. Angle Θ (see Fig. 107)	Θ	30	
43. End connection: $P_p/\cos\Theta$	$*c$	19.3	

*Symbols marked with an asterisk (*) are usually of temporary significance; they may be used in other places, to stand for other quantities.

	Symbols		
Mean turn, etc.:			
44. Coil mean turn: $2(a + b + c)$	C	62.85	49.5
45. Conductor section	M	73,600	25,950
46. Number of paths	....	6	2
47. Armature resistance at 1.05 ohms/circular mil-inch	R_a	0.00748	0.177
48. Circular mils per ampere	....	508	590
IR drops, generated voltage:			
49. Armature *IR* drop	....	6.5	15.5
50. Interpole *IR* drop	....	4.00	2.0
51. Series field *IR* drop			
52. Brush drop	....	2.0	
53. Total armature circuit drop	....	12.5	
54. Generated voltage	E_g	217.5	.223 210
Slot and conductor data:			
55. Round, square, or rectangular conductor?	....	rectangular	
56. Bare width, conductor	*a	0.09375	
57. Bare depth (or size if round)	*b	0.625	.238
58. Section, circular mils [45]	M	73,600	25,950
59. Double cotton covered width (if double cotton covered)	*a		
60. Double cotton covered depth (if double cotton covered)	*b		
61. Conductors across slot	*m	4	
62. Conductors down slot	*n	2	
63. Conductors per slot: mn [31]	....	8	
64. Allowance across slot (Table VI)	x	0.175	
65. Bands or wedges?	....	wedges	
66. Allowance down slot (Table VI)	y	0.390	
67. Slot width: $am + x$	s	0.550	.510
68. Slot depth: $bn + y$	d	1.64	1.25
Tooth data:			
69. Slot pitch, top [16]	$s + t$	1.340	1.00
70. Slot width [67]	s	0.550	.510
71. Tooth-top width	t	0.790	.490
72. Tooth-root width	t_r	0.650	.312
73. Tooth ratio: t/t_r	k	1.212	1.57
74. Average gap density [27]	B_{ga}	49,000	38,500
75. Maximum gap density	B_{gm}	70,000	53,000
76. Maximum tooth-root density	B_{rm}	150,000	170,000
Tooth loss:			
77. Unit tooth loss (Fig. 119)	W	0.104	.11
78. Frequency	f	22.5	20
79. Gross iron length [14]	L_g	7.625	
80. Tooth depth [68]	d	1.64	
81. Number of teeth [15]	S	75	
82. Tooth loss	I	1,430	202
Core loss:			
83. Flux per pole [26]	Φ	4.84	1.49
84. Tooth root diameter: $D - 2d$	D_1	28.72	11.5
85. Inside core diameter	D_2	18.72	7
86. Radial core depth	*c	5.00	2.25
87. Core section: $0.93cL_g$	*a	35.5	11.15
88. Average core density: $\Phi/2a$	....	68,200	66,700
89. Unit core loss per pound (Fig. 21)	W	0.09	
90. Frequency [78]	f	22.5	20
91. Weight: $0.28L_g(D_1^2 - D_2^2)\pi/4$	*w	796	0
92. Core loss: wfW 1.25	II	2,000	218

	Symbols		
Slot loss:			
93. Slot length [10]	L_a	8.375	6.7
94. Ampere per conductor [36]	I_c	145	44
95. Conductor section, circular mils [45]	M	73,600	25,950
96. Number of conductors [29]	Z	600	704
97. Eddy current coefficient (Art. 309)	f_e	1.15	1.15
98. Slot copper loss: $(ZL_aI_c^2\ 1.05f_e)/M$	III	1,734	361
99. Total of (I + II + III) losses	W_l	5,264	781.5
Dissipation: outer surface:			
100. Outer core area: πDL_a	A_1	767	234
101. Peripheral velocity [7]	V	3,765	2200
102. Dissipation coefficient (Fig. 120)	k_{d1}	0.058	.043
103. Average temperature rise of air	θ_1	6	5
104. Rise of iron above air: $\Theta - \theta_1$	Θ_1	34	35
105. Watts dissipated: $\Theta_1A_1k_{d1}$	W_1	1,510	352
Dissipation: ducts, ends:			
106. Core end area (one): $(D^2 - D_2^2)\pi/4$	$*a$	529	115.6
107. Width of fingers [13]	$*m$	0.375	.375
108. Number fingers per plate [15]	S	75	44
109. Finger area, one plate: $mS(D - D_2)$	$*b$	374	115.5
110. Number of ducts [12]	n	2	1
111. Radial area: $a(2n + 2) + b(n + 2)$	A_2	4,670	810
112. Duct air velocity: $V/4$	v	941	550
113. Dissipated coefficient (Fig. 39, short ducts)	k_{d2}	0.025	.016
114. Average temperature rise of air	θ_2	9	7
115. Rise of iron above air: $\Theta - \theta_2$	Θ_2	31	33
116. Watts dissipated: $\Theta_2A_2k_{d2}$	W_2	3,620	427
Dissipation: Inner Surface:			
117. Inside core area: $L_g\pi D_2$	A_3	449	
118. Inside core air velocity: $V/4$	....	941	
119. Dissipated coefficient (Fig. 35, round duct)	k_{d3}	0.013	
120. Average temperature rise of air	θ_3	0	
121. Rise of iron above air: $\Theta - \theta_3$	Θ_3	40	
122. Watts dissipated: $\Theta_3A_3k_{d3}$	W_3	233	
Armature temperature rise:			
123. Total watts dissipated at $\Theta = 40$, $W_1 + W_2 + W_3$	W_d	5,363	804
124. Approximate temperature rise: $40(W_l/W_d)$	Θ_p	39.3	
End-connection temperature rise:			
125. Section width of group coil [67]	s	0.55	
126. Section height: $d/2$, approximately (68)	....	0.84	
127. Section area (surface) per axial inch	$*a$	2.78	
128. Watts per axial inch: Number of conductors × $1.05\ I_c^2/M$	$*w$	1.20	
129. Watts per square inch: w/a	....	0.43	
130. Class of ventilation (Fig. 121)	....	II	
131. Dissipated coefficient	k_d	0.028	
132. Temperature rise: $(w/a)/kd$	. .	15	

Gap-and-tooth magnetization data: gap design:

133. Dimensions, ratios:

t: 0.790 [71] t_r: 0.650 [72] k: 1.212 [73] s: 0.550 [67]

$s + t$: 1.340 [16] t/s: 1.435 s/t_r: 0.845

CARTER'S COEFFICIENTS

134	g_1	0.13	s/g_1	4.23	C_1	1.22	C_1g_1	0.1585
135	g_2	0.23	s/g_2	2.39	C_2	1.137	C_2g_2	0.2611
136	g_3	0.33	s/g_3	1.665	C_3	1.10	C_3g_3	0.3630

$$k_1 = \frac{0.93t_r}{s + t} = 0.451 \qquad k_2 = 1.066$$

(k_2 taken as unity up to B_r = 140,000; at 160,000, see Fig. 95)

MAGNETIZATION DATA

	B_r	$B_g = B_rk_1k_2$	Gap NI			Tooth NI		Gap + tooth NI		
			g_1	g_2	g_3	$d = 1$	$d = 1.64$	g_1	g_2	g_3
137	120,000	54,100	2,680	4,420	6,150	170	280	2,960	4,700	6,430
138	140,000	63,100	3,130	5,170	7,170	600	980	4,110	6,150	8,150
139	160,000	77,000	3,830	6,300	8,750	1,800	2,950	6,780	9,250	11,710

140. Gap and tooth NI, M_{gt} (Fig. 123): 4,400

GAP SPECIFICATIONS (From Fig. 123)

		Preliminary to field coil design	Revised values, following design of field coil
141	g_e	0.13	No change
142	g_t	0.32	No change
143	X	20 per cent (Art. 242)	No change
144	g_a	0.22	No change

	Symbols		
Stability: design of stabilizing winding:			
145. Gap and tooth NI [140]	M_{gt}	4,400	
146. Full load B_{ga} [27] (Fig. 123)	....	49,000	
147. No load B_{ga} (Fig. 123)	....	54,300	
148. Difference in per cent of no load value	....	10	
149. Full load generated volts [54]	....	217.5	
150. No load generated volts	....	228	
151. Difference in per cent of no load value	....	4.6	
152. Per cent speed drop, no load to full load, approximately	....	−5.4	
153. Stabilizing NI per cent per pole to be at least	....	5.4	
154. Stabilizing NI as per cent of M_{gt}	....	238	
155. Stabilizing turns per pole	....	½	
156. Stabilizing NI per pole	....	435	
157. Remainder of M_{gt}, furnished by shunt coil	M'_{gt}	3,965	
NOTE.—Put series stabilizing turn around outside of outer end of shunt coil.			
Pole piece:			
158. Pole arc, inches [18]	T	12.07	
159. One-half pole arc, degrees	α	21.6	
160. Pole arc chord: $D \sin \alpha$	....	11.38	
161. Pole core width	W_p	8.0	
162. Dimension (Fig. 124)	$*a$	1.375	
163. Dimension (Fig. 124)	$*b$	1.8	
164. Dimension (Fig. 124)	$*c$	1.0	
165. Dimension (Fig. 124)	$*e$	0.937	
166. Pole core length: $L_g - 2g_a$	L_p	7.935	
167. Pole core section: $W_p L_p$	....	63.4	
Pole leakage, NI:			
168. Leakage inches: $L_p + W_p/2$	$*L_l$	12	
169. Leakage ϕ per axial inch per NI (Table VII)	$*\phi_{ml}$	7.5	
170. Approximate leakage NI estimated from M_{gt}	$*M_F$	5,400	
171. Leakage flux/10^6: $L_l \phi_{ml} M_F/10^6$	Φ_l	0.49	
172. Useful flux/10^6 [26]	Φ	4.84	
173. Pole core flux: $\Phi_l + \Phi$	Φ_p	5.33	
174. Pole core density	....	84,000	
175. Pole core NI per inch	....	35	
176. Pole core path length: $H + 1$	....	8	
177. Pole core m.m.f.	M_p	280	
Yoke:			
178. Yoke density	....	85,000	
179. Yoke axial length: $1.7L_a$	....	14.25	
180. Yoke thickness	....	2.2	
181. Yoke mean radius (from sketch)	....	25	
182. Yoke path length	L_y	13.1	
183. Yoke NI per inch	....	35	
184. Yoke NI	M_y	460	
Shunt m.m.f.:			
185. Gap and tooth NI from shunt coil [157]	M'_{gt}	3,965	
186. Desired shunt NI per pole: $M_y + M_p + M'_{gt}$	....	4,705	
Tentative shunt field coil design:			
187. Coil thickness, assumed	T	1.0	
188. Coil height (checked from drawing)	H	7.0	
189. Coil inside perimeter	P	32.17	
190. Add, to get mean turn: πT	....	3.14	
191. Mean turn	C	35.31	

	Symbols	
Tentative shunt field coil design:		
192. Volts per coil	E	38.35
193. Resistivity	....	1.05
194. Shunt NI to be obtained [186]	....	4,705
195. Wire section desired: 1.05 $NI\ C/E$	M	4,550
196. Next larger A.w.g. section, No. 13	M_1	5,170
197. Next smaller A.w.g. section, No. 14	M_2	4,109
If M differs from nearest obtainable by:		
(a) 6 per cent or less, use the nearer section and respecify air gap.		
(b) more than 6 per cent, design a two-wire coil.		
Preliminary design of coil in terms of desired (M) section:		
198. Diameter (bare) of M wire	D	0.0675
199. Add, for enameled single-cotton-covered insulation	....	0.0075
200. Diameter of insulated wire	D^1	0.075
201. Turns per layer: H/D'	t	93.3
202. Layers: T/D'	l	13.3
203. Turns: lt	N	1,244
204. Coil resistance: $1.05NC/M$	R	10.12
205. Current	I_f	3.79
206. Watts	W	145
207. Coil area: $2(H + T)C$	A	566
208. Dissipated coefficient: (Fig. 122)	k_d	0.0092
209. Temperature rise	Θ	28
Final shunt field coil design:		
210. Estimate of T for proper temperature rise	T	0.8
211. Mean turn	C	34.69
212. M/M_1	....	0.88
213. P/T	....	40.2
214. T_1/T: (Fig. 29)	....	0.58
215. T_1	T_1	0.51
216. Insulation diameter, No. 13	D_1'	0.0795
217. Layers, No. 13: T_1/D_1' (tentative)	l_1	6.4
218. Layers, No. 13, used	l_1	7
219. T_1, revised	T_1	0.556
220. T_2: $T - T_1$	T_2	0.324
221. Insulated diameter, No. 14	D_2'	0.0716
222. Layers, No. 14; T_2/D_2' (tentative)	l_2	4.5
223. Layers, No. 14, used	l_2	5
224. Mean turn, inner coil: $P + \pi T_1$	C_1	33.92
225. Turns per layer, No. 13	t_1	88
226. Turns, No. 13	N_1	616
227. Ohms, No. 13	R_1	4.25
228. Mean turn, outer coil	C_2	34.94
229. Turns per layer, No. 14	t_2	98
230. Turns, No. 14	N_2	490
231. Ohms, No. 14	R_2	4.37
232. Shunt coil ohms: $R_1 + R_2$	R	8.62
233. Current	I_f	4.45
234. Coil watts	W	171
235. Coil area	A	541
236. Temperature rise	Θ	34.4
237. Total turns	N	1,106
238. NI obtained by two-wire design	NI	4,920

COMMUTATION

	Symbols		
239. Commutation type (Table IX)	....	5	
Leakage; core density; gap clearance:			
240. Interpole face width: $0.9(s + t)$	$*w$	1.215	
241. Face density selected	$*B$	30,000	
242. Flux per axial inch: wB	$*\phi_1$	36,500	
243. Unit leakage (Table VII)	ϕ_{il}	11	
244. Reaction NI: $(N - 2)I_s/2$	M_r	6,100	
245. Tentative total NI: 1.5 M_r	M_i	9,150	
246. Leakage per axial inch: $M_i\phi_{il}$	$*\phi_2$	100,600	
247. Core flux per axial inch: $\phi_1 + \phi_2$	$*\phi_3$	137,100	
248. Core width	W_i	2.25	
249. Core maximum density: ϕ_3/W_i	....	63,000	
250. Tentative excess NI: 0.5 M_r	M_e	3,050	
251. Tentative k_i (Table VIII)	k_i	1.08	
252. Tentative gap m.m.f.: M_e/k_i	M_{ig}	2,820	
253. Gap ϕ per axial inch per NI: ϕ_1/M_{ig}	ϕ_i	12.9	
254. Ratio t/s	....	1.435	
255. Ratio g/t (Fig. 106)	....	0.45	
256. Gap clearance: $(g/t)t$	g	0.355	

COMMUTATION FLUXES

	Symbols		
Slot flux:			
257. Conductor current [36]	I_c	145	
258. Conductors per slot [31]	....	8	
259. Active slot current	I_s	1,160	
260. (Table IX)	k_{cs}	0.75	
261. Commutated slot current: $K_{cs}I_s$	I_{cs}	870	
262. Slot width [67]	s	0.55	
263. Slot top density	B_s	5,060	
264. Slot depth [68]	d	1.64	
265. Effective slot flux per axial inch	ϕ_s	2,770	
266. Slot flux-linkage change: $4L_a\phi_s$	Φ_s	92,800	
Flux over slot: under interpole:			
267. (Fig. 109)	ϕ_{s1}	11	
268. Flux per axial inch: $I_{cs}\phi_{s1}$	....	9,600	
269.	L_i	.....	13.45
270. Linkage change: $L_iI_{cs}\phi_{s1}$	Φ_{s1}	.....	129,000
Flux over slot: not under interpole:			
271.	ϕ_{s2}	6	
272. Flux per axial inch: $I_{cs}\phi_{s2}$	....	5,200	
273. $(2L_a - L_i)$	....	.....	3.3
274. Linkage change: $(2L_a - L_i)(I_{cs}\phi_{s2})$	ϕ_{s2}	.....	17,000
End-connection flux:			
275. Slot depth minus wedge $= 2h$	a	1.44	
276. Group coil section width $= s$	b	0.55	
277. Ratio $2h/s$ or a/b	....	2.62	
278. (Fig. 107)	ϕ_{ecx}	0.8	
279. Flux per inch: $I_{cs}\phi_{ecx}$	....	700	
280. Ratio h/s	....	1.31	
281. (Fig. 107)	ϕ_{ecy}	1.3	
282. Flux per inch: $I_{cs}\phi_{ecy}/2$	....	570	
282*a*.	X	0.625	
283. Space between end connections	$*a$	none	
284. $s + a$	....	0.55	
285. $s + t_r$	....	1.20	
286. Sin Θ: $(s + a)/(s + t_r)$	....	0.458	
287. Θ, degrees (angular)	Θ	27.3	
288. Tan Θ	....	0.515	
289. One-half pole pitch (18)	$*b$	8.4	
290. b tan Θ	Y	4.33	
291. Slot pitch [69]	....	1.34	
292. Slot pitch times basic ϕ_{er} (Fig. 108)	ϕ_{er}	0.402	
293. Reaction NI [244]	M_r	6,100	
294. Flux per axial inch	....	2,450	
295. Average flux per axial inch from plot like Fig. 107*e*	ϕ_e	2,000	
296. $X + Y$	....	4.95	
297. Linkage change: $4(X + Y)\phi_e$	Φ_e	39,000	
Interpolar space reaction flux:			
298.	ϕ_a	2	
299. Flux per axial inch: $M_r\phi_a$	....	12,200	
300. $(2L_a - L_i)$ [273]	....	.....	3.3
301. Linkage change: $(2L_a - L_i)M_r\phi_a$	Φ_a	.....	40,000
Reversal flux:			
302. (Fig. 106)	ϕ_r	8.5	
303. [252]	M_{i0}	2,820	
304. Flux per axial inch: $M_{i0}\phi_r$	....	24,000	
305.	L_i	.....	13.45
306. Linkage change: $L_iM_{i0}\phi_r$	Φ_r	.....	323,000
Check:			
307. To check solution: $\Phi_s + \Phi_{s1} + \Phi_{s2} + \Phi_e + \Phi_a$	....	.....	317,800

REVISION OF INTERPOLE FIGURES

	Symbols	
308. Half, or full number of interpoles?	....	Full
309. Axial length, each pole	L	7.0
310. Flux, 1 interpole to arm.: $M_{ig}L\phi_i$	....	128,000
311. Leakage per pole: $M_i\phi_{il}(L + W_i/2)$	....	850,000
312. Interpole flux including leakage	....	978,000
313. Interpole flux in per cent of main pole flux	%	20
314. M_{ig} in per cent of M_m	%	55
315. Corrected k_i (Table VIII)	k_i	1.04
316. Interpole excess NI: k_iM_{ig}	M_e	2,950
317. Reaction equalizing NI [244]	M_r	6,100
318. Interpole coil NI	NI	9,050
Interpole coil design:		
319. Interpole core radial dimensions (Fig. 124)	....	7.625
320. Current [4]	I_a	870
321. Turns	N	11
322. Height of coil	H	7.0
323. Type of coil winding	....	Strap on edge
324. Layers	....	1
325. Turns per layer	t	11
326. Conductor's radial dimensions: $(H/t) - 0.006$	....	0.63
327. Coil thickness, assumed	T	1.25
328. Inside perimeter (core perimeter + 1 in.)	P	19.5
329. Mean turn: $(P + \pi T)$	C	23.4
330. Coil ohms (at 1.05 resistivity)	ohms	0.00027
331. Coil watts	W	204
332. Dissipation coefficient (Fig. 122)	k_d	0.0092
333. Coil area: $2(H + T)C$	A	386
334. Temperature rise: $(W)/(Ak_d)$	Θ	57.5 (high)

(Above dimensions, clearance, etc., checked against or obtained from sketch, Fig. 124)

Commutator and brush design:		
335. Commutator diameter	....	24
336. Number of bars [33]	....	300
337. Bar pitch	....	0.251
338. Brush arc, inches	....	0.75
339. Bar pitches in brush arc	....	3.0
340. Length, one brush	....	1.5
341. Brushes per stud	....	6
342. Square inches per stud	....	6.75
343. Amperes for one stud	....	290
344. Amperes per square inch	....	43
345. Volts drop per pair of brushes	....	2
346. Electrical contact loss	....	1,740
347. Pounds per square inch pressure	....	1.5
348. Friction coefficient	....	0.25
349. Velocity, feet per minute	....	2,550
350. Frictional watts loss	....	875
351. Total watts loss	W	2,615
352. Commutator length	....	10.5
353. For riser cooling effect, add	....	2.0
354. Effective commutator cooling length	....	12.5
355. Effective cooling area	A	941
356. Dissipation coefficient (Fig. 120)	k_d	0.047
357. Temperature rise: $(W)/(Ak_d)$	Θ	59

SUMMARY OF LOSSES; CORRECTED CURRENTS; EFFICIENCY

	Symbols	Watts	Totals
358. Series (stabilizing) winding, approximately	....	100	
359. Interpole	....	1,224	
360. Armature copper, slot part [98]	....	1,734	
361. Armature copper, end connections	....	4,150	
362. Commutator, electrical loss [346]	....	1,740	
363. Armature circuit variable losses, total	....	.....	8,948
364. Commutator, friction [350]	....	875	
365. Bearing and windage, at 1.8 per cent	....	3,360	
366. Iron losses	....	3,390	
367. Armature fixed losses, total	....	.....	7,625
Corrected values:			
368. Output: horsepower × 746	....	.....	186,500
369. Armature input	....	.....	203,073
370. Corrected armature current	....	882	
371. Corrected variable losses	....	.....	8,960
372. Armature fixed loss, total	....	7,625	
373. Field watts	....	1,026	
374. Total fixed losses	....	.....	8,651
375. Output	....	.....	186,500
376. Motor input	...	.....	204,111
377. Efficiency, per cent	....	91.3	
378. Line current	....	886.5	

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